subgroups cannot involve two operators whose orders are powers of prime numbers and whose product has an order which is a power of another prime number. In particular, a solvable group cannot involve two such operators.

The second question to which we referred above is as follows: Is there a group which is not the product of some one of its possible sets of non-conjugate Sylow subgroups? It is well known that a necessary and sufficient condition that a group is the direct product of its Sylow subgroups is that we arrive at identity by forming the successive groups of inner isomorphisms, but no general criterion as regards whether a group is a product of a set of non-conjugate Sylow subgroups seems to have been found.

THE MATHEMATICS OF MAHĀVĪRĀCĀRYA.

The Ganita-Sāra-Sangraha of Mahāvīrācārya with English Translations and Notes. By M. RAṄGĀCĀRYA, M.A., Rao Bahadur, Professor of Sanskrit and Comparative Philology in the Presidency College, and Curator of the Government Oriental Manuscripts Library, Madras. Sanskrit text and English translation. Madras, Government Press, 1912. 27+325 pp.

IT was announced at the Fourth International Congress of Mathematicians, at Rome, in 1908, that Professor Raṅgācārya had for a number of years been engaged in the laborious task of translating a work of great importance in the history of mathematics, the Gaṇita-Sāra-Sangraha of Mahāvīr the Learned. Now, after four years more, the work has been brought to completion, and the mathematical world is the debtor to Professor Raṅgācārya for his arduous labor and to the Government Press for publishing the volume that is before us.

We have so long been accustomed to think of Pataliputra on the Ganges and of Ujjain over towards the western coast of India as the ancient habitats of Hindu mathematics, that we experience a kind of surprise at thinking that other centers equally important existed among the multitude of cities of that great empire. We have known for a century, thanks chiefly to the labors of such scholars as Colebrooke and Taylor, the works of Aryabhata, Brahmagupta, and Bhāskara, and have come to feel that to these men alone are due the noteworthy contributions to native Hindu mathematics. Of course a little reflection shows this conclusion to be an incorrect one. Other great schools, particularly of astronomy, did exist, and other scholars taught and wrote and added their quota, small or otherwise, to make up the sum total. It has, however, been a little discouraging that native scholars under the English supremacy have done so little to bring to light the ancient material known to exist, and to make it known to the Western world. This neglect has not been owing to the lack of material, for Sanskrit manuscripts are known, as are also Persian and Arabic and Chinese and Japanese, that are well worth translating, from the historical standpoint. It has rather been owing to the fact that it is hard to find a man like Professor Raṅgācārya, with the requisite scholarship, who could afford to give his time to what is necessarily a labor of love.

Mahāvīrācārya probably lived in the court of one of the old Rāshṭrakūta monarchs who ruled over what is now the kingdom of Mysore, and whose name is given as Amoghavarsha Nirpatuṅga. He ascended the throne in the first half of the ninth century A. D., so that we may roughly fix the date of the treatise in question as about 850, or between the dates of Brahmagupta and Bhāskara, though nearer to the former. There are four or five manuscripts of this author's work known, three of the oldest being in Madras. One of the Madras copies is written on paper in Grantha characters and contains the first five chapters. The other two are written on palm leaves in the Kanarese characters used in Mysore in Mahāvīrācārya's time. In all cases the language is Sanskrit. There is another manuscript in Kanarese characters at Mysore, and still another in a Jaina monastery at Mudbidri in South Canara. All of these have been used in making the translation, and all were necessary in the reading and in arriving at the meaning of many of the obscure passages.

In general it may be said that Mahāvīrācārya seems to have known the work of Brahmagupta. It would have been strange if this were not so, for the Brahmasphutasiddhānta was probably generally recognized in his time as a standard authority. Mahāvīrācārya seems to have made the effort to improve upon the work of his predecessor, and certainly did so in his classification of the operations, in the statement of rules, and in the nature and number of problems. As a result his work became well known in southern India, although there is no definite proof that Bhāskara, living in Ujjain, far to the north, was familiar with it. The work itself consists of nine chapters. The first is introductory, and contains seventy stanzas on terminology. It opens, as is usual in oriental treatises, with an invocation, in this case apparently to the author's patron deity: "Salutation to Mahāvira, the Lord of the Jinas, the protector (of the faithful), whose four infinite attributes, worthy to be esteemed in (all) the three worlds, are unsurpassable (in excellence). I bow to that highly-glorious Lord of the Jinas, by whom, as forming the shining lamp of the knowledge of numbers, the whole of the universe has been made to shine." This is followed by "An appreciation of the Science of Calculation" of which I venture to quote three stanzas: "In all those transactions which relate to worldly, Vedic, or (other) similar religious affairs, calculation is of use. In the science of love, in the science of wealth, in music and in the drama, in the art of cooking, and similarly in medicine and in things like the knowledge of architecture: in prosody, in poetics and poetry, in logic and grammar and such other things, and in relation to all that constitutes the peculiar value of (all) the (various) arts; the science of computation is held in high esteem." The terminology relates chiefly to the measures used, the names of the operations, numeration, and negatives and zero. Of the operations with numbers, eight are given, addition (except in series) and subtraction (even with fractions) being omitted as if presupposed. One interesting feature is the law relating to zero, which is stated thus: "A number multiplied by zero is zero, and that (number) remains unchanged when it is divided by, combined with, (or) diminished by zero." That is, the law known to Bhāskara, of dividing by zero, is not here recognized, division by zero being looked upon as of no effect. The law of multiplication by negatives is stated, and the imaginary number is thus disposed of: "As in the nature of things a negative (quantity) is not a square (quantity), it has therefore no square root."

The second chapter treats of arithmetical operations, the first being multiplication, and this being followed by division, squaring, square root, cubing, cube root, summation of series, in which is included some treatment of arithmetical and geometrical progressions, and Vyutkalita (that is, the summation of a series after a certain number of initial terms, *ista*, have been cut off).

Chapter III treats of fractions, following the same order as Chapter II. The most noteworthy feature is that relating to the inverted divisor, which is set forth as follows: "After making the denominator of the divisor its numerator (and vice-versa), the operation to be conducted then is as in the multiplication (of fractions)." It is curious that this device, which from another source we know to have been used in the East, became as it were a lost art until rediscovered in Europe in the sixteenth century.

Chapter IV consists of miscellaneous problems in fractions, which, however, include certain questions involving quadratic equations. For example: "One fourth of a herd of camels was seen in the forest; twice the square root (of that herd) had gone on to mountain slopes; and three times five camels (were), however, (found) to remain on the bank of a river. What is the (numerical) measure of that herd of camels?" This evidently requires the finding of the positive root of the equation $\frac{1}{4}x + 2\sqrt{x} + 15 = x$, or, in general, the solution of an equation of the type $x - (bx + c\sqrt{x} + a) = 0$, the rule for which is given. The chapter also contains various other types of equations involving some knowledge of radical quantities.

Chapter V relates to the rule of three, simple and compound, direct and inverse, with applications to interest, barter, and mensuration.

Chapter VI is entitled "Mixed Problems," and is interesting from the considerable use made of rules that would now be expressed in algebraic formulas, particularly with reference to the various computations of interest and to the solution of indeterminate equations.

Of the latter a single example may suffice to show the nature of the problems. "Into the bright and refreshing outskirts of a forest, which were full of numerous trees with their branches bent down with the weight of flowers and fruits, trees such as jambu trees, lime trees, plantains, areca palms, jack trees, date palms, hintala trees, palmyras, punnāga trees, and mango trees—(into the outskirts), the various quarters whereof were filled with the many sounds of crowds of parrots and cuckoos found near springs containing lotuses with bees roaming about them—(into such forest outskirts) a number of weary travellers entered with joy. (There were) sixty-three (numerically equal) heaps of plantain fruits put together and combined with seven (more) of those same fruits, and these were equally distributed among twenty-three travellers so as to have no remainder. You tell me now the numerical measure of a heap of plantains." Problems of this sort are solved by a process of calculation known as Vallikā-Kuttīkāra, a kind of division or distribution employing a creeper-like chain of figures, and the patience shown by Professor Raṅgācārya in interpreting the long and complicated rule will strike the reader of this work as worthy of the highest praise.

A complex kuttīkara, known as Sakala-kuttīkara, is also given, an example of which is as follows: "A certain quantity multiplied by six, then increased by ten, and then divided by nine, leaves no remainder. Similarly, a certain other quantity, multiplied by six, then diminished by ten, and then divided by nine, leaves no remainder. Tell me quickly what these two quantities are which are to be multiplied (by the given multiplier here)." The case has no new interest, however, since it resolves itself into two simple problems, $\frac{1}{9}(6x + 10) =$ integer, and $\frac{1}{9}(6x - 10) =$ integer, instead of the problem $(ax + by)/c =$ integer. Cases, however, of the type $ax + by + cz + dw = p$, $\Sigma a = n$, are given, and others involving several variables.

A single example will suffice to show their general nature: "Four merchants who had invested their money in common were asked, each separately, by the customs officer what the value of the commodities was, and indeed one eminent merchant among them, deducting his own investment, said that it was twenty-two; then another said that it was twenty-three; then another, twenty-four; and the fourth said that it was twenty-seven, each of them deducting his own invested amount. O friend, tell me separately the value of the commodity owned by each." This is of course determinate.

Chapter VII relates to the measurement of areas, and naturally reminds one of a similar chapter in Brahmagupta. It is, however, distinctly in advance of the latter. Mahāvīr makes the same mistake as Brahmagupta with respect to the formula for the area of a trapezoid, in that he does not limit it to a cyclic figure. The same error enters into his formula for the diagonal of a quadrilateral, which he gives as

$$\sqrt{\frac{(ac + bd)(ab + cd)}{ad + bc}} \quad \text{or} \quad \sqrt{\frac{(ac + bd)(ad + bc)}{ab + cd}}.$$

For π he uses $\sqrt{10}$, a common value all through the East and also in medieval Europe, although Aryabhaṭa* had long before this time given the approximation $62832/20000 = 3.1416$. Bhāskara had also given the latter value, in the reduced form $3927/1250$.

Chapter VIII relates to "calculations regarding excavations," a common title in India for the treatment of the mensuration of solids. The rule for the sphere is interesting. The approximate value is given as $\frac{9}{2}(d/2)^3$, and the accurate value as $\frac{9}{10} \cdot \frac{9}{2}(d/2)^3$, which means that π must be taken as $3.03\frac{2}{3}$, which is somewhere near $\sqrt{10}$.

Chapter IX relates to shadows, the primitive trigonometry of the gnomon.

Professor Raṅgācārya has added to the value of the work by an extensive appendix in which he gives the Sanskrit numeral words; the Sanskrit words used in the translation, with an explanation of their meaning,—a most helpful list; the answers to all of the problems; and the tables of measures used in the work.

Such is a brief outline of the work. It is sufficient, however, to show that we shall have, in Professor Raṅgācārya's labors, the most noteworthy single contribution to the history of Hindu mathematics that has been made for nearly a century. What light it will throw upon the relation of Bhāskara's Lilāvati to works of his predecessors, upon the relation of the schools of Pataliputra and Ujjain to each other and to that of Mysore, upon the knowledge of Greek mathematics in the East, and upon the state of algebra in India at about the time that Al-Khowārazmi was writing his Al-jebr w'al-muqābala in Baghdad, it is impossible as yet to say.

DAVID EUGENE SMITH.

* Mr. Kaye thinks a later mathematician of the same name.