Nearly a quarter of a century ago Professor Klein expressed the hope that he might sometime be able to treat the whole theory of elliptic functions from this point of view,* and the present text, when completed, may be regarded as in a sense a fulfillment of that hope.

The Jacobi functions, $\operatorname{sn} w$, $\operatorname{cn} w$, $\operatorname{dn} w$, now appear as elliptic functions of the second stage and the remainder of the volume is devoted to their properties, their relations to the Weierstrass functions, and to the properties of the allied theta functions.

Dr. Fricke's style is well known the world over and needs no comment. The text is free from errors, due credit being given by the author to Fräulein Dr. H. Petersen for careful proof-reading. The book appeared from the Teubner press "in spite of all the difficulties of the time, without unusual interruptions." In conclusion we may express the hope that nothing will interfere with the publication of the two remaining volumes.

L. WAYLAND DOWLING.

Problems in the Calculus, with Formulas and Suggestions. By DAVID D. LEIB, Ph.D. Boston, Ginn and Company, 1915. xii+224 pp. Price $1.00.

THE first impression conveyed by the title of this book, that it may be a collection of problems of the sort discussed by Professor Archibald in the BULLETIN of June, 1914, is corrected by the first few words of the preface. The book is said to be "the outgrowth of lists of problems prepared by the author to supplement the textbook," and in it we find "a supplementary list of workable problems on any topic ordinarily included in a general course in the calculus."

Preceding each set of exercises is a brief statement of the formulas and methods applicable thereto, with warnings against some common errors. In fact it seems to the reviewer that too careful a classification and too definite directions are given to permit, much less encourage, the development of initiative, versatility, and flexibility on the part of the student. When he has worked through a reasonable number of examples in his text-book, where properly enough most of them are carefully classified, the best additional preparation for the

* Evanston Colloquium (Aug. 28–Sept. 9, 1893). Lecture X.

unassorted difficulties he will meet in physics, mechanics, or elsewhere would seem to be furnished by examples in which the method of attack is left for him to discover. This is especially true in the case of formal integration; many students can " turn the crank " when they know what crank to turn, but are helpless when it comes to selecting a suitable method for themselves. This book of problems would be more helpful to them if the author had written each problem on formal integration for instance on a separate piece of paper and had secured the cooperation of a playful kitten in completing the arrangement.

The intentional omission of the answers to a large number of examples should assist the development of students, but one would hardly make that claim for the inaccuracy which is threatened by the author's statement that " no one has formally verified the answers." The reviewer has not done so in many cases, but would call attention to errors in parts (*b*), (*e*), (*f*), (*g*), (*i*) of example 3, exercise XCIX and in examples 3 and 15 in exercise CII. The last-named example is the approximate integration of

$$\int_0^{\frac{1}{2}\pi} \frac{dx}{1 + \cos x}.$$

In the first place, it hardly seems worth while to use approximate rules for more than one or two examples that can be done easily by direct integration; or at least it is not worth while to preserve many such problems and answers in a book. In the second place, it is hardly worth while to employ the methods of approximate integration unless it is done accurately enough to give better results than inspection, counting squares, or guessing in any other form. The answer given for this problem, 1.057 as compared with the correct 1, is surely beyond permissible limits of error. It apparently results from a hasty use of the trapezoidal rule, which inspection shows to be inexact in this case. The same unwise use of this rule is suggested in example 3 of the same exercise, to find by four ordinates the value of $\int_0^1 \sqrt{1 + x^4}\, dx$. If this is treated as an area, the slope of the curve at the upper point is $\sqrt{2}$, and the slope of the upper boundary of the last trapezoid is less than 1; obviously the trapezoidal rule will be inaccurate. The answer given, 1.07, seems to be the result of hasty work; accurate use of the trapezoidal rule yields 1.103, as compared with the exact value, 1.0894, obtained by the aid of Γ functions and verified by Simpson's rule with five ordinates. The error due to the trapezoidal rule is nominally only 1.3 per cent., but inspection shows the area to be a unit square surmounted by a small area, and for this latter even an accurate use of the rule suggested results in an error of about 15 per cent. Such approximate integration seems unsatisfactory to the reviewer, although it must be admitted that other books appear to sanction it.

The points open to criticism—the too orderly arrangement, inaccuracies in certain answers, and undesirable features in a few problems—are flaws which limit rather than destroy the usefulness of the book. It is certain that many teachers will find it convenient to put in the hands of students for supplementary work, and more will find it a very satisfactory source from which to draw, for classroom work or tests, problems which will be new to their students.

R. W. BURGESS.

Vom periodischen Dezimalbruch zur Zahlentheorie. By ALFRED LEMAN. Leipzig, Teubner, 1916. iv+59 pp. Price 80 Pfennige.

THE aim of this little book is to present the main properties of periodic decimals as material for a concrete introduction to the more elementary topics of the theory of numbers. The concept congruence is introduced on page 16 and Fermat's theorem is presented, illustrated and proved on pages 21–23, —each topic being a natural sequel to concrete questions and results concerning periodic decimals.

Periodic fractions to bases other than 10 are treated briefly in the final chapter 8, although on page 51 this part of the theory is said to be treated in chapters 8 and 9.

There is a list of about 15 papers, including most of the earliest ones. There is no mention of the early MS. by Leibniz; Henry Goodwyn's tables, 1816–23; the papers by Poselger, 1827; Bredow, 1834; Midy, 1836; Catalan, Thibault, and Sornin in *Nouvelles Ann. Math.*, 1842, 1843, 1846; Desmarest's text, 1852; Hudson's excellent paper in Oxford, *Cambridge and Dublin Messenger of Mathematics*, 1864, pages 1–6—not to cite various earlier papers and a hundred later ones. The topic is not exhaustively treated as claimed.