introduced using the concepts of segments and order as modified by the introduction of ideal elements. The proof of the fundamental theorem of von Staudt is then based on continuity. Curves and envelopes of the second order, poles and polars with respect to a curve are then studied in considerable detail, with the use of metrics for some properties.

An involution on a form is defined as a cyclic projectivity of order 2 and conjugate imaginary points, lines and planes are introduced from the elliptic involution. An entire chapter is devoted to the focal properties of conics. Collineations, dualities, affinities, and polarities are treated briefly.

The figures are fairly well done, the typography is very good, and the volume as a whole is neat and attractive. The large number of exercises scattered throughout the book add much to its utility as a textbook. The little historical notes although very brief are also valuable and stimulating.

In the opinion of the reviewer, such a book as this one, which is avowedly not concerned with the more logical phases of the subject, would be more valuable to many if it gave at least references to such books as that of Veblen and Young, where such treatment could be found. On the whole, however, the book will no doubt be of much service in beginning courses in the subject.

F. W. OWENS.

Introduction to the Elementary Functions. By RAYMOND BENEDICT McClenon, with the editorial cooperation of William James Rusk. Boston, Ginn and Company, 1918. x + 244 pp.

Under the above title the authors present in book form their idea of the content of a required course in mathematics for freshmen in a small college. As presented, the text is the result of five years of teaching of the course in such a college. The elementary functions are those from plane trigonometry, plane analytic geometry, and the elements of differential calculus. These subjects are combined with such review topics from algebra as is necessary to bind the topics presented into an organic whole. The chapters are not sections taken from a single one of the principal subjects, but are mixtures. The calculus covers only the differentiation of algebraic functions with applications to rate and maxima and minima problems.

The treatment of topics is informal and inductive, there being a noticeable absence of formal proofs. Naturally the reviewer does not agree with the arrangement of material nor with its content, but we do agree with an avowed object of the book, namely that of coaxing students in a small college, or elsewhere for that matter, into a further study of non-required mathematics. We object to the form of the discussion of the quadratic in one unknown (pages 46, 47) wherein the authors seem to discard complex roots, as roots. Of course problems in analytic geometry and elsewhere lead to equations with complex roots, which are not interpretable in terms of the real elements presented in the problem. But this does not seem to be the meaning of the authors.

C. F. CRAIG.

## NOTES.

THE following mathematical papers have appeared in recent numbers of the Proceedings of the National Academy of Sciences: volume 5, number 1 (January, 1919): "A theorem in power series, with an application to conformal mapping," by T. H. GRONWALL; number 3 (March): "Tables of the zonal spherical harmonic of the second kind  $Q_1$  (z) and  $Q_1'$  (z)," by A. G. Webster and W. Fisher; number 4 (April): "On the real folds of abelian varieties," by S. Lefschetz; "Covariants of binary modular groups," by O. E. Glenn; "The general solution of the indeterminate equation: Ax + By + $Cz + \ldots = r$ ," by D. N. Lehmer; number 6 (June): "On the most general class L of Fréchet in which the Heine-Borel-Lebesgue theorem holds true," by R. L. Moore; "On a certain class of rational ruled surfaces," by A. EMCH; number 7 (July): "On the twist in conformal mapping," by T. H. GRONWALL; "Groups involving only two operators which are squares," by G. A. MILLER; "Real hypersurfaces contained in abelian varieties," by S. Lefschetz.

The concluding (June) number of volume 20 of the Annals of Mathematics contains the following papers: "Relations between abstract group properties and substitution groups," by G. A. Miller; "The complete quadrilateral," by J. W.