NEW EDITION OF OSGOOD ON FUNCTIONS

Lehrbuch der Funktionentheorie. By W. F. Osgood. Vol. I, fifth edition, 1928. 14+818. Vol. II, part 1, second edition, 1929. 7+307 pp. Leipzig and Berlin, Teubner.

The second edition of the first volume of this work was reviewed by E. B. Van Vleck in this Bulletin, (vol. 20 (1913), pp. 532–546). No reviews of the intervening editions have appeared in this journal. The first edition of the first half of the second volume was also reviewed by Professor Van Vleck, in this Bulletin (vol. 33 (1927), pp. 358–365). Both of these reviews are adequate and authoritative in giving an excellent idea of the plan of the work and of its chief characteristics. As a consequence we can limit ourselves in this review to the changes which have been made in these editions. In point of size, the fifth edition of the first volume contains some fifty odd pages more than the second. The added material is scattered through the volume, including changes in expressions, added exercises, added explanations, and new material.

The added material in the earlier chapters of the book include such matters as a separate section devoted to the definite (Cauchy) integral, its definition and properties, paragraphs on Abel's theorem on the continuity of a power series at a point of convergence on the circle of convergence, on iterated integrals, on the convergence of products of power series (it is puzzling why this was omitted in the early editions, when division and other arithmetic operations were included) and a section devoted to the Poincaré theta series. In the chapter on the logarithmic potential function the section on the motion of an incompressible fluid has been considerably expanded. The section covering the matter of isolated singularities of harmonic functions and the symmetry of Green's functions has been rearranged to advantage. Further, a paragraph giving the Arzelà theorem on compactness of a sequence of continuous functions has been added. The last chapter dealing with the uniformisation problem, aside from a number of minor changes, has been augmented by a section deriving, in detail, results concerning the analysis situs of the Riemann surfaces involved in the proof. Further, the final paragraphs, dealing with the uniformisation of an arbitrary analytic function, and the proof of the existence theorem for a many-valued function on an arbitrary region of definition, have been considerably revised. This has helped to make the subject matter more intelligible, the developments clearer, and have added materially to the usefulness of the volume, both as an introduction to the theory of functions, particularly the uniformisation problem, and as a work of reference.

Most of the exercises added to the later editions are of the thought provoking variety, frequently giving points of theory not covered in the text. One finds, for instance, as exercises, to prove that the sum of the residues of a rational function is zero, to discuss the many-valuedness of z^{α}, $(\alpha = a+ib)$, to prove that a monogenic analytic function can be at most denumerably infinitely many-valued, and so on. Putting questions of this character into problem form is certainly desirable, and could be carried to even further extremes.

If one were in a quibbling mood, one might take objection to the introduction of the definite integral on the basis of the most primitive form, of equal subdivisions of the fundamental interval, as given in first courses in calculus. To be sure, it is adequate for the class of continuous functions, to which integration processes are restricted in this work. But certainly a mind which can cope with the intricacies of a proof of the Jordan curve theorem, and finds a proof of the Green's integration theorem based thereupon desirable, should be able to understand at least the Riemannian definition of integration.

Perhaps too, we might have expected some of the sets of references to have been brought up to date, for example, those on the Jordan curve theorem. On the other hand, it is a little disconcerting, when infinite derivates of real-valued continuous functions are admitted, to find a reference (p. 22) to a recent nowhere differentiable function, non-differentiability being restricted to non-existence of a finite derivative, when E. H. Moore in 1900* called attention to a function which in a few lines can be shown to be non-differentiable in the sense of having at no point even an infinite derivative.

Another puzzling thing is that the author does not utilize the notion of total differential, as given on page 240, to advantage. It simplifies statements of differentiation properties of functions of two variables and gives a more elegant form to the condition for the existence of a derivative of a function of the complex variable. These more or less minor matters do not alter the fact that we have in this volume an example of fine and rigorous exposition.

The second edition of the first part of the second volume seems to have grown from 242 pages to 307 pages. Closer examination, however, reveals the fact that the type is larger, and the spacing wider, giving a more open and attractive page. The improvement in appearance is very commendable.

The contents of the volume have undergone only very slight changes. The first chapter is almost unchanged. The second chapter has received additional explanatory material, in a few places. Two sections, 14 and 15, dealing with the zeros of a single-valued function on a manifold, and the singularities of a pseudo-algebraic manifold, have been rewritten, considerably enlarged and improved. In the third chapter, the changes have been made in connection with the extension of the Weierstrass theorem that an everywhere meromorphic function is a rational function, to manifolds extended so as to include infinite regions, to which is then added the proof of the theorem that the most general one-to-one transformation of extended space into itself is a rational one, giving an elegant and satisfying conclusion to this part of the theory of functions.

Apropos of Professor Van Vleck's remark on this work in English, the following incident may be of interest. At a midwestern University, in a list of new books on mathematics received in the library of the university, occurred Osgood's *Theory of Functions*, translated into English. Investigation disclosed that the office secretary had by mistake sent out as received a list of books *wanted* by students.

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* Transactions of this Society, vol. 1 (1900), pp. 77 ff. The same example was rediscovered independently by Hahn: Jahresbericht der Vereinigung, vol. 26 (1918), pp. 281–284.