BOUTROUX ON MATHEMATICAL IDEALS

L'Idéal Scientifique des Mathématiciens dans l'Antiquité et dans les Temps Modernes. By Pierre Boutroux. Paris, Felix Alcan, 1920. 274 pp.

What conception do mathematicians have of their science? What plan do they follow in their research? What principles direct their activity? What is the goal which they seek? These are the questions which the author of this book sets himself and to which he seeks an answer. Few probably will deny the value and importance of such questions nor the desirability that the devotees of mathematics should ask them of themselves and that they should have some clear notions as to the answers. In the vast complexity of modern mathematics, it is surely highly desirable that the man who is working in some particular corner and possibly on a very restricted problem should occasionally pause in his work in order to gain in proper perspective a view of the whole. Whither is he tending? What is his object? What constitutes real progress in mathematical investigations? What is important and why?

Although everyone will probably admit the importance of such considerations, it is to be feared that relatively few have seriously set themselves these questions. Still fewer perhaps have been able to formulate satisfactory replies. Indeed the present work shows how very difficult it is to answer the questions proposed.

The author sets himself a very definite problem. While his inquiry is in the nature of the case philosophical in the broad sense of the term, he is not at all concerned with the question of what place mathematics has in a general system of philosophy. He is concerned merely with the philosophy of mathematics as such. Nor is he concerned at all with the metaphysical aspects of the problem, but purely and simply with answers to the questions proposed. Furthermore he seeks an objective answer. He would eliminate as far as possible all personal bias and seek to discover answers to his questions in the actual work and progress of mathematics itself. His method therefore is, as he says, historical and critical. By a careful examination of the development of mathematics through the centuries he seeks to discover what the leading tendencies were and are. In a word, as indicated in the title of the work, what scientific ideals mathematicians of the past and present have set themselves. He is well aware of the difficulties of his problem and indeed there are many places in the book where the reader will be inclined to differ from the author in his placing of emphasis on such matters. There can be no doubt, however, that the author has written a very stimulating book which may be highly recommended to everyone interested in the questions discussed and should prove of special value to the young investigator starting upon his career and seeking orientation in his chosen field.

The author distinguishes three great epochs in the development of mathematics as bearing upon his problem: the epoch of Greek mathematics, the epoch beginning near the end of the 17th century and continuing for 150 years thereafter, and the present epoch beginning about the middle of the last century. The ideals governing the ancient Greeks were very largely æsthetic in character. They dealt with ideal concepts which had no concrete reality. The beauties they sought they considered to be inherent in the objects of study and not to be added to or subtracted from by the human intellect. Moreover, to satisfy their sense of the beautiful, their results had to be simple, harmonious. Another great ideal with which they furnished posterity was that of the geometric demonstration and of the deductive logical system. These ideals governed the development of mathematics through several centuries. They had inherent limitations, however, which the author analyzes with care. A geometric entity did not "exist" for them unless it could be "constructed," and quite obviously their ideals of simplicity and of the rôle that intuition should play in their discoveries involved serious limitations. Herein as well as in their contempt for practical applications may be sought the reasons why the Greeks never developed an algebra.

Preparation for the second epoch came through the introduction of algebra in the middle ages. In direct contrast to Greek ideals, its origin is to be found in practical applications. Algebra came to Western Europe in the form of practical rules for computation with practically no scientific foundation. Moreover progress depended largely on the absence of scientific scruples. The faith of the early investigators was superb. All through the 17th century and beyond, they were governed by the hope that they held in algebraic methods the mechanical key to all science. The new epoch as such may be said to begin with the publication of Descartes' *Geometry*, and with the invention of the calculus by Newton and Leibnitz. Descartes introduced a new conception in place of the euclidean demonstration, a new method which involved the discovery of geometric properties by indirection; that is, by the application of algebraic methods. The new conception of the character of mathematics introduced by the work of Descartes, Newton, and Leibnitz is the idea of synthesis, the idea of putting together simple elements in such a manner as to form progressively compounds of a more and more complicated character. Algebra is considered not as a collection of results but as a method of combination and discussion. In glancing back over the activities of the 150 years following the invention of the calculus, we can readily grasp the enthusiasm of the man working with his new tools. It is hardly an exaggeration to say that mathematical investigation at that time ceased to be a profession and became an industry. No limits were seen to the power of the new methods and all that seemed to be necessary was to proceed systematically in the building up of the edifice from its simple elements to more and more complicated and extensive structures. Leibnitz's dream of a general combinatory calculus whereby all problems of human thought should be capable of solution by an appropriate operational symbolism was a not unnatural consequence of the situation in which he found himself. This era of synthesis is then governed by the ideal that the perfect mathematical science is constructive and mechanical, the calculations of which are performed, so to speak, automatically.

The course of events, however, proved the limitations of the new methods. Blocked in its triumphant progress, it is only natural that a critical spirit should develop. The study of the logical foundations of the science became prominent. The author interprets even this phase of activity as belonging in part to the era of synthesis, the idea being that the axioms and postulates sought for were merely in order to provide a secure foundation for the edifice which had been and was being built. It would seem, however, to the reviewer, at least, that the development of postulational methods, especially in their latter course, belongs to the third epoch rather than to the second not merely chronologically, but also in spirit.

This third epoch is again sharply contrasted with the preceding in that it is characterized by analysis rather than by synthesis. The modern mathematician is like a chemist who analyzes an extremely complicated situation and seeks the elements of which it is compounded.* Our present epoch then is characterized by a frank recognition of the limitations of logic alone. Other intellectual activity than that of mere logic is necessary for further progress. Such activities are especially experimentation, the careful analysis of special cases, and above all the recognition of the power of intuition or insight. The modern mathematician must be constructive in the domain of ideas, not merely in the mechanical putting together of simple elements already existing. Progress at present demands the development of new points of view for classifying and interpreting the baffling new problems which present themselves.

In his final chapter on the present mission of mathematics, the author attempts to appraise the manifold and apparently conflicting tendencies that are at present in existence. An extended and interesting discussion of the relation of mathematics to theoretical physics leads to the rather obvious conclusion that the demands of the applications of mathematics cannot furnish the sole or even the principal guide to further progress. The author then takes up the claims of those who would find the desired guide post in the æsthetic or artistic element in mathematics, only to reject this also. He admits that this orientation of our science has been fruitful in that it has served to introduce a large number of new ideas. But, he says, it merely raises the fundamental question in another form: "What precisely does the mathematician mean by 'beautiful,' 'elegant,' 'remarkable?'"

The author thus admits himself unable to give any satisfactory answer

* It may be desirable at this point to caution the reader against a possible confusion of terms. The text of our review would seem to make sufficiently clear the sense in which we (and the author) are using the words "synthesis" and "analysis." The possible confusion arises from the fact that the method used in the second epoch and which would seem to be essentially synthetic in character has received the name of "mathematical analysis."

to the questions he has set himself. He has recourse finally to the advice often given to the young aspirant toward mathematical research: Study the great masters! They had a certain flair for recognizing the valuable and important directions of advance. And the fact remains that, even though we cannot find any simple rules which govern the directions of progress, progress does exist. Our science has advanced and is continually advancing in spite of the lack of any conscious direction.

The author obviously laid down his pen after writing the last word of his interesting little book with a feeling of discouragement and dissatisfaction. The reader shares this feeling;—but, in spite of it, he feels that the writing and the reading has been worth while. The questions raised are of fundamental importance and of the greatest interest. The fact that they remain to a large extent unanswered is merely a challenge to the future. The reviewer has a feeling that the answer may possibly be found in a more vigorous attack on the question which the author himself raises but which he dismisses with a few words. Just what is implied by the words "beautiful," "elegant," "remarkable" as used by the mathematician? Just what is the "flair" which the great masters possess? Is it not possible that this flair is essentially artistic in its nature and that the development of mathematical science is governed largely by laws analogous to those that govern the development of the fine arts?

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SHORTER NOTICES

Ueber Spiralen. By Archimedes. Translated and annotated by Arthur Czwalina-Allenstein. Leipzig, Akademische Verlagsgesellschaft, 1922. 71 pp.

This German translation of Archimedes's classic work on spirals (Ostwald's Klassiker, No. 201), which is now published on account of the fact that Nizze's German translation of 1824 has long been out of print, is of no significance for the American student, as we have Heath's admirable translation.* The supplement (pp. 61–71) gives a reconstruction of a possible method by which Archimedes may have been led to his results; the method is ingenious and plausible, but it has the serious defect that it considers the ratio of an area to a volume, which would have been anathema to a Greek of the classical period; so that we can hardly be convinced, in the absence of evidence, that even so original a genius as Archimedes would have hit upon this particular method. It is probably as well to confess that we are entirely ignorant of the way in which Archimedes did arrive at his admirable results.

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* *The Works of Archimedes,* edited in modern notation with introductory chapters by T. L. Heath. Cambridge, 1897.