This result may be of interest for an analysis of the sound produced when a vortex oscillates about a state of uniform motion. It must be remembered, however, that the above analysis is only approximate, for velocities are treated as small in the derivation of the wave equation.

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SHORTER NOTICES.

Lectures on the Philosophy of Mathematics. By JAMES BYRNIE SHAW. Chicago, The Open Court Publishing Company, 1918. viii + 206 pp.

THE purpose of Professor Shaw's book is a discussion of the evergreen question: What is mathematics? While in his first chapter the author develops in a highly exalted style various aspects of this subject, the greater part of the subsequent chapters may be said to be essentially devoted to two more specific questions, viz., what influences operate and have operated in the development of mathematics, and how may existing mathematics be concisely described. With the treatment of these questions, perhaps not always recognized as explicit and distinct, Chapters II to XIII are taken up, together occupying 140 pages. To the first question the author gives a positive answer, viz.: "Mathematics is a creation of the mind and is not due to the generalization of experiences or to their analysis; nor is it due to an innate form or mold which the mind compels experience to assume, but is the outcome of an evolution, the determining factors of which are the creative ability of the mind and the environment in which it finds the problems which it has to solve in some manner and to some degree." The second question is answered in a negative sense; as the various fields and principles of mathematics are discussed, the conclusions are reached that mathematics is not wholly arithmetic, nor geometry, nor logistic; that mathematics can not be completely characterized as a theory of invariance, nor as a theory of functions, etc., however important each of these principles may be. The closest approach to a satisfactory answer to our second question the author appears to find in the statement that mathematics is a theory of equations. On page 152 we read: "This is the most important central principle of mathematics, namely, that of inversion, or of creating a class of objects that will satisfy certain defining statements. If the mathematician does not find these at hand in natural phenomena, he creates them and goes on in his uninterrupted progress. This might be considered to be *the* central principle of mathematics, for with the new creation we start a new line of mathematics, just as the imaginary started the division of hypernumbers, just as the creation of the algebraic fields started a new growth in the theory of numbers."

Chapters XIV, XV and XVI take up respectively "Sources of mathematical reality" (15 pages), "The methods of mathematics" (17 pages) and "Validity of mathematics" (10 pages). There is finally given on pages 196 and 197 a double entry table for the classification of existing mathematics, presenting in schematic form the division of the whole field on the basis of which the discussion is carried on in Chapters II to XIII. This very interesting and suggestive mode of dividing mathematics is not the least valuable part of the book. Each mathematical topic is classified as to its structure and as to its central principle. Structurally, mathematics falls into two main divisions, static mathematics and dynamic mathematics. In the former group we find arithmetic, geometry, tactic and logistic; in the latter group are placed algorithms, algebra, transmutations and inferences. The central principles of mathematics are taken to be form, invariance, functionality and inversion.

The book is written in an easy, flowing style; sometimes, as for instance in the first six pages in a flowering style, reminiscent of an earlier literary epoch. In places it is sketchy, so much so as to be of little value to the non-professional reader for whom the book is intended, and to be somewhat irritating to the mathematician. A rather strong example of this is found on page 26, where we find the following statement: "By introducing a 'measure of a set' Lebesgue and others have found a means of handling sets satisfactorily"; others are to be found on pages 90, 91, and at the end of chapter VII. This floweriness and sketchiness of style have in a few instances led to what seems to the reviewer to be an overstatement of perfectly sound positions. As such we would characterize the assertion that "Nothing whatever in our sense-data tells us that the earth is rotating" (page 156), and the claim that "we find in mathematics that subject whose results have lasted through the vicissitudes of time and are regarded universally as the most satisfactory truths the human race knows" (page 154). Why is Lie relegated to the remote period of the Norsemen?

While the author "cherishes the hope that the professional philosopher too may find some interest in these lectures," he addresses himself primarily to "students of fair mathematical knowledge," such as may be secured through an ordinary college course in mathematics. The book is intended to be non-technical from the philosophical as well as from the mathematical point of view. Indeed it seems to the reviewer that the author has taken the term "philosophy of mathematics" in the more or less popular sense, in which it means general discussion about the nature of mathematics, rather than in the sense of systematic, rigorous, scientific discussion. This being assumed, and the assumption is not intended as a criticism of the value of the book, it is understood why one finds nowhere a statement of the author's general philosophical position, of his beliefs and convictions on some of the fundamental questions of philosophy, on which many of his conclusions are based, and a knowledge of which is indispensable for an understanding and just appraisal of his work. Hints are found scattered throughout the book, as for instance on pages 30, 55, 59, 88, etc.; and perhaps the most definite statement of a philosophical credo is that on page 166: "The mind, it is true, as Kant insisted, organizes experience, but it does this by methods that are evolutionary. It originates schemes from its own activity, and makes a choice of which of several equally valid schemes it will use." But it seems clear that a systematic, critical examination of the philosophical position and of the conclusions derived therefrom would be entirely out of keeping with the purpose of this book.

While granting without hesitation, particularly at the present stage of development of the subject, the value of a general, let us say intuitional, discussion of the philosophy of mathematics, the reviewer would like to use this opportunity to call attention to the scientific aspects of this subject. To these may be said to belong (*a*) the study of the logical structure of mathematics; (*b*) the philosophical discussion of concepts to which mathematics gives rise, such as infinity, countability; (*c*) the introduction of a mathematical point of view, and as far as possible, of mathematical methods into the discussion of problems belonging to the domains of epistemology, of ethics, and to other philosophical fields. For progress in the treatment of these questions the closest cooperation between philosophers and mathematicians is essential. Such cooperation will certainly be stimulated by the reading of Professor Shaw's book.

ARNOLD DRESDEN.

Functions of a Complex Variable. By THOMAS M. MACROBERT. London and New York, Macmillan, 1917. xiv + 298 pp.

THIS book is designed for students who have acquired a good working knowledge of the calculus and desire to become acquainted with the theory of functions of a complex variable and its principal applications. The material has been well chosen for accomplishing this purpose. The first two chapters a e intended to familiarize the student with the geometrical representation of complex numbers in a plane and with simple rational and irrational functions of a complex variable. Some fundamental properties of holomorphic functions are established in Chapter III and these are used in defining certain elementary transcendental functions. Chapters IV to VII deal in order with the theory and practice of integration, convergence of series and the Taylor and Laurent expansions, uniformly convergent series and infinite products, and various summations and expansions. Up to this point (page 131) the book gives to the student an introduction to the elementary parts of the classic theory of functions of a complex variable with some of the simpler applications.

The theory of the gamma functions is presented in Chapter VIII (pages 132–159) in a way which is not very satisfying. The development of the initial elementary properties of elliptic functions is to be found in Chapters IX, X, XI (pages 160–207) in an exposition which is pleasing and particularly well suited to the needs of the young learner. The remaining chapters (IX to XV, pages 208–276) are devoted to differential equations, in part to general theorems and in other part to special cases, such as the equations of Legendre and Bessel. This division of the book also is well suited to the needs of the beginner.

Besides numerous worked examples the book contains many