By the use of Theorems III and IV, we derive (DM) from (L). Let $r_{in}-r'_{in}$ define a sequence of horizontal functions on I. The limit function is obviously a null function whose integral is zero. Conclusions 1 and 2 of (DM) follow immediately from (L).

Conversely we derive (L) from (DM). By Theorem IV, $f(x)$ is bounded and integrable in I. Let $h_n(x)$ be a set of horizontal functions on I with respect to I_{in} approaching $f(x)$ as a limit function almost everywhere on I. Identify $r_{in}=h_{in}$. Also let $H_n^{(k)}(x)$ be a horizontal function of index n on I associated with I_{in} and having $f_k(x)$ as a limit function almost everywhere on I. Identify $r'_{in}=H_{in}^{(n\,k)}$ and Conclusion 2 of (L) follows from (DM).

It is worthwhile emphasizing the role which horizontal functions, together with the principle isolated by R. L. Moore, are destined to play in the theory of functions of a real variable. They may be made the basis for a concise and elegant treatment of the greater part of the theory. The bulk of a book like Hobson's *Theory of Functions of a Real Variable*, volume I, may be reduced to one-third or less by their use. Schlesinger in the book under review has made a beginning in this direction.

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FUBINI AND ČECH, PROJECTIVE DIFFERENTIAL GEOMETRY

Geometria Proiettiva Differenziale, Vol. I. By G. Fubini and E. Čech. Bologna, N. Zanichelli, 1926. 388 pp.

"Un nuovo indizio dei sentimenti fraterni che vanno sempre più legando fra loro i vari rami della matematica!"

These words of Segre were chosen by Wilczynski to be inscribed on the title page of his prize memoir *Sur la théorie générale des congruences*. Thus the founder of the American school of projective differential geometry indicated that he was studying the projective differential properties of a geometric configuration by the means of the invariants and covariants of a completely integrable system of linear homogeneous partial differential equations under a certain continuous group of transformations, in the sense of Lie. The same sentiment would be no less appropriate as a motto for the new book by Fubini and Čech, since these distinguished protagonists of the Italian school of projective differential geometry define a configuration by means of differential forms, after the manner of Gauss, and employ the absolute calculus of Ricci.

Those who know the absolute calculus only as it is used in the theory of relativity will be interested to see this geometric application of it. And those who know only Wilczynski's method of attacking a problem in projective differential geometry will be eager to learn this new theory. Wilczynski's method is particularly adapted to certain types of problems and has a power and elegance of its own. But it has some inconveniences. For instance, certain calculations become quite laborious, which are accomplished more easily and efficiently by the tensor analysis.

The first volume of the treatise before us is dedicated to the dean of differential geometers, Luigi Bianchi, and is devoted to the geometry of curves and surfaces in space of three dimensions. The projective definition of a surface by means of three differential forms of the first order, two quadratic and one cubic, is due to Fubini. Of these, one, F_2, vanishes for the asymptotics, and another, F_3, which is apolar to F_2, vanishes for the curves of Darboux. The ratio $F_3 : F_2$ is called the *linear projective element*. For this Bompiani has furnished a geometric interpretation; Čech has given an algebraic theory of the invariants thereof. The extremals of the integral $\int F_3 : F_2$ are called *pangeodesics*.

Fubini has defined a *projective applicability*, two surfaces being projectively applicable in case they have the same linear projective element. Moreover, for a non-ruled surface, Fubini has obtained a normalization of the proportionality factor of the homogeneous coordinate system such that his *normal coordinates*, if the asymptotic net is parametric, satisfy a certain canonical system of differential equations of the same form in covariant derivatives as Wilczynski's much used canonical equations in ordinary derivatives. Introducing the normalized form of F_2 as a projective substitute for the squared arc element, or first form of Gauss, Fubini defines a *projective normal*, which G. M. Green also discovered independently, projective geodesics, projective lines of curvature, etc. For this projective metric of Fubini, a geometric interpretation was furnished by Wilczynski.

The principle of duality plays its proper role, particularly in the formulation of the differential equations for the determination of a surface when the three forms that define it are given. The authors prefer, when using a local coordinate system at a point of a surface, to define the system so that a point and a plane whose corresponding local coordinates are equal are respectively pole and polar with respect to the quadric of Lie, called by Wilczynski the osculating quadric. With this convention the condition of united position of point and plane does not have its usual simple form.

An entire chapter is devoted to the theory of space curves, another to the theory of ruled surfaces, and still another to the transformation of surfaces by means of Weingarten congruences. The calculation of the integrability conditions for a curved surface in arbitrary parameters, by means of covariant derivatives, deserves special mention, as does also the treatment of various interesting classes of surfaces.

The book contains surprisingly few typographical errors, and these are for the most part easily corrected. One might wish that the type setter had been consistent in his use of italics for mathematical symbols. Occasionally the differentials du and dv and some other symbols are not in italics. The notation is well chosen, although one of my students expressed the wish that the authors had used c_{rst} instead of a_{rst} for the coefficients of F_3, since they had already used a_{rs} for the coefficients of F_2. The style of exposition is clear, concise, and direct.

This treatise fills a long felt want. There is no other of its kind. Hereafter the projective differential geometer must know his Fubini and Čech, and every student of geometry has now a new reason for acquiring at least a reading knowledge of *la lingua italiana*.

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