ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

231. Miss Beatrice Aitchison: *Concerning the mapping of locally connected continua on to simple arcs*. Preliminary communication.

Mazurkiewicz has determined a necessary and sufficient condition to be satisfied by a dendrite D, so that there exists a real continuous function over D, taking any value only a finite number of times at most and transforming D into a simple arc (Fundamenta Mathematicae, vol. 18). Results already obtained indicate that, by methods being developed in the present paper, based on the cyclic element structure of locally connected continua, it will be possible to find a sufficient condition for such a continuum C to be transformed by a similar function into a simple arc. If the space C has a countable number of end points it can be transformed by a function of this type into a dendrite D, which satisfies a part of Mazurkiewicz's conditions. If under known suitable conditions on C, D can be shown to satisfy all of these conditions, and hence can be transformed into a simple arc in this definite way, the space C can also be transformed into a simple arc by this special type of function. (Received September 2, 1932.)

232. Dr. E. Sperner: *On topological transformations of a plane without invariant points.*

In the first part of this paper new and simple proofs are given for the theorems of L. E. J. Brouwer concerning path curves and transformation fields of sense-preserving topological transformations of a plane without invariant points. In the second part necessary and sufficient conditions are derived for a topological transformation of the plane to be topologically equivalent to a translation. (Received September 2, 1932.)

233. Professor H. P. Robertson: *Possibilities for a universe with non-negative pressure.*

A survey is made of the long-range behavior of all spatially homogeneous and isotropic space-times suitable for relativistic cosmology subject to the restriction $p \geqq 0$. (Received August, 30, 1932.)

234. Professor A. D. Michal and Mr. J. L. Botsford: *Normal representations and linear connections.*

In a preceding paper the authors considered a theory of differential invariants of an affine connection and a linear connection with the aid of "geodesic representations of order r" (see abstract 38-9-220, this Bulletin, and Proceedings of the National Academy, August, 1932). In the present paper the properties of "normal representations" are used to develop a slightly different approach to the subject. (Received September 1, 1932.)

235. Professor A. D. Michal and Dr. R. S. Martin: *A set of expansions in vector space.*

The authors consider a complete linear vector space S closed under multiplication by elements of A, the real or complex number system. There is postulated a bilinear operation of composition such that the elements of S form a ring with respect to addition and composition. A sequence of abstract polynomials is defined by means of the recursion formulas $B_n(T)=a_{n-1}(T)T - B_{n-1}(T)T$; $a_n(T)=[B_n(T)]/n$ where $[\cdots]$ is a linear continuous operation on S to A which is called contraction and has the property $[T_1T_2]=[T_2T_1]$. The properties of the functions $D(T)\equiv\sum_0^\infty a_n(T)$ and $B(T)\equiv\sum_1^\infty B_n(T)$ are considered. Among the results are, for example, that B and D are analytic and Fréchet differentiable functions of T for $\|T\|<1$ ("analytic" is used in the sense previously defined by one of the authors; see R. S. Martin, California Institute, thesis, 1932), and that $D(T)$ does not vanish for $\|T\|<1$. (Received August 31, 1932.)

236. Professor G. C. Evans: *Note on the gradient of the Green's function.*

Let T be a bounded, open, simply-connected region in the plane or in space, $g(M, P)$ its Green's function with pole at M, and let $I=\int_T|\nabla g(M, P)|dp$. In the plane, it is shown by means of the inequalities of Schwarz and Hardy, using complex variables, that I is bounded independently of the position of M; in fact, $I\leqq(2\pi Tg)^{1/2}(1-e^{-g})^{-1}$, where g is an arbitrary positive number. The minimum value of this expression is $2.22\ (\pi T)^{1/2}$, whereas for the circular region of measure T, $I=2(\pi T)^{1/2}$. The length of the level curve $g(M, P)=g$ is $\leqq(2\pi Tg)^{1/2}(e^g-1)^{-1}\leqq(2\pi T/g)^{1/2}$. In space, the subset F, from each point of which T is star-shaped, is null or is closed with respect to T. Making use of Gergen's lower bound for the absolute value of the gradient of the Green's function for a star-shaped region (American Journal of Mathematics, vol. 53 (1931), pp. 746-752), it may be shown that for all points of F, $I\leqq 4\pi g^{-1}+(4\pi gT)^{1/2}$, g any positive number. In particular, $I\leqq 6.35(\pi^2T)^{1/3}$. If T is convex, of course $F=T$. (Received August 12, 1932.)

237. Professor J. H. Binney: *An elliptic system of integral equations on summable functions.*

The functions $\phi=D_y\psi+(2\pi)^{-1}\int_T r^{-1}(\cos(r, y)dB(e)-\cos(r, x)dA(e))$, $\theta=D_x\psi+(2\pi)^{-1}\int_T r^{-1}(\cos(r, x)dB(e)+\cos(r, y)dA(e))$ where ψ is an arbitrary solution of Laplace's equation in the simply connected plane region T, and $A(e)$, $B(e)$ are additive functions of point sets in T, form a system of solutions of the pair of integral equations $\int_s \phi dy + \theta dx = A(s)$ and $\int_s -\theta dy + \phi dx = B(s)$, where $A(s)$ and $B(s)$ are functions of curves of limited variation corresponding to the functions of point sets $A(e)$ and $B(e)$ respectively, for every curve s in T for which $\int_T r^{-1}|dA(e)|$ and $\int_T r^{-1}|B(e)|$ represent summable functions on s; in particular for almost all rectangles in T. Conversely, if ϕ and θ are summable superficially in T and are solutions of the pair of integral equations on almost all rectangles, they may, except for removable discontinuities at most on a set of superficial measure zero, be expressed in the form given above. (Received August 12, 1932.)

238. Professor B. A. Bernstein: *On Nicod's reduction in the number of primitives of logic.*

Nicod, using Sheffer's "stroke" operation, obtained a drastic reduction in the number of primitives for the theory of "elementary" propositions in *Principia Mathematica*. The reduction consists in deriving eight of the ten *Principia* primitive propositions from a set of three postulates. But Nicod does not discuss the consistency or the independence of his postulates; fails to consider the *Principia* primitives (two propositions and two ideas) not involved in the above eight propositions, and fails to prove that his postulates are derivable from the *Principia* primitives. The author supplies Nicod's omissions. He shows that Nicod's postulates, when rewritten in customary mathematical language, are consistent and independent; that the four *Principia* primitives ignored by Nicod are redundant; and that Nicod's postulates are derivable from those of the *Principia*. In connection with the last item is pointed out the fact that if a mere derivation of a postulate set S from a smaller set S' constitute a reduction of S, then a reduction in the number of the *Principia* primitives exists that is far more simple and far more drastic than is the Nicod reduction. (Received August 18, 1932.)

239. Professor A. D. Campbell: *Note on cubic surfaces in the Galois fields of order* 2^n.

In this paper a study is made of the cubic surfaces in the Galois fields of order 2^n. The Hessian of any such surface vanishes identically. For certain cubic surfaces the first polar of every point in space is a double plane. The twenty-seven straight lines on a cubic are obtained by a new short method. There turn out to be four, three, two, one or no double points on a cubic. The equation of the cubic in plane coordinates is written down and is found to be of the twelfth degree or of the ninth degree. (Received September 16, 1932.)

240. Dr. I. O. Horsfall: *Transformations associated with the lines of a cubic, quadratic, or linear complex.*

Two equations, $\sum x_i f_i(p) = 0$ and $\sum x_i F_i(p) = 0$ $(i = 1, 2, 3, 4)$, bilinear in x_i and the line coordinates p_{ik}, define an extensive type of cubic complex and also map the complex on the space (x) so that each line (l) is mapped by a point

(x) on (l). The cubic complex of lines joining corresponding points of the general cubic involutorial transformation is included as a special case. The method is also applied to two known cases of the quadratic complex and the linear non-special and special complex. (Received September 6, 1932.)

241. Mr. George Comenetz: *Translation families of heat curves.*

A flow of heat by conduction in the plane defines a family of *heat curves*: the ∞^2 isothermals throughout the flow. When has such a *heat family* a specially simple geometrical structure? (See Kasner, *Geometry of the heat equation*: First paper, Proceedings of the National Academy of Sciences, vol. 18 (1932), p. 475.) We discuss the problem of determining the heat families which consist merely of the ∞^2 copies by translation of a single curve. In a special case we find explicit results for the equations of the families in terms of elementary functions. (Received October 8, 1932.)

242. Mr. Aaron Fialkow: *The geometry of degenerate heat families.*

Professor Kasner has defined "heat family" and "degenerate heat family" in a paper published in the Proceedings of the National Academy of Sciences, vol. 18 (1932), No. 6. In this paper, he has shown that there are three distinct types of degenerate heat families defined by the equations of Laplace, $\Delta u=0$, Poisson (special), $\Delta u=1$, and Helmholtz-Pockels, $\Delta u=\pm u$. A geometric characterization of the Laplace type is known. In his seminar of 1917, Kasner indicated that the other two types must each possess two fourth-order geometric properties. In our work, we derive these two properties of each type, and furthermore prove that they are sufficient to characterize these types. One geometric equation, $\gamma_1(d^2\gamma/ds^2-d^2\gamma_1/dn_1ds_1)+(d\gamma_1/dn_1-\gamma_1^2)(d\gamma/ds+d\gamma_1/ds_1)=0$, which is common to both types, is shown to characterize completely the more general equation $\Delta u=f(u)$. (Received October 6, 1932.)

243. Professor A. K. Mitchell: *On teleparallelism in Finsler space.* Preliminary report.

Synge, Taylor, and Berwald have considered parallel displacement of tensors in Finsler space and have derived equations corresponding to those of Levi-Civita for parallel transport in Riemannian space. In the present paper the equations of transformation, satisfied by the coefficients of connection used by Synge and Taylor, are considered. A definition is made of coefficients, for the Finsler space, which correspond to those used by Einstein in his equations of teleparallelism and which lead to equations of parallelism for Finsler space analogous to those, for Riemannian space, used by Einstein in his unified field theory. (Received September 20, 1932.)

244. Dr. Leo Zippin (National Research Fellow): *Characterization of the closed 2-cell.*

We give the following topologic characterization of the closed 2-cell (a set of points homeomorphic with a plane circle plus its interior): the closed 2-cell is a compact continuous curve C containing a simple closed curve J and at least one arc spanning J (this shall mean an arc having its end points and these only on J) *such that every arc spanning J irreducibly separates C*. (Received September 27, 1932.)

245. Professor W. A. Wilson: *On unicoherency about a simple closed curve.*

If a metric space Z contains a simple closed curve J, we say that Z is unicoherent about J if, for every division of J into open arcs α and β by points a and b and for every decomposition of Z into closed sets P and Q such that $\alpha \cdot Q = \beta \cdot P = 0$, some component of $P \cdot Q$ joins a and b. This paper discusses some general properties of this extension of the idea of unicoherent continuum, including several equivalent definitions of the extension. It is shown that unicoherency about every simple closed curve contained in the space is the necessary and sufficient condition for a compact metric continuum to be a unicoherent continuum, that every compact metric space Z unicoherent about a simple closed curve J contains a set P irreducible with respect to the property of being closed and unicoherent about J, and finally that, if Z is a locally connected compact metric continuum irreducible with respect to the property of being closed and unicoherent about a simple closed curve J and every simple arc ab such that $(ab) \cdot J = a + b$ disconnects Z, then Z is the homeomorphic image of a two-dimensional simplex. (Received September 23, 1932.)

246. Mr. Victor Perlo: *On the distribution of Student's ratio for samples from non-normal populations.*

We present the exact sampling distribution of Student's ratio, t, comparing sample mean with sample standard deviation, for samples of three drawn from a rectangular population; also the probability P that t will exceed a given value. Letting p be the value corresponding to P for samples drawn from a normal population, for which the distribution of t, i.e., Student's distribution, is known for samples of n, we also obtain the limit of the ratio P/p as t approaches infinity, for samples of n. Since this limit is the upper bound of the ratio P/p for finite values of t, we have a limit within which we may use Student's distribution to calculate P. Then we show that the limiting ratio P/p for samples drawn from rectangular distributions is the upper bound of the corresponding limiting ratio for samples drawn from any unimodal symmetrical distribution, and with a suitable correcting factor, for any unimodal distribution, indicating the use of the limit of P/p, suitably corrected for particular curves, is the bounding factor for the use of Student's distribution for samples drawn from non-normal populations. (Received October 8, 1932.)

247. Professor B. H. Camp: *The converse of Spearman's two-factor theorem.*

There have been several attempts to prove, or to disprove, the converse of Spearman's two-factor theorem. As shown by Irwin, these various methods of proof result ultimately in the same expression for the so-called general factor.

Although the several proofs are necessarily alike in many respects, the different authors appear to have different pictures in mind at the background of their analytical demonstrations. The same may be said of the proof presented in this paper. In addition, I have inserted a certain necessary but hitherto neglected hypothesis, have investigated the possibility of the use of other than linear functions as the basis of the formation of the general factor function, have given a numerical example in which this factor is not unique, and have discussed more fully the important additional question raised by Piaggio as to whether this factor is "almost" unique. (Received September 29, 1932.)

248. Mr. Archie Blake: *Canonical expressions in Boolean algebra.*

In a finite Boolean algebra various canonical expressions can be defined, including the well known *developed* form and a new *simplified* form. These forms always exist uniquely, and the simplified form has the further important property of being the simplest expression that includes formally all terms that it includes at all. Constructions are given for the simplified form, and applications are indicated. (Received September 7, 1932.)

249. Dr. K. E. Rosinger: *A new theorem in combinatorial analysis with application to a classification of triadic relations.*

Given a K of n distinct elements, $n \geqq 3$; let m represent the number of triads formed from the elements of K such that (a) each triad consists of distinct elements, (b) no two triads have more than one element in common, (c) the order of elements within the triad is irrelevant. Then $m \leqq (n^2-n)/6$. By means of the theorem just stated, it is proved that if a triadic relation $Rabc$ is such that it implies every permutation of its terms, the relation between the variables and their values must remain constant. Graphically, this means that the logical matrix can be rotated arbitrarily without changing its significance only with reference to a fixed set of coordinates. The proof utilizes the following two lemmas. (I) If $(n^2-n)/6$ is an integer, then $n \neq 3x+2$, $x \geqq 0$. (II) If $n \neq 3x+2$, $x \geqq 0$, then $n^2-n \equiv 0$, mod 6. (Received October 5, 1932.)

250. Mr. E. H. Hadlock: *On the progressions associated with a ternary quadratic form.*

H. J. S. Smith (Collected Mathematical Papers, vol. 1, p. 470) has proved that there exists a primitive ternary quadratic form if and only if the generic characters associated with the form satisfy one of three formulas according as f is properly primitive, f is improperly primitive, or F is improperly primitive. The purpose of this paper is to express these formulas in terms of the progressions associated with the form (see B. W. Jones, *A new definition of genus for ternary quadratic forms*, Transactions of this Society, vol. 33 (1931), pp. 92–110). This leads to the fact that the number of sets of progressions of a certain kind is odd or even according as f is positive or indefinite. It is also found that with every positive form there are associated infinitely many progressions of numbers not represented by f. (Received October 3, 1932.)

251. Dr. S. S. Cairns: *The generalized theorem of Stokes.*

Using the theorem of abstract 38-11-257, we prove the generalized Theorem of Stokes for an orientable regular bounded topological n-manifold M_n in $(n+r)$-space. We use the method employed in 3-space by O. D. Kellogg (*Foundations of Potential Theory*, chapter IV). Let $(y_1, \cdots, y_{n+r})$ be rectangular cartesian coordinates in our $(n+r)$-space. Consider a set of $\binom{n+r}{n-1}$ functions, $Y_{i_1\cdots i_{n-1}}$, of $(y_1, \cdots, y_{n+r})$, defined and of class C' on M_n, where the i's are numbers of the set $(1, \cdots, n+r)$ arranged in order of increasing magnitude. Using generalized direction cosines, $(\gamma_{j_1\cdots j_n})$ (abstract 38-3-114), of the tangent n-space to M_n at any point, and also $(\beta_{i_1\cdots i_{n-1}})$, of the tangent $(n-1)$-space to the boundary, B_{n-1}, of M_n, we formulate the theorem thus: $\int_{M_n}\sum D_{j_1\cdots j_n}\gamma_{j_1\cdots j_n}d\tau=\int_{B_{n-1}}\pm\sum Y_{i_1\cdots i_{n-1}}\beta_{i_1\cdots i_{n-1}}d\sigma$, where the summations are over all the sets $(j_1\cdots j_n)$ and $(i_1\cdots i_{n-1})$, respectively, and where $D_{j_1\cdots j_n}=\sum_{k=1}^{n}(-1)^{k+1}(\partial/\partial y_{j_k})(Y_{j_1\cdots j_{k-1}j_{k+1}\cdots j_n})$. The alternative signs are given because a change of orientation on M_n(or B_{n-1}) alters the signs of all the γ's (or all the β's). This is equivalent to the usual formulation, but has the advantage of being formally independent of any parametric representation of M_n. (Received September 7, 1932.)

252. Mr. J. G. Deutsch: *On the existence and equality of repeated Riemann integrals.*

In this paper, a necessary and sufficient condition is determined for the existence of the repeated Riemann integrals of a function of two variables. A necessary and sufficient condition that these integrals not only exist but be equal is also established. These conditions are of the nature of simple comparison tests and specialize to yield the results of Young, Lichtenstein, and Gillespie. (Received October 6, 1932.)

253. Dr. W. J. Trjitzinsky: *Analytic theory of linear q-difference equations.*

In the past the analytic theory of linear q-difference equations has been developed under the assumption of restricted roots of the characteristic equation (Carmichael, Birkhoff). In the present paper the author develops the analytic theory for the general case of unrestricted roots. The main results are embodied in the Fundamental Existence Theorem. It is proved that in all cases there exist complete sets of analytic solutions with the asymptotic form (in certain regions) of the formal series solutions. This is proved on the basis of a certain method of contour q-difference summation. The importance of this paper, in the theory of q-difference equations, is analogous to that of the joint paper by G. D. Birkhoff and the author in the theory of difference equations (Acta Mathematica, vol. 59 (1932)). The present paper will appear in the Acta Mathematica. (Received October 6, 1932.)

254. Dr. Wladimir Seidel: *On the distribution of values of bounded analytic functions.*

If $f(z)$ is a bounded analytic function in the unit circle $|z|<1$, the limit $\lim_{r\to 1}f(re^{i\theta})=f^*(e^{i\theta})$, $z=re^{i\theta}$, exists by a theorem of Fatou for all values of θ in the interval $0 \leqq \theta \leqq 2\pi$ save perhaps in a set of measure zero. This paper principally deals with the case when $|f^*(e^{i\theta})| = 1$ for almost all points of either the whole circumference or an arc of the circle $|z| = 1$. Among the theorems proved is the following: *If $f(z)$ is a bounded analytic function in the circle $|z| < 1$ such that $|f^*(e^{i\theta})| = 1$ for almost all points of the circumference $|z| = 1$, and if it omits a value α, $|\alpha| < 1$, in the circle $|z| < 1$, then there exists a radius on which $f(z)$ approaches the value α.* By means of this theorem we can study the conformal representation of certain types of multiply-connected regions on a circle, the representation being defined by Blaschke products. The following extension of Schwarz's reflection principle is also obtained: *Let $w = f(z)$ be a bounded analytic function in the unit circle $|z| < 1$: $|f(z)| < 1$. If $|f^*(e^{i\theta})| = 1$ for almost all points of an arc A of the circle $|z| = 1$ and if $f(z)$ does not assume in the unit circle $|z| < 1$ values which form a set of positive superficial measure in the circle $|w| < 1$, then $f(z)$ may be continued analytically beyond the arc A in accordance with the relation $f(1/\bar{z}) = 1/\overline{f(z)}$.* (Received October 8, 1932.)

255. Mr. L. A. MacColl: *On the distributions of the zeros of certain analytic functions.*

The subject of this paper is the distribution of the zeros of a function of the form $f(z) = \sum_{j=0}^{J} \exp(\lambda_{jN} z^N + \cdots + \lambda_{j1} z + \lambda_{j0})$, where J and N are positive integers, and the λ's are constants. The problem is a generalization of one studied by Wilder, Tamarkin, Pólya, Schwengler, and others. The λ's determine a set of critical rays in the z-plane; these are classified as primary, secondary, etc. When $N = 1$ only primary critical rays occur. Each zero of $f(z)$ is a point of one or more of a set of half-strips, which are equal in number to the critical rays, and each of which extends in the direction of a different one of these rays. Asymptotic expressions are given for the numbers of zeros in the various half-strips and within the large circle $|z| = r$. Whereas when $N = 1$ these expressions are the same, to within constant factors, for the several half-strips, in the general case this is not so. (Received October 4, 1932.)

256. Dr. Selby Robinson (National Research Fellow): *Properties related to the Borel property and to the closure of derived sets.*

It is shown that in a space having properties B and D of Hausdorff and in which every point is interior to a set having the property of Borel-Lebesgue, the interior of every set is open. We have theorems on the relations of various sorts of reducibility of coverings to each other and to the closure of sets. In terms of set functions introduced by Chittenden, we give two necessary and sufficient conditions that the interior of every set in a space (P, K) shall be open. If a space (P, K) has the latter property, the points of condensation of any set form a closed set. (Received October 10, 1932.)

257. Dr. S. S. Cairns: *The triangulation of regular topological n-manifolds in $(n+r)$-space.*

The theorem below extends and amplifies a previously announced result (National Academy Proceedings, vol. 16 (1930), pp. 488–491, Theorem 2).

For *n-spread,* the term *topological n-manifold* is substituted. A *regular bounded topological n-manifold* means a compact connected n-dimensional point set, M_n, made up of (1) a set M_n' of points each of which has a regular n-cell (loc. cit.) for one of its neighborhoods on M_n, and (2) a set B_{n-1} of limit points of M_n' none of which has such a neighborhood on M_n, where B_{n-1} is the sum of a finite number of distinct regular unbounded topological $(n-1)$-manifolds. Theorem: A regular topological n-manifold (bounded or unbounded) in euclidean $(n+r)$-space is coincident with a simplicial n-complex. The vertices of each i-cell $(i=1, \cdots, n)$ of the complex determine an i-simplex in the $(n+r)$-space. The set of all simplexes so determined is a simplicial polyhedron, whose approximation to the given manifold can be made arbitrarily close. (Received September 7, 1932.)

258. Dr. Hassler Whitney (National Research Fellow): *Cross sections of curves in 3-space.*

Let F be a regular family of curves (see National Academy Proceedings, March, 1932, pp. 275–278) filling a region in euclidean 3-space. Using a theorem by the author (to appear in the Transactions of this Society; see abstract 37-11-354) it is shown that there is a cross section through any point of the region which is a closed 2-cell. Hence the curves in the neighborhood of a point are equivalent to a family of parallel straight lines. (Received October 7, 1932.)

259. Dr. Gordon Pall: *The structure of the number of representations function in a binary quadratic form.*

To extend the results of a recent paper (abstract No. 37-5-229) to any integral binary quadratic form of non-zero discriminant, there are studied fully properties of sets of representations (abstract No. 38-1-25) in a binary quadratic form, that is, representations equivalent through automorphic transformations. In particular the problem of representing a number by trial in an indefinite form is simplified. (Received October 10, 1932.)

260. Dr. S. B. Littauer and Professor Marston Morse: *A characterization of fields in the calculus of variations.*

The present paper is concerned with a geometric characterization of conjugate points and focal points in the calculus of variations. In the plane one can prove that conjugate points of a point P on an extremal g are the points on g whose neighborhoods are not covered in a one-to-one manner by the family of extremals issuing from P with directions near that of g at P. The methods used to prove this theorem in the plane are inadequate if $n>2$. We are able, however, to prove the following theorem: *In the analytic case, a necessary and sufficient condition that a point Q on an extremal g be a conjugate point of a point $P(\neq Q)$ on g is that the family of extremals through P issuing from P with directions near that of g at P fail to cover the neighborhood of Q in a one-to-one manner.* In the proof of this theorem no assumption is made as to the nature of the envelope, and herein lies the power as well as the difficulty of the theorem. The sufficiency of the condition is of course well known. The theorem is of particular interest because the vanishing of the Jacobian of a transformation of n real variables does not necessarily imply that the transformation fails to be one-to-one. Focal points are characterized in a similar manner. (Received October 10, 1932.)

261. Dr. C. B. Morrey (National Research Fellow): *A class of representations of manifolds.*

The representation $x^i = x^i(u, v)$, $i = 1, \cdots, N$, of a surface S is said to be of class L if the $x^i(u, v)$ are absolutely continuous in the sense of Tonelli and either (A) $|x_u^i|^p$ and $|x_v^i|^q$, $i = 1, \cdots, N$, are summable, $1/p + 1/q \leqq 1$, p, $q > 1$, or (B) $|x_u^i| < M$ and $|x_v^i|$ summable or $|x_v^i| < M$ and $|x_u^i|$ summable, $i = 1, \cdots, N$. It is first shown that the Lebesgue area, $L(S)$, of a surface S is given by the usual integral formula. Green's formula for space and Stokes's formula are extended to situations where the surfaces involved are of class L and the function $P(x, y, z)$ is very general. Many of these results are extended to n-dimensional manifolds. If a representation is of class L and $E = G$, $F = 0$ almost everywhere, it is said to be generalized conformal. It is shown that if the generalized conformal representations $x^i = x_n^i(u, v)$ of S converge in the mean (i.e., the functions do) to the continuous representation $x^i = x^i(u, v)$ of S, and $L(S_n)$ approaches $L(S)$, then the limit representation is generalized conformal. (Received October 10, 1932.)

262. Mr. J. G. Deutsch: *Partial derivates.*

In this paper we show that a classification of the partial derivates of measurable functions of two variables, analogous to the Denjoy-Young-Saks classification of the derivates of functions of one variable, is not, in general, possible. This contradicts certain theorems given by G. C. Young (Proceedings of the London Mathematical Society, (2), vol. 20 (1921–22), pp. 182–88). (Received October 6, 1932.)

263. Mr. J. G. Deutsch: *Functions having the property* (N).

It is known that, if a continuous function has the Lusin property (N) on an interval, then that interval contains a set at which the function has a derivative and on which is assumed almost every value assumed on the interval. We show further that this set can be so taken as to have almost every value assumed on the interval assumed the same number of times on the set. This added fact provides a vehicle for obtaining another proof of Mlle. Bary's theorem on the absolute continuity of functions having the property (N). (Received October 6, 1932.)

264. Dr. J. J. Gergen and Dr. S. B. Littauer: *Continuity and summability for double Fourier series.*

Let $f(u, v)$ be an even-even function, integrable in the Lebesgue sense over the square $(0, 0; \pi, \pi)$, and doubly periodic with period 2π in each variable. Let $\sigma_{m,n}^{a,b}$ be the (m, n)-Cesàro mean of order (a, b) of the Fourier series of f at the origin. Let $f^{\alpha,\beta}(x, y) = [\alpha\beta/(x^\alpha y^\beta)]\int_0^x (x-u)^{\alpha-1} du \int_0^y (y-v)^{\beta-1} f dv$ be the mean of order (α, β) of f with respect to the origin. The first object of this paper is to study the relations between the limiting processes $\lim \sigma_{m,n}^{a,b}$ as $(m, n) \to \infty$ and $\lim f^{\alpha,\beta}(x, y)$ as $(x, y) \to (+0, +0)$. By applying Wiener's method of quasi-Tauberian theorems we obtain, for a certain general class of functions, results analogous to those obtained by Wiener in sharpening the familiar theorem of Hardy and Littlewood on continuity and summability of simple Fourier series. Incidentally, we show that under certain conditions the Riesz and Cesàro methods of summability for double series are equivalent. We also obtain results in connection with the iterated limiting processes $\lim \lim \sigma_{m,n}^{a,b}$ as, first, $m \to \infty$, and then $n \to \infty$, and $\lim \lim f^{\alpha,\beta}(x, y)$ as, first, $x \to +0$, and then $y \to +0$. (Received October 8, 1932.)

265. Professor E. V. Huntington: *A set of independent postulates for Principia Mathematica.*

Given the following primitive ideas: a class K ["propositions"]; a subclass T ["true" propositions]; a binary operation, $a+b$ ["a or b"]; a unary operation, a' ["not -a"]. Any system $(K, T, +, ')$ which satisfies the following seven postulates may be called a "*Principia* system." (1) If a and b are in K, then $a+b$ is in K. (2) If a is in K, then a' is in K. (3) If a, b, etc., are in K, then $b'+(a+b)$ is in T. (4) If a, b, etc., are in K, then $(a+b)'+(b+a)$ is in T. (5) If a, b, c, etc., are in K, then $(b'+c)'+[(a+b)'+(a+c)]$ is in T. (6) If $a+b$ is in T, then at least one of the elements a and b is in T. (7) If a' is in T, then a is not in T. Here Postulates 1–5 correspond precisely to "formal," and 6–7 to "informal" statements in the *Principia*, but 6 and 7 are not deducible from the formal part of the *Principia*. All the propositions, "formal" and "informal," in Section A of the *Principia* are deducible from Postulates 1–7. (Received October 6, 1932.)

266. Dr. E. J. McShane (National Research Fellow): *Integrals over surfaces in parametric form.*

The author has previously (Annals of Mathematics, vol. 32, p. 460) investigated the semi-continuity of double integrals of the calculus of variations taken over surfaces $x=x(u, v)$, $y=y(u, v)$, $z=z(u, v)$, where the functions $x(u, v)$, etc., are Lipschitzian. In the present paper the theorems previously obtained are extended to a wider class of surfaces, including, e.g., surfaces for which $x(u, v)$, etc., are absolutely continuous in the sense of Tonelli and the partial derivatives $\partial x/\partial u$, etc., are summable together with their squares. For the same wider class of surfaces it is shown that the area (in the sense of Lebesgue) is given by the classical double integral. (Received September 20, 1932.)

267. Dr. E. J. McShane (National Research Fellow): *Parametrizations of saddle surfaces, with application to the problem of Plateau.*

If the surface S: $x=\bar{x}(u, v)$, $y=\bar{y}(u, v)$, $z=\bar{z}(u, v)$ is bounded by a Jordan curve and has finite area, and the functions $\bar{x}(u, v)$, etc., are monotonic in the sense of Lebesgue, then there exists a representation $x=x(u, v)$, $y=y(u, v)$, $z=z(u, v)$, $u^2+v^2 \leqq 1$, of the surface S satisfying the following conditions: (1) the functions $x(u, v)$, etc., are absolutely continuous in the sense of Tonelli; (2) the partial derivatives $\partial x/\partial u$, etc., are summable together with their squares; (3) for almost all points (u, v) we have $E=G$, $F=0$, where E, F, G have their usual meanings. This theorem, together with the previously established result that (1) and (2) ensure that the area is given by the classical double integral, leads readily to a solution of the problem of Plateau. (Received September 20, 1932.)

268. Dr. W. J. Trjitzinsky: *A property of indefinitely differentiable classes.*

Let classes C of functions $f(x)$, indefinitely differentiable on an interval (a, b), be specified by the law of increase of the absolute values of the derivatives. The author proves that no non quasi-analytic classes C exist whose members are uniquely determined by the values of the derivatives (of all orders) at $n(\geqq 2)$ points of (a, b). (Received October 6, 1932.)

269. Professor J. L. Walsh: *On the approximation of analytic functions by rational functions of best approximation.*

Let the function $f(z)$ be analytic in the interior of the Jordan region C and continuous in the corresponding closed region. Let $R_{mn}(z)$ be a (necessarily existent) function of the form $(a_0z^m+a_1z^{m-1}+ \cdots +a_m)/(b_0z^n+b_1z^{n-1}+ \cdots +b_n)$, $b_0z^n+b_1z^{n-1}+ \cdots +b_n \neq 0$, of best approximation in the sense of Tchebycheff to $f(z)$ in C. Then any infinite sequence of the $R_{mn}(z)$, for which the first subscript has no lower limit less than the number of zeros of $f(z)$ interior to C, converges to $f(z)$ uniformly in the closed region C. Overconvergence may take place. The doubly infinite array of the $R_{mn}(z)$ is in many respects analogous to the Padé table. (Received October 6, 1932.)

270. Professor W. A. Wilson: *On topological characterizations of the two-dimensional simplex.*

Let Z be a locally connected compact metric continuum containing a simple closed curve J and satisfying the following conditions: (a) if ab is a simple arc in Z such that $(ab)\cdot J=a+b$, $Z-ab$ has exactly two components and these contain the respective components of $J-(a+b)$; (b) if α and β are disjoint open arcs of J and K is a closed sub-set of $Z-(\alpha+\beta)$ of which no component meets both components of $J-(\alpha+\beta)$, there is a simple arc ab in $Z-K$, from a point a in α to a point b in β, which has no other points on J. It is shown that the methods used by H. Whitney in his combinatorial characterization of the two-dimensional simplex can be employed with little change to prove that the continuum Z defined above is the homeomorphic image of a two-dimensional simplex. (Received October 3, 1932.)

271. Professor Edward Kasner: *Families of heat surfaces.*

This paper considers the geometry of Fourier's equation for the conduction of heat in three dimensions, extending some of the two-dimensional results of earlier papers (see Proceedings of the National Academy of Sciences, vol. 18 (1932), pp. 475–480). A heat family of surfaces contains in general ∞^2 surfaces, one for each time and each temperature. There are only three types of degenerate families, for which there are merely ∞^1 surfaces, and these are defined by the equations of Laplace, Poisson, and Helmholtz or Lamé. No doubly-infinite family can be composed of planes. (Received October 10, 1932.)

272. Professor Harold Hotelling: *Analysis of a multivariate complex into principal components.*

The problem of extracting and measuring the common elements in an aggregate of statistical variables, and particularly of determining whether the number of independent common elements can be taken as less than the number of observed variables, has received considerable attention from E. B. Wilson, C. Spearman, T. L. Kelley, E. L. Thorndike, and other mathematical psychologists. The same logical and mathematical considerations apply in other fields also, as Wilson and Shewhart have pointed out. The present paper is concerned with the selection of a system of independent components of which the first, among all linear functions, makes the maximum contribution to the total variance, and the others, in sequence, make the maximum contributions to the residual variances in turn. This is shown to be geometrically equivalent to the selection of the principal axes of an n-dimensional ellipsoid in the space of the scatter diagram. An iterative process is offered which reduces the otherwise enormous calculation necessary for the application of this idea to a moderate amount, even for numerous variables. The sampling problems associated with this type of analysis are examined, and certain approximate solutions found, which are capable of distinguishing between cases in which Spearman's single-general-factor theory is relevant and those in which it is not. The problem of errors of measurement is also considered. (Received October 8, 1932.)

272. Dr. Stanislas Saks: *On some functionals.*

The author gives a new proof and various extensions of the following theorem of Hahn: If $\{f_n(x)\}$ is a sequence of integrable functions and if the sequence of the integrals $\int_E f_n(x)dx$ converges for every measurable set E, then the indefinite integrals $F_n(t)=\int_a^t f_n(x)dx$ are equally continuous in every finite interval. (Received September 7, 1932.)

273. Professor Virgil Snyder: *On a series of Cremona involutions defined by a pencil of ruled surfaces.*

Consider a pencil of surfaces $|F_n|$ through a rational curve r to multiplicity $n-2$. Let the points M of r and the surfaces of the pencil $|F|$ be in $(1, k)$ correspondence. A point P fixes a surface F of the pencil and this in turn a point M on r. The line PM meets F_p, in one residual point P'. The relation between P, P' is an involutorial birational transformation of space. The case in which r is a straight line has been solved by Carroll (American Journal, vol. 54 (1932)). That in which r is a proper curve and the residual base simple and irreducible has been considered by Black (Transactions of this Society, vol. 34

(1932)). The latter includes that of the (ruled) quartics through a double space cubic curve r. The present paper discusses all possible cases in which every surface of the pencil is ruled. The interest lies in the rôle played by the fundamental lines of the second kind, and the contact conditions along the directrix curve. The new transformations include a number of well known types, but furnish infinitely many new ones in which m (number of generators through a point of p), $n-2$, k can each take any positive integral value. (Received October 19, 1932.)

274. Dr. A. T. Craig: *Distributions of functions of middle items.*

In the present paper, laws of probability are determined in accord with which various functions of the middle n items of samples consisting of $2k+n$ items are distributed. Among the functions considered are the arithmetic mean, the average of the extreme values, and the range. An experimental study of certain aspects of this problem has been made by E. S. Pearson (Biometrika, vol. 20A, 1928). (Received November 3, 1932.)

275. Mr. F. A. Brandner: *A test of the significance of the difference of the correlation coefficients of normal bivariate samples.*

By using the method of likelihood, the author has procured a criterion to test the significance of the difference of the correlation coefficients of two normal bivariate samples. The ratio of the chance of procuring the samples with the added hypothesis that the correlation coefficients are identical, to the chance of procuring them with only the condition that they come from normal bivariate populations, is set up as a test. This ratio proves to be a function of R. A. Fisher's well known expression z_1-z_2. Thus the criterion z_1-z_2 is shown to be a logical test to use in case nothing is known as to the values of the remaining parameters. (Received October 31, 1932.)

276. Professor E. S. Allen: *A generalized definition of probability.*

In view of the fact that there is never any certainty that a frequency sequence encountered in nature converges, it is natural to ask what can be regarded as probability in case of divergence. The present paper shows that the sequence itself satisfies Reichenbach's axioms of probability with one slight alteration; and investigates the modifications required in the resulting theory. (Received November 4, 1932.)

277. Mr. Deane Montgomery: *Sections of point sets.*

This paper considers the problem of characterizing a plane point set, E, in terms of properties of subsets of E, called horizontal and vertical sections of E, which are the products of E and given horizontal and vertical lines. The theory is developed for the more general case in which E is a subset of the combinatorial product of metric spaces. Theorems are given concerning the Borel classification of E when its vertical sections are a certain type of closed or open set and its horizontal sections are F or O of class α. The category of E is related to the categories of its sections by theorems additional to those of Kuratowski and Ulam (Fundamenta Mathematicae, vol. 19, p. 246). Certain applications are made to functions of two variables continuous in each of them. (Received November 3, 1932.)

278. Professor H. R. Pyle: *The conditions for conformality in the elliptic and hyperbolic geometries.*

This paper develops the differential equations which are the necessary and sufficient conditions for conformality in the elliptic and hyperbolic plane geometries. These equations contain the Cauchy-Riemann equations as the special case in which the curvature constant is infinite. They lead to a generalization of the Laplace equation which reduces to that equation in the euclidean case. The function $W=\phi+i\psi$, conformal in the non-euclidean plane, is shown to be the reciprocal of a harmonic polygenic function, although it is not itself harmonic. In order to find the above conditions, a transformation which carries points from the non-euclidean plane into corresponding points in the euclidean plane is used. Standard forms of functions conformal in the non-euclidean plane are found in terms of functions conformal in the euclidean plane. (Received October 26, 1932.)

279. Professor M. H. Ingraham: *On the reduction of a matrix to its rational canonical form.*

Let A be an nxn matrix with coefficients in a field F. Let $L=L(\xi_1\cdots\xi_p)$ be the linear set consisting of the totality of vectors of the form $\sum_1^p f_i(A)\xi_i$ where the f's are polynomials with coefficients in F and the ξ's are $nx1$ matrices (vectors) with elements in F. If η is a vector, we say that $g(A)\eta\equiv 0 \bmod L$ if $g(A)\eta$ is in L. It is shown that for each vector η there is a function not identically 0 of minimum degree effective as g. This function divides all other polynomials effective as g. There is an η which maximizes the degree of the corresponding minimum function g, and the minimum function g_1, corresponding to any other vector η_1, divides g. From these considerations one readily develops the reduction of A to a canonical form by transformations in F. (Received November 2, 1932.)

280. Professor H. T. Davis: *A technique for the study of the interaction of economic series.*

The present paper, the product of computations made by the Cowles Commission for Research in Economics, sets forth a technique which is applicable to the study of the interaction effects of several economic series. Let us assume that the effect of three series, (y_i), (y_i'), and (y_i''), upon one another is desired. A relationship exhibited by correlation coefficients of magnitude greater that .5 will be assumed to exist between the three series. First and second derivative curves, (Δy_i), $(\Delta^2 y_i)$, are constructed for each series by first computing the differences and then smoothing these by means of moving averages. The three series, the original and the two derivative series, are thus reduced to approximately the same degree of smoothness. The nine series obtained in this manner are then correlated into three difference equations by correlating the second derivative curves in turn with the other eight. This system of equations is then studied for its oscillations and the results obtained are interpreted in the light of the periodogram for each series. The method has been applied to the three series: Dow Jones averages, pig iron production, and Bradstreet commodity price index, for the pre-war data. (Received November 3, 1932.)

281. Professor David Moskovitz: *Certain irregular non-homogeneous linear difference equations.*

This paper treats the non-homogeneous linear difference equation $\sum_{k=0}^{n} a_k(x)g(x+n-k)=b(x)$ in which $a_k(x)$ and $b(x)$ are rational functions, or only of a rational character at infinity, and for which the associated homogeneous equation does not belong to the so-called "regular" case, its characteristic equation being permitted to have zero or infinite roots. Under various assumptions analytic solutions are obtained and their asymptotic forms studied. Two methods are employed, one analogous to that used by K. P. Williams (Transactions of this Society, vol. 14, p. 209) in studying the non-homogeneous equation whose associated homogeneous equation is regular; this method extends the results obtained by Williams to the class of equations here studied, and certain of Williams' results are amplified. The second method transforms the non-homogeneous equation to a homogeneous equation of one higher order, which is shown to belong to the same irregular case as the homogeneous equation associated with the non-homogeneous equation under study. The results of C. R. Adams (Transactions of this Society, vol. 30, p. 507) concerning the solutions of homogeneous difference equations in this irregular case may be applied at once. (Received October 31, 1932.)

282. Dr. M. C. Gray: *Mutual impedance of long grounded wires when the conductivity of the earth varies exponentially with depth.*

Assuming that the conductivity of the earth increases exponentially with depth $(-z)$ according to the formula $\gamma=\gamma_0 e^{-z/c}$, where γ_0 is the conductivity at the surface $z=0$, this paper gives a formula for the mutual impedance, per unit length, of two long parallel wires, grounded at their end-points. The earth is supposed flat, semi-infinite in extent, and the inductivities of both earth and air to be equal to that of free space ν. The frequency, $\omega/(2\pi)$, is assumed to be sufficiently small to allow all displacement currents to be neglected. The wires are at heights h and H above the earth's surface, and the horizontal separation between their vertical planes is y. The propagation constant at the surface is $\Gamma_0=(i\omega\nu\gamma_0)^{1/2}$. For $c=\pm\infty$ this formula reduces to that given by J. R. Carson (Bell System Technical Journal, vol. 5, pp. 539–554) for a uniformly conducting earth, while if c approaches zero through negative values it agrees with that given by O. Mayr (Elektrotechnische Zeitschrift, vol. 46, pp. 1352–1355) for an earth consisting of a conducting layer at the surface only. (Received November 1, 1932.)

283. Professor D. L. Holl: *The deflection of a rectangular plate, supported at two opposite edges, due to a point load.*

A. E. H. Love (Proceedings of the Royal Society, London, vol. 118 (1928)) solved this problem for a *centrally* loaded plate by conformally mapping the rectangle upon a unit circle and determining five biharmonic functions satisfying the necessary boundary conditions. The functions are represented by infinite series and in some case the constants determined are expressed by infinite sums. In this paper the solution is effected by dividing the plate into three rectangles, one of which is uniformly loaded on a rectangular portion of it. For these three sections, suitable solutions of $\nabla^4 w = f(x, y)$ are chosen so as to satisfy the edge conditions and the necessary continuity properties at the adjacent sections. The solutions are in series form with closed expressions for the constants. By enlarging the loaded area or by shrinking it to remain a finite point load, the solutions include the cases of uniformly loaded plates or infinite plate strips as well as the rectangular finite plate or infinite strip with a concentrated load at any point. (Received October 31, 1932.)

284. Professor W. C. Brenke: *On the summability and generalized sum of the series of Legendre polynomials* $\sum n^p X_n(x)$.

In this paper it is shown that, for positive integral values of p, the series $\sum n^p X_n(x)$ is summable by the Hölder mean value process of order p, that is, summable (H, p), for the range $-1 < x < 1$. It is summable $(H, p+1)$ for this range and including $x = -1$. The sum is obtained in closed form. For $x = -1$ this gives the sum of the oscillating series $\sum (-1)^n n^p$. (Received November 1, 1932.)

285. Professor Dunham Jackson: *Problems of approximation with integral auxiliary conditions.*

This note discusses the modification to which a problem of closest approximation according to the criterion of least mth powers is subjected if it is required by way of auxiliary condition that certain definite integrals involving the approximating function agree exactly in value with the corresponding integrals in terms of the function to be approximated. It is shown that hypotheses which would naturally be formulated to insure convergence in the absence of the auxiliary conditions remain adequate when the additional conditions are imposed. (Received October 21, 1932.

286. Professor J. V. Atanasoff: *Solution of Dirac's equation without specialization of the operators.*

Sauter, in two papers in the Zeitschrift für Physik (vol. 63, p. 803; vol. 64, p. 295), has given a simple direct method for reducing Dirac's equation to a system of differential equations in ordinary numbers without specializing the Dirac operators. However, Sauter's solution for ψ possesses a more complicated structure than the situation demands. As a result an additional hypothesis is needed in forming the quantities quadratic in the ψ's. In the present paper the ψ's are assumed to depend in a linear way upon the characteristic operands of the Dirac operators. In this way results equivalent to Sauter's are obtained without the need of his hypothesis. (Received November, 2, 1932.)

287. Mr. C. P. Wells: *Separation of partial differential equations in two unknowns.*

A definition of a partial differential equation that is separable in specified coordinates is given. This definition is first applied to the study of the coordinates that separate Laplace's equation in two independent variables. The principal result here is that if u, v separate this equation, there exist functions U of u alone and V of v alone which satisfy the Cauchy-Riemann equations. This definition is also applied to a self-adjoint partial differential equation of the elliptic type, in the study of which extended Cauchy-Riemann equations are employed. These equations have much the same relation to this problem that the original Cauchy-Riemann equations have to the separation of Laplace's equation but an additional condition must be imposed before the conditions are sufficient to insure the separation of the equation. (Received November 2, 1932.)

288. Mr. Fred Robertson: *The fractional differentiation operator.*

The function of x defined by $\int_0^x \{s^{n-1}e^{-sz}/\Gamma(n)\}ds$, $(n>0)$, where z is the operator d/dx arises in the calculus of operators. The author considers some properties of these functions of x when n assumes negative values. The fractional derivative ${}_0D_x{}^\alpha u(x)$ is expressed in the operational form $\{[x^{-\alpha}/\Gamma(-\alpha)] \cdot\sum_{r=0}^{\infty}(-1)^{r+1}[x^r z^r/(\alpha-r)r!]\}\rightarrow u(x)$ where α replaces $-n$ and is not an integer. This formula is very convenient for computing fractional derivatives. The theory of these fractional derivatives is developed by means of a second order partial differential equation containing a parameter α. (Received November 5, 1932.)

289. Mr. R. H. Cameron: *Almost periodic transformations.*

The aim of this paper is to define and study a class of one-to-one uniformly continuous transformations of the points of a closed set in a complete metric space. The definition of these transformations, called almost periodic transformations, is analogous to Bohr's definition of an almost periodic function and Walther's definition of an almost periodic sequence. On the basis of this definition, certain fundamental theorems are proved: for instance, *the product of any two permutable almost periodic transformations is almost periodic.* Next, the notion of a power of an almost periodic transformation is very much extended, and the properties of these new powers are shown to be similar to those of ordinary powers defined by iteration. Finally, a special case of an almost periodic transformation called a mono-basal transformation is defined, and it is shown that every almost periodic transformation can be expressed as a finite or infinite product of mono-basal transformations. (Received October 28, 1932.)

290. Mr. Joseph Lev: *The effects of linear transformations on the divergence of bounded sequences and functions.*

This paper is a new approach to a problem previously studied by Hurwitz and Knopp. The *limit circle* of a bounded sequence of complex terms is defined as the circle of least radius which contains within or on its boundary the limit points of the sequence. If the transformation is given by $y(t) = \sum_{i=1}^{\infty} K_i(t) X_i$, where the sequence $[x_i]$ is bounded, and the coefficients $K_i(t)$ are suitably restricted, then the limit points of $y(t)$, as t approaches t_0, lie in a circle whose center and radius are expressed linearly in terms of the center and radius of the limit circle of the sequence. The new concept permits a study of a wider class of transformations than those used by earlier writers. Similar results are obtained when linear transformations are applied to the complex function $f(x)$ of the real variable x, and the limit circle of $f(x)$ at a point is defined as in the case of sequences. Applications are made to the study of Cauchy products and convergence factors in the theory of infinite series. (Received October 27, 1932.)

291. Professor N. H. McCoy: *On quasi-commutative matrices.*

Two square matrices x and y, of order n, may be said to be *quasi-commutative* if $xy - yx$ is commutative with both x and y. It is proved that if z is a given matrix of order n, a necessary and sufficient condition that there exist quasi-commutative matrices x and y such that $xy - yx = z$, is that the elementary divisors of z be of the form $\lambda^{n_1}, \lambda^{n_2}, \cdots, \lambda^{n_k}$, with $n_i - n_{i+1}$ either zero or unity and $n_k = 1$. Several well known properties of commutative matrices are shown to hold also for quasi-commutative matrices. In particular, if $f(x, y)$ is a scalar polynomial in x and y, then the roots of $f(x, y)$ are of the form $f(\alpha_i, \beta_j)$, where α_i is a root of x and β_j a root of y. Some additional properties of quasi-commutative matrices are obtained by means of known theorems in the algebra of quantum mechanics, as the ring of polynomials in x and y is homeomorphic to the ring of polynomials in the infinite matrices (or operators) p and q of quantum mechanics. (Received November 3, 1932.)

292. Professor J. L. Walsh: *On approximation by non-vanishing analytic functions.*

Let the functions $\phi(z)$ be analytic and univalent (i.e., schlicht) interior to the Jordan region C, and let $\phi(z)$ have at least one (hence precisely one) zero interior to C. Let $w = \delta$ be the boundary point (or one of the boundary points) nearest the origin of the region into which C is transformed by $w = \phi(z)$. Then for the particular function $f(z) = \phi(z) - \delta$, which is analytic and different from zero interior to C, we have $|f(z) - \phi(z)| = |\delta|$, z in C. If any function $F(z)$ analytic interior to C is such that we have $|F(z) - \phi(z)| \leq \delta' < |\delta|$, z in C, then $F(z)$ has precisely one zero interior to C. (Received October 26, 1932.)

293. Dr. G. T. Whyburn: *On the existence of totally imperfect and punctiform connected subsets in a given continuum.*

A set containing no compact perfect subset [continuum] is totally imperfect [punctiform]. Sierpinski has shown that in any euclidean space $E_n (n > 1)$ the complement of any totally imperfect set is connected. In this paper it is shown that Sierpinski's theorem may be so generalized as to read as follows: In any locally compact continuum M, the complement of every totally imperfect set augmented by the set of all local separating points of M is connected. Using this proposition, we obtain the following. (1) Let M be a locally compact continuum and let L be the set of all local separating points of M; then in order for M to contain, for each $x \epsilon M$, a totally imperfect connected set containing x it is necessary and sufficient that L be countable. (2) Also if M is locally connected, it will contain, for each $x \epsilon M$, a punctiform connected set containing x if and only if L is punctiform. (3) In order that the hereditarily locally connected continuum H contain no punctiform connected subset, either of the following conditions is necessary and sufficient: (a) that the set of local separating points of no sub-continuum of H be punctiform, (b) that every cyclicly connected sub-continuum of H have a free arc. (Received November 3, 1932.)

294. Dr. G. T. Whyburn: *Characterizations of certain curves by continuous functions defined upon them.*

Čech has shown (Fundamenta Mathematicae, vol. 17) that any compact continuum M upon which there can be defined a real, continuous function which is not constant on any infinite subset of M is a particular kind of regular curve. Mazurkiewicz (Fundamenta Mathematicae, vol. 18) has given necessary and sufficient conditions that a given acyclic locally connected continuum (dendrite) have the property of admitting such a function to be defined on it. In the present paper it is shown that, by varying the restrictions on the function, regular, rational, and 1-dimensional curves (Menger-Urysohn sense) may be characterized. Indeed it is shown that in order for a compact continuum M to be (1) a regular curve, (2) a rational curve, (3) a 1-dimensional curve, it is necessary and sufficient that there exist on M a real, continuous function which is not constant on any subcontinuum of M and such that there exists an everywhere dense set of its values each of which it takes only (1) a finite number, (2) a countable number, (3) any number, respectively, of times. (Received November 3, 1932.)

295. Professor A. D. Campbell: *Plane quartic curves in the Galois fields of order* 2^n.

Plane quartic curves in the Galois fields of order 2^n have many peculiarities. Every point has a cuspidal polar cubic with respect to such a quartic. The tricuspidal quartic does not exist. There are in general only seven bitangents to a quartic, which arrange themselves respectively as the six sides of a complete quadrangle, and the line containing the diagonal points of this quadrangle. Also there are quartics having each four tangents at undulations, and three bitangents that form the same configuration as the above mentioned seven bitangents. The Plücker equations for plane algebraic curves do not hold for a quartic. We find even a quartic with a cusp P and with every line through a certain other point proving to be a bitangent to the curve. Finally, we note that only a very few types of quartics can be first polars of points with respect to a quintic. (Received November 5, 1932.)