from the assumption of the existence of such an elementary series. There is, however, no ground for believing that either of these problems can be solved.*

Hence it has become evident that also the theorem of Bernstein, and with it the positive part of the theory of potencies, does not allow an intuitionistic interpretation.

So far my exposition of the fundamental issue, which divides the mathematical world. There are eminent scholars on both sides and the chance of reaching an agreement within a finite period is practically excluded. To speak with Poincaré: "Les hommes ne s'entendent pas, parce qu'ils ne parlent pas la même langue et qu'il y a des langues qui ne s'apprennent pas."

SHORTER NOTICES.

Essai de Géométrie analytique modulaire a deux Dimensions. By GABRIEL ARNOUX. Paris, Gauthier-Villars, 1911. xi + 159 pp.

WITH respect to a given prime m, modular space of two dimensions contains just m^2 distinct points (x, y), where $x, y = 0, 1, \cdots, m - 1$. One identifies (x_1, y_1) with (x, y) if $x_1 \equiv x$, $y_1 \equiv y \pmod{m}$. The distance of (x, y) from the origin is an integer if $x^2 + y^2$ is a quadratic residue of m, but is a Galois imaginary if $x^2 + y^2$ is a non-residue. If we join the origin to one of our points and take the sine or cosine of the angle made with the x-axis, we obtain either an integer or a Galois imaginary modulo m; but the tangent is always an integer modulo m.

The set of points (x, y), where x, y range over all sets of integral solutions of $F(x, y) \equiv 0 \pmod{m}$, is called the modular curve $F \equiv 0$. The book under review is devoted chiefly to functions F of the first or second degree, the methods being analogous on the whole to those of ordinary analytic geometry. Homogeneous coordinates are not used.

* Such belief could be based only on an appeal to the principium tertii exclusi, i. e., to the axiom of the existence of the "set of all mathematical properties," an axiom of far wider range even than the axioms of inclusion, quoted above. Compare in this connection Brouwer, "De onbetrouwbaarheid der logische principes," *Tijdschrift voor Wijsbegeerte,* 2e jaargang, pp. 152–158.

There is one disadvantage of this theory as compared to the ordinary analytic geometry. In the latter case the curve uniquely determines the (algebraic) function. But obviously we can find as many polynomials in x, y as we please, each passing through the same given points, finite in number. The real points on a modular curve $F \equiv 0$ do not therefore form an adequate picture of $F \equiv 0$. To this end and for the purpose of investigating intersections and all but the most trivial questions, we must introduce also the imaginary points of $F \equiv 0$, i. e., solutions of $F(x, y) \equiv 0 \pmod{m}$ in which x (and likewise y) is a root of any congruence modulo m with integral coefficients. The aggregate of the resulting infinitude of points gives an adequate representation of the function. If the author had recognized this point of view and had succeeded in materializing a suitable graphical representation of this infinitude of points, he would have made a substantial contribution to modular geometry. But in confining himself to real points, the author goes no further than earlier writers.* The author and his collaborators G. Tarry and Laisant are apparently not familiar with the history of Galois imaginaries, as there is no mention of Galois when such imaginaries (of the second order) are used and since a particular case of Galois' generalization of Fermat's theorem is attributed on page 148 to Tarry.

L. E. DICKSON.

La Logique déductive dans sa dernière Phase de Développement. Par ALESSANDRO PADOA. Paris, Gauthier-Villars, 1912. 106 pp.

THIS treatise is an adaptation of a course of lectures given by the author at Geneva, under the auspices of the university. The author had previously lectured on the subject in Brussels, Pavia, Rome, Padua, Cagliari, and presented memoirs before the congresses at Rome, Leghorn, Parma, Padua, and Bologna. The treatise contains an explanation, with abundant examples, of the symbols of logic as used in the Formulario Matematico, of Peano, some study of their properties, analysis of their relations, and their reduction to a minimum number. The author expresses his point of view very well in the following:

* Veblen and Bussey, "Finite projective geometries," *Trans. Amer. Math. Soc.*, vol. 7 (1906), p. 241. As the title shows, these authors were interested only in definite finite geometries and not in general modular geometry, so that the criticism of Arnoux's text does not apply to them.