this quotient group. This implies that this quotient group is abelian and of type (2, 2) when $p = 2$, and when $p > 2$ it must contain at least p invariant cyclic subgroups of order p^2. As this is contrary to the fact that G contains $p + 1$ conjugate sets which involve generating operators of its maximal cyclic subgroups, we have proved that we arrive at an absurdity by assuming that G does not involve any operator of order p^{m-1}, $m > 3$.

When $p > 2$ there are only two non-cyclic groups of order p^m which involve operators of order p^{m-1}, $m > 3$, and each of these clearly contains maximal cyclic subgroups of order p^α which are transformed into themselves by more than $p^{\alpha+1}$ operators of G. Hence it results that the three non-cyclic groups of order 2^m which were considered above in the second paragraph are the only non-cyclic groups of order p^m in which every maximal cyclic subgroup is transformed into itself by at most p times as many operators of the group as there are operators in this maximal subgroup. This completes the proof of the theorem in question, and hence we can assume that *every non-cyclic group of order p^m, with the exception of the three of order 2^m which involve one and only one cyclic subgroup of order 2^{m-1}, contains at least one maximal cyclic subgroup of order p^α which is transformed into itself by more than $p^{\alpha+1}$ operators of the group.*

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AN EXPRESSION FOR THE GENERAL TERM OF A RECURRING SERIES.

BY PROFESSOR TSURUICHI HAYASHI.

PROFESSOR Arthur Ranum has given in the BULLETIN, volume 17, No. 9, June, 1911, pages 457–461, an explicit form of the general term of a recurring series rationally in terms of the first few terms and the constants of the scale of relation. I will give here another more explicit and more convenient form without demonstration.

Let $u_0 + u_1 + u_2 + \cdots + u_n + \cdots$ be any recurring series of order n, and let

$$u_m = a_1u_{m-1} + a_2u_{m-2} + \cdots + a_nu_{m-n} \qquad (m \geqq n)$$

be its scale of relation. Then for any value of m not less than n,

$$u_m = \sum_{\nu=0}^{\nu=n-1} u_\nu A_\nu,$$

where

$$A_\nu = \sum a_1^{\alpha_1} a_2^{\alpha_2} \cdots a_n^{\alpha_n} \cdot \frac{(\alpha_1+\alpha_2+\cdots+\alpha_n-1)!}{\alpha_1!\,\alpha_2!\cdots\alpha_n!} \cdot (\alpha_n+\alpha_{n-1}+\cdots+\alpha_{n-\nu}),$$

the summation extending over all the positive integral and zero values of $\alpha_1, \alpha_2, \cdots, \alpha_n$, for which

$$\alpha_1 + 2\alpha_2 + 3\alpha_3 + \cdots + n\alpha_n = m - \nu.$$

Since the formula is true when $m = n$, and assuming it to be true for the values $n + l$, $n + l - 1, \cdots, n+1, n$ of m, we can prove it by mathematical induction to be true for the next value $n + l + 1$, without much difficulty, unless $l < n$. Even for the case $l<n$, a slight attention leads us to the same result.

More generally, it may also be shown that for any value of m greater than λ

$$u_m = \sum_{\nu=\lambda-n+1}^{\nu=\lambda} u_\nu A_\nu,$$

where

$$A_\nu = \sum a_1^{\alpha_1} a_2^{\alpha_2} \cdots a_n^{\alpha_n} \cdot \frac{(\alpha_1+\alpha_2+\cdots+\alpha_n-1)!}{\alpha_1!\,\alpha_2!\cdots\alpha_n!} \cdot (\alpha_n+\alpha_{n-1}+\cdots+\alpha_{\lambda-\nu+1}),$$

the summation extending over all the positive integral and zero values of $\alpha_1, \alpha_2, \cdots, \alpha_n$ for which

$$\alpha_1 + 2\alpha_2 + 3\alpha_3 + \cdots + n\alpha_n = m - \nu.$$

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