function it would be better to emphasize the notion of *correspondence* rather than dependence. Graphs of the elementary functions are given and a short treatment of fitting functions to empirical data is included. In many places the notion of a limit is used but nowhere is limit defined. The fundamental relations of increments, derivatives and differentials are well presented. We are glad to find theorems on infinitesimals which are needed in setting up problems and which are frequently neglected when the calculus is approached from the standpoint of limits.

The part on differential equations seems too short. Integration by partial fractions is omitted. An illustrative problem would have emphasized the importance of the constant of integration. Some illustrations of the use of the definite integral are needed to make plausible their statement that "The concept of the definite integral is the most useful concept in the application of the calculus."

The promised parts on "Functions of Two or More Variables," "Numerical Computation," and on "Elementary Dynamics" have not yet appeared so far as the reviewer is aware.

ERNEST B. LYTLE.

*Einführung in die höhere Mathematik.* By HANS VON MANGOLDT. Vol. I: *Anfangsgründe der Infinitesimalrechnung und der analytischen Geometrie.* 1911. xiv+477 pp. Vol. II: *Differentialrechnung.* 1912. xi+566 pp. Vol. III: *Integralrechnung.* 1914. x+485 pp. Leipzig, S. Hirzel.

THE volumes before us constitute an important contribution toward the solution of a problem which is of great concern to the majority of teachers of mathematics. That problem has to do with the amount and arrangement of the mathematical methods to be included in courses for students of physics and of engineering. We are of course agreed that there must be included the analytical geometry of two and of three dimensions, differential and integral calculus. But though we realize the deficiencies of the traditional college courses in these subjects, we are not well agreed as to the remedy. There are on the one hand those who claim that the student in question need only learn when and how certain formulas should be used, on the other hand those who would teach all mathematical courses merely as if they were an

end in themselves, with no regard at all to the future interests of the student. Between these extremes the advocates of the middle course have had a hard time. Many more or less successful attempts have been made to solve the problem by combinations of the old courses, eliminating some of the established chapters here and there, and introducing the fundamental notions of analytic geometry and the calculus at earlier stages, mainly with a view to saving time that had hitherto been employed either in solving problems in several different ways, or in the study of subjects which had no immediate application to physics and engineering. Most of the combinations which have come to the reviewer's attention seem to him to fall short in the lack of a central idea which should weld the whole course into an harmonious whole. The old courses have been mixed without being unified.

The book under review is centered on the differential and integral calculus. The first volume is introductory. Assuming the student acquainted with elementary geometry, trigonometry and algebra, the author first discusses certain chapters in advanced algebra, viz. permutations and combinations, the binomial theorem, probabilities, determinants, irrational numbers, this latter including a clear and sound exposition of Dedekind's theory. These matters occupy 177 pages. He then takes up the fundamental notions of analytic geometry, with emphasis on the idea of a function, concluding the volume with a full account of limits and continuity. The chapters on analytic geometry include both two and three dimensions, but are confined to problems on lines and planes, leaving all curved lines and surfaces to be treated in the succeeding volumes in connection with the calculus. The chapter on functions is unusually full and thoroughly modern, based on a brief, but clear, exposition of point-sets. In this chapter are included the interpolation formulas of Lagrange and Newton.

The second and third volumes are characterized by the same thoroughness of treatment and soundness of method that I have tried to indicate in this too short account of the first volume. In the second volume, the subject of infinite series receives a full algebraic treatment in a chapter of 75 pages in addition to a long chapter on Taylor's theorem. In the case of functions of several variables, the condition for validity of $\delta^2 f/\delta x \delta y = \delta^2 f/\delta y \delta x$ is adequately presented. The

volume closes with 87 pages on functions of a complex variable, including the fundamental theorem of algebra and conformal representation.

The third volume, on the integral calculus, includes such matters as Cauchy's theorem, the theorem of Gauss on transformation of a volume integral into a surface integral, Green's and Stokes's theorems, with a short discussion of vectors in connection with the latter. There is an admirable chapter on improper integrals, that is, on integrals of discontinuous functions and integrals with infinite limits. The final chapter, on differential equations, is confined to those equations which commonly occur in the applications, but contains a good treatment of the geometric interpretation of a differential equation and the existence proof.

The volumes contain very much more material than could possibly be included in any ordinary university course even in Germany, but at the same time there is scarcely anything which should not be of essential value to every student of physics or of engineering. As a supplementary hand-book, to which the teacher could refer for a sound and clear discussion of fundamental principles, it is all that could be desired, and is quite the best book of this kind, so far at least as the students referred to are concerned, which has come to the reviewer's attention.

M. W. HASKELL.

*Functions of a Complex Variable.* By E. J. TOWNSEND. New York, Henry Holt and Company, 1915. vii + 384 pp. 8vo. Price $4.00.

THERE has been a noticeable dearth of text-books in the English language on the elements of the theory of functions of a complex variable. The student with an easy command of German and French has found a rich and delightful literature, while his companion still in the period of language difficulties has had little opportunity to choose his reading in function theory in accordance with his individual tastes and requirements. But within the last three years there have appeared English translations of the classic works of Burkhardt and Goursat, a very full volume by Pierpoint, and the book we have before us for review.

The most obvious advantage of Townsend's treatment seems to be the absence of the synoptic character common to