Finally, let us say that there can be no doubt that Forsyth has rendered his colleagues a distinct service in adding this book to his already long list of useful publications. It will be of definite value to a large number of persons interested in the theory of functions of complex variables.

R. D. CARMICHAEL.

SHORTER NOTICES.

The Geometrical Lectures of Isaac Barrow. Translated, with notes and proofs, by J. M. CHILD, B.A. (Cantab.), B.Sc. (Lond.). The Open Court Publishing Company, Chicago, 1916. xiv+218 pp.

"ISAAC BARROW was the first inventor of the Infinitesimal Calculus; Newton got the main idea of it from Barrow by personal communication; and Leibniz also was in some measure indebted to Barrow's work, obtaining confirmation of his own original ideas, and suggestions for their further development, from the copy of Barrow's book that he purchased in 1673."

"The above is the ultimate conclusion that I have arrived at, as the result of six months' close study of a single book, my first essay in historical research. By the 'Infinitesimal Calculus,' I intend 'a complete set of standard forms for both the differential and integral sections of the subject, together with rules for their combination, such as for a product, a quotient, or a power of a function; and also a recognition and demonstration of the fact that differentiation and integration are inverse operations.'"

These are the opening paragraphs of the preface to this edition of Barrow's Lectures. While the rest of the book does perhaps not justify the claims of the preface, it furnishes a very welcome addition to the generally available information concerning Barrow. It presents, in abridged form, a translation by Mr. Child of the "Lectiones Geometricæ" of 1670, of which a first English translation was published by Edmond Stone in 1735. Numerous notes, bearing upon Mr. Child's thesis, are scattered throughout the text, proofs have been added in a number of places, and there is an introduction of

32 pages. There we find a brief account of the life and work of Barrow, of the work on infinitesimal calculus, done before his time and of the mutual influence of Barrow and Newton; a description of the original text, an argument to show that Barrow obtained his results analytically, although he published them in geometrical form, and finally a list of the analytical equivalents of his chief theorems.

Mr. Child appears to base his conclusions on the following points: (1) Barrow gave in his "Lectiones Geometricæ" a complete treatment of the elements of both the differential and integral calculus, including the fundamental theorem; (2) he obtained his results by means of some analytical device, essentially the same as that used by Newton; (3) the essential parts of the "Lectiones Geometricæ" were completed before 1664, the year in which "Barrow first came into close personal contact with Newton"; (4) Leibniz bought a copy of Barrow's work in 1673. The present reviewer believes that Mr. Child's work definitely establishes the first of these points and furnishes strong arguments for the second and third, without, however, settling them absolutely. Additional arguments for the conclusion concerning Leibniz are reserved for the future.

It seems not at all unlikely therefore that we shall have to place Barrow at least on a par with, if not above, Newton and Leibniz among the inventors of the calculus. Just where the Japanese mathematician Seki, whose "circle principle" antedates the work of Barrow, should be placed is a matter to be decided by historians who are concerned with questions of priority and with the influence of one mind upon another. To them I must also leave it to judge in how far Mr. Child's conclusions are justified by the evidence he has so far presented.

But entirely apart from their bearing upon historical questions, Barrow's lectures are of interest to the modern reader. In the first five lectures are developed various properties of curves, which are thought of kinematically, viz., as generated by a point descending with increasing velocity along a line which moves uniformly and parallel to itself. A tangent to a curve is spoken of as a line which has one point in common with the curve and which lies entirely on one side of it. In his fourth lecture, Barrow then shows that the slope of the tangent is equal to the slope of the curve at the point of tangency. He is thereby led to conceive of the tangent as "a prolongation of an element of the curve," which is "to be considered as composed of an infinite succession of infinitesimal straight lines."

The remaining seven lectures are devoted to a purely geometrical treatment of problems of tangents and areas, not, as was the case in the work of most of Barrow's predecessors, for special curves, but for general classes of curves. One of the characteristics of Barrow's method, and a consequence of its geometrical character, is the constant use of what we now know in our textbooks as the "subtangent" and "polar subtangent," and it is somewhat of a surprise to find these good friends outside a textbook. So we have in Lecture VIII the theorem, that if $\varphi(x) = a/f(x)$, then the subtangents for points with the same abscissa on the curves $y = f(x)$ and $y = \varphi(x)$ are equal but opposite in sign. In Lecture IX it is shown that if $[\varphi(x)]^m = a^{m-n}[f(x)]^n$, then the subtangents for corresponding points on the curves $y = \varphi(x)$ and $y = f(x)$ are in the ratio $m : n$; i. e., Barrow derives a formula for the differentiation of a fractional power. Each theorem on curves in cartesian coordinates is followed by a corresponding one dealing with curves given in polar coordinates. There are given formulas for the differentiation of the sum, the product and the quotient of two functions, for the differentiation and integration of $\tan \theta$, of $\sqrt{x^2 + a^2}$, etc. Let us close by quoting two theorems, appearing in Lectures X and XI respectively and constituting together the essence of the fundamental theorem of the calculus: "Let ZGE be any curve of which the axis is AD; and let ordinates applied to this axis, AZ, PG, DE, continually increase from the initial ordinate AZ; also let AIF be a line such that, if any straight line EDF is drawn perpendicular to AD, cutting the curves in the points E, F, and AD in D, the rectangle contained by DF and a given length R is equal to the intercepted space $ADEZ$; also let $DE : DF = R : DT$, and join DT. Then TF will touch the curve AIF"; and "Again, let AMB be a curve of which the axis is AD and let BD be perpendicular to AD; also let KZL be another line such that, when any point M is taken in the curve AB, and through it are drawn MT a tangent to the curve AB, and MFZ parallel to DB, cutting KZ in Z and AD in F, and R is a line of given length, $TF : FM = R : FZ$. Then the space $ADLK$ is equal to the rectangle contained by R and DB."

ARNOLD DRESDEN.