(3) if O is a point of K and P is any point not belonging to K, then P can be joined to O by an arc having no point except O in common with K.

Every point set K that satisfies these conditions is an open curve.

- 12. If p is a prime of the form 4n+3, consider the number of quadratic residues included in the set  $1, 2, \dots, 2n+1$ . In the present note Mr. Vandiver proves a theorem which sets forth a connection between this number and the number of quadratic residues in any set defined by h + [ah] < p, where a is a fixed integer less than p-1 and h ranges over the set  $1, 2, \dots, p-1$ , the expression [ah] denoting the least positive residue of ah, modulo p. Analogous theorems are also found concerning the distribution of higher power residues.
  - 13. Consider the indeterminate congruence of Lagrange

$$(x-1)(x-2)\cdots(x-(p-1))\equiv x^{p-1}-1\pmod{p},$$

where x is an indeterminate and p is a prime integer. Mr. Vandiver obtains some generalizations of this relation such that the set  $1, 2, \dots, p-1$  modulo p is replaced by all the incongruent residues of a composite ideal modulus which are prime to the modulus. The paper will appear in the *Annals of Mathematics*.

F. N. Cole, Secretary.

## CORRECTION.

The following regretable errata in the reports of the summer meeting and colloquium of the Society, published in the November Bulletin, have been brought to the attention of the Secretary:

I. In the report of the summer meeting, page 65, it is stated that Professor C. N. Moore's paper appeared in full in the October Bulletin. A paper with the same title did appear in the October Bulletin, but it was read at the annual meeting held last January. The abstract of Professor Moore's summer meeting paper is printed below, with apologies to the author.

- 10. In Professor Moore's paper an example is given of a function continuous in an interval  $0 \le x \le 1$ , whose development in Bessel's functions is not summable  $(C\lambda)$  at the point x = 0, for any value of  $\lambda$  in the interval  $0 \le \lambda < \frac{1}{2}$ .
- II. In the report of the colloquium, pages 85–88, Professor Veblen's subject matter appears distributed in six "Lectures," whereas only five lectures were actually delivered by each author. The headings in Professor Veblen's synopsis represent certain divisions of the subject matter, not the division into lectures, and the word "Lecture" should have read "Section."

## THE MAXIMUM NUMBER OF CUSPS OF AN ALGE-BRAIC PLANE CURVE, AND ENUMERATION OF SELF-DUAL CURVES.

BY PROFESSOR M. W. HASKELL.

(Read before the San Francisco Section of the American Mathematical Society, October 24, 1914.)

It is well known that the double points of a rational algebraic curve can not in general all be cusps, and the maximum number of cusps has been determined in certain special cases. It is not difficult to find the maximum number from the consideration that none of the numbers in Plücker's equations can be negative.

Let m be the order and n the class, d the number of double points, k of cusps, i of inflexions and t of double tangents. We may first assume d = 0. In this case

$$2t = [m^2 - 9 - 3k][m^2 - 2m - 3k]$$

and the following inequalities must be satisfied:

$$(1) 3k < m(m-1),$$

$$(2) 8k \le 3m(m-2),$$

(3) 
$$2k \leq (m-1)(m-2),$$

(4) Either  $3k \le m^2 - 9$  and  $3k \le m^2 - 2m$  simultaneously or else