of Rademacher and to the "probabilistic" interpretation of the latter. Chapter 5 (pp. 149–194) is devoted to the theory of convergence (almost everywhere, unconditional, · · ·), divergence, and summability of orthogonal series. The climax of this chapter is reached in an elegant proof of the fundamental theorem of Rademacher-Menchoff. A systematic use of "Lebesgue's functions" associated with orthogonal expansions deserves a special mention. Chapter 6 (pp. 195–242) deals with orthogonal expansions in various spaces (L^P, C, M) different from L^2. Among various topics treated here we mention the relationships between the closure and completeness of an orthogonal system; the theorems of Young-Hausdorff and of Paley; the theory of "multipliers" transforming orthogonal expansions of functions of various classes into each other; and a discussion of various singularities which occur in orthogonal expansions. Chapter 7 (pp. 243–260) reveals various remarkable properties of "lacunary" series. Chapter 8 (pp. 261–298), the last chapter, is of somewhat mixed character, being devoted partly to biorthogonal expansions, and partly to polynomials orthogonal relative to a given weight-function.

The exposition, which is in general clear and concise, in some places shows a tendency to be either somewhat vague, or so condensed that it will be difficult to follow for a reader who is not well versed in the field. The number of misprints (in addition to those mentioned in a list of 16 Errata) and of slips of pen or thought is not entirely negligible. Thus on page 6 the reader is told that every point set can be decomposed into a sum of a perfect set and of an at most denumerable set; on page 19 the norm of the functional $\int_a^b dg$ is stated to be $\int_a^b |dg|$; the condition [874] on page 280, which is sufficient for the completeness of the system of orthogonal polynomials relative to the weight function $w(t)$, is obviously not satisfied in the case of Laguerre polynomials, contrary to the assertion preceding this condition. In order to avoid footnotes the authors are using a new scheme of cross references, which, according to the reviewer's experience, does not represent an improvement over the customary system.

J. D. TAMARKIN

Introduction à la Théorie des Fonctions de Variables Réelles. Parts I and II. By Arnaud Denjoy. (Actualités Scientifiques et Industrielles, nos. 451 (55 pp.) and 452 (57 pp.).) Paris, Hermann, 1937.

These are two of the brochures in the section on *Sets and Functions*, under the editorship of Denjoy, who here writes Parts I and II on the introduction to real function theory. Having himself gained important results (on derived numbers, totalization, and so on), Denjoy has paused to scan the field of sets and real variables. From the brevity of each brochure it is clear that his treatment of the various topics is necessarily skeletal (it omits all proofs), but, we believe, is interesting and successful.

Early in Part I he advances reasons of a physical nature for studying non-analytic, even discontinuous, functions. Then comes a foremention of names and topics to be considered: Cauchy, Abel, Riemann (on convergence and integration), Cantor (sets, transfinite numbers), Baire (classification of functions), Borel, Lebesgue (measure and integration), · · · ; and a mention of general analysis. Chapter 2 deals with the geometry of Cartesian point sets, that is, familiar point set theory (including measure). The author here makes distinction between descriptive (topological) ideas and metric ideas, which distinction is also carried over to Chapter 3 on the analysis of functions. Examples of descriptive notions are continuity, convergence, the Baire classification; of metric notions, derived numbers, differentials. Functions of bounded variation illustrate a combination of the two ideas. Brief mention is also made of the theorem on polynomial approximation to continuous functions, of Tchebycheff polynomials, and of summability of sequences.

Part II, beginning with integration, applies the theory. Brief treatments of the Riemann, Lebesgue, and Stieltjes integrals are given, and mention is made of the Denjoy totalization process. Chapter 2 takes up trigonometric series. The importance of such series in leading to a general conception of function (attributed to Riemann) is indicated. There follow remarks concerning the determination of the coefficients, uniqueness, tests for convergence, the Fejér method of summation; then a short note on general orthogonal functions. Chapter 3 deals with quasi-analytic functions (dear to the heart of Denjoy). He sets the general problem leading to such functions, and states the solution of Carleman. A last chapter discusses functionals: continuity, linearity, differential of a functional, functions of sets.

Denjoy had looked upon the handiwork of those who have made sound contributions to real variable theory, and saw that it was good.

I. M. SHEFFER

Sur les Fonctions d'une Variable Complexe Représentables par des Séries de Polynômes. By M. Lavrentieff. (Actualités Scientifiques et Industrielles, no. 441.) Paris, Hermann, 1936. 60 pp.

This brochure is in the section of its series on *The Theory of Functions*, edited by Paul Montel. Baire found the condition that a function of a real variable be of class one (representable as a series of continuous functions). Using the approximation of continuous functions by polynomials (Weierstrass), one may say that Baire determined when a function is the sum of a series of polynomials. The extension of this problem to the complex plane is the subject of Lavrentieff's work. He treats the problem from its classical beginnings to the latest advances, thus providing a valuable résumé.

Chapter 1 is introductory. Baire's work is taken as starting point, after which the first general advance to the complex plane, the well known Hilbert-Runge theorem, is stated and proved. In the short second chapter are stated some needed theorems on the correspondence of boundaries under conformal mapping.

One of the problems dealt with in Chapter 3 concerns the convergence of sequences of (holomorphic, in particular, polynomial) functions that are bounded in their set, together with a uniqueness theorem (to the effect, for example, that two functions which are limits of such sequences are identical in a given region if they coincide on certain point sets of the boundary). Another problem treated is that of finding necessary conditions and sufficient conditions that a function have a polynomial expansion (of given type) on a closed point set. An extension to harmonic sequences is indicated.

In the last chapter special point sets M and M^* (which space does not allow to describe adequately) are introduced, and it is shown that these sets are important in the statement of general theorems on the representation of holomorphic functions by series of polynomials.

I. M. SHEFFER

Diophantische Approximationen. By J. F. Koksma. (Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 4, no. 4.) Berlin, Springer, 1936. 8+157 pp.

It is remarkable material on diophantine approximations with which the author presents us in this volume. The bibliography, which seems to be unusually complete,