

La Théorie des Surfaces et l'Espace Réglé: (Géométrie Projective Différentielle).

By L. Godeaux. (Actualités Scientifiques et Industrielles, No. 138.) Paris, Hermann, 1934. 36 pp.

In this little pamphlet the author has collected and blended those papers in projective differential geometry which regard the line as the space element. At its end is a bibliography consisting of four treatises and eighty memoirs, of which thirty-eight are from the pen of the author.

The starting point is that of the four solutions of two partial differential equations of the second order in terms of two variable parameters u, v as developed by Wilczynski. These are interpreted as homogeneous point coordinates x in S_3 which generate a surface (x) .

The tangents to the u and v curves are interpreted as point coordinates of a quadric primal Q in S_5 . The line joining these points u, v lies on the primal. It is the image of the pencil of tangent lines to the surface (x) at x . The points u, v are consecutive in a series of Laplace (Bompiani) which in the general case is unlimited in both directions. It is self polar as to the primal Q . To this series corresponds in S_3 a series of quadrics having the property that any two consecutive ones touch in four points.

Associated with each point x is a Lie quadric Φ , which belongs to the series of quadrics just mentioned. As u, v vary Φ has an envelope. Apart from the four points of contact, the Lie quadrics have at most five distinct characteristic points. Demoulin has considered the case in which they have just two; this case is featured extensively. It gives rise in S_5 to a series of Laplace of period six. If the Lie quadrics have three characteristic points there is conservation of the asymptotes on the three nappes of the envelope. The Lie quadrics of the envelope have also three characteristic points. Cases in which the Lie quadrics have four or five characteristic points are treated in less detail.

The pamphlet is well written, is free of typographical errors, and furnishes a good insight into this fascinating problem.

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