SHORTER NOTICES.

First Course in Calculus. By E. J. Townsend, Professor of Mathematics, and G. A. Goodenough, Professor of Mechanical Engineering, University of Illinois. New York, Henry Holt and Company, 1908. xii + 466 pp.

Essentials of Calculus. By the same authors and publishers, 1910. xii+355 pp.

The larger of these books is intended as a text in calculus in courses given to engineering and college students in our stronger universities and technical schools. In preparing the smaller volume "the authors have had in view the needs of those colleges and technical schools in which the time devoted to calculus is limited to a three-hour course for a year."

These books are sufficiently alike to permit of a common characterization. They are perhaps unique among our texts on the calculus in that they are a collaboration by a pure mathematician and a mechanical engineer. The result is "that more attention is given to elementary applications to mechanics than is usual and perhaps less to geometry, it being the thought of the authors that the two should stand in about the same relative importance." However "the attempt has been made to select such problems and applications as arise in actual practice of an engineer without introducing technical difficulties beyond the experience of the average sophomore student who has had the usual course in high school physics. The book has not been written, however, solely from the point of view of the engineer. The applications are such as the general student will find both helpful and stimulating in showing the broad use of the calculus in practical problems."

On reading these volumes these claims of the authors seem to be well borne out. To a teacher who is asked many times each year "what is the use of all these theorems and processes?" such applications will be most welcome. The authors have avoided one of the commonest pitfalls, viz., the introduction of real applied problems which are too difficult in their essential character, or which are so long and bungling in statement as entirely to discourage the student and thus to

serve no good purpose whatever except in case of the most brilliant students.

This idea of exhibiting the calculus to the beginner in its many and varied uses has largely determined the character of the books. Thus integration is begun unusually early. "The book has not been divided into differential and integral calculus. The student is made familiar with integration as soon as he learns to differentiate, thus making it possible to introduce early a broader field of simple applications of the calculus."

In general the more difficult parts come late in the book. The arrangement is pedagogical rather than logical,—but not illogical. Thus series and expansion of functions begin on page 310 in the larger book and on page 291 in the smaller. Still later comes a general treatment of indeterminate forms, plane curves with such topics as order of contact, osculating circles, envelopes, evolutes, singular points, and a brief chapter on differential equations. Special methods of integration are scarcely touched. Envelopes and order of contact are entirely omitted in the smaller volume. Functions of two variables are treated more fully than usual on account of the many important applications.

The authors have set out to do a perfectly definite thing. What is uppermost in their minds is not the calculus as an abstract science with the traditional grouping of subjects and distribution of emphasis. It is rather such a science plus the young student of average caliber who meets it for the first time and whose interest it is sought to engage. The inclusion and exclusion of subject matter and the distribution of emphasis is determined rather by what are thought to be the needs and capacities of such students than by any a priori notion as to what logically belongs to a first course in calculus.

Effort is made to develop the theory in the form and terminology which is in constant use by those who are applying the calculus to practical ends. A good example of this is § 128 of the Essentials of Calculus, which deals with exact and inexact differentials. These are treated in a manner which falls directly into use in physics and thermodynamics.

Considered as a whole the authors have done a distinctive piece of work which is bound to influence teaching and the future editions of texts.

N. J. Lennes.