$$s_1 = abcdefgh \cdot ijlmnop, \quad s_2 = aiem \cdot cogk.$$

When s_1 is of order 8, s_2 must be of order 4 since s_2^2 is not commutative with s_1. When s_1 is of order 4, the order of s_2 is either 4, 2, or 1, as may be readily seen from the following substitutions:

$$s_1 = aecg, \quad s_2 = abcd \cdot efgh\,; \quad s_1 = abcd, \quad s_2 = ac.$$

Finally, when s_1 is of order 2, the order of s_2 is evidently 2 or 1. Hence the theorem: *If two non-commutative operators satisfy the relations* $s_1^{-2}s_2s_1^2 = s_2^3$, $s_2^{-2}s_1s_2^2 = s_1^5$, *their orders are one of the following pairs of numbers:* 8, 4; 4, 4; 4, 2; 2, 2.

When s_1 is of order 8, H is abelian and of order 8. From the following equations it results that s_1^2 is transformed into its inverse by $(s_1s_2)^2$:

$$(s_1s_2)^{-1}s_1^2s_1s_2 = s_2^{-1}s_1^2s_2 = s_1^2 \cdot s_1^{-2}s_2^{-1}s_1^2 \cdot s_2 = s_1^2s^2,$$

$$(s_1s_2)^{-2}s_1^2(s_1s_2)^2 = s_2^{-1}s_1s_2^2s_1s_2 = s_2 \cdot s_2^{-2}s_1s_2^2 \cdot s_1s_2 = s_2s_1^{-2}s_2 = s_1^{-2}.$$

Hence the order of G is a multiple of $8 \cdot 4 \cdot 2 = 64$ whenever s_1 is of order 8. That the order of G may be exactly 64 results directly from the given substitutions, as they generate an imprimitive group of degree 16 and order 64. From the properties of the dihedral group it results that s_1, s_2 may be so selected that the order of G is an arbitrary multiple of 64 and that the third derived of each one of these groups is identity. The categories of groups which result when the orders of s_1, s_2 have the other possible sets of values are still more elementary and their fundamental properties are easily derived from the general theorems of the preceding section.

THE SOLUTION OF AN INTEGRAL EQUATION OCCURRING IN THE THEORY OF RADIATION.

BY PROFESSOR W. H. JACKSON.

(Read before the American Mathematical Society, December 30, 1909.)

PROFESSOR Arthur Schuster * has discussed the propagation of heat by radiation when the isothermal surfaces are

* "The influence of radiation on the transmission of heat." *Phil. Magazine*, Feb., 1903.

parallel planes and the radiation is homogeneous and everywhere normal to the isothermal surfaces.

If either of these two last restrictions is discarded, an equation is obtained of the form

$$E - \frac{1}{[k(\kappa)]^2}\frac{\partial^2 E}{\partial m^2} = \int_0^1 E d\kappa.$$

By a series of substitutions, the solution of this equation can be led back to the solution of an ordinary integral equation of the second kind with symmetric kernel. Each step is directly suggested by physical conceptions. Write

$$A = E - \frac{1}{k}\frac{\partial E}{\partial m}, \quad B = E + \frac{1}{k}\frac{\partial E}{\partial m},$$

whence

$$2E = A + B.$$

Put further

$$\theta(m) = \int_0^1 E d\kappa,$$

so that

$$A + \frac{1}{k}\frac{\partial A}{\partial m} = B - \frac{1}{k}\frac{\partial B}{\partial m} = \theta(m).$$

By the ordinary device of an integrating factor $e^{\pm km}$ these equations lead to

$$A = A_0(\kappa)e^{-km} + k\int_0^m e^{-k(m-\xi)}\theta(\xi)d\xi,$$

$$B = B_0(\kappa)e^{-k(m'-m)} + k\int_m^{m'} e^{-k(\xi-m)}\theta(\xi)d\xi,$$

where $0 < m < m'$ and A_0, B_0 are arbitrary functions of κ.

By addition, putting

$$2E_0 = A_0(\kappa)e^{-km} + B_0(\kappa)e^{-k(m'-m)},$$

we have

$$2E = 2E_0 + k\int_0^{m'} e^{-k|\xi-m|}\theta(\xi)d\xi.$$

Finally, integrating from 0 to 1 with respect to κ and writing

$$2K(x) = \int_0^1 ke^{-k|x|}d\kappa, \quad \theta_0(m) = \int_0^1 E_0 d\kappa,$$

we have

$$\theta(m) = \theta_0(m) + \int_0^{m'} K(\xi - m)\theta(\xi)d\xi.$$

The auxiliary function θ is the temperature and the obvious physical mode of solution is Liouville's method of successive substitutions.*

A case of special interest physically is that in which k is defined by

$$k\kappa = 1.$$

Is there any method of numerical computation better than approximate integration?

HAVERFORD COLLEGE,
March, 1910.

GRASSMANN'S PROJECTIVE GEOMETRY.

Projektive Geometrie der Ebene unter Benutzung der Punktrechnung dargestellt. Von HERMANN GRASSMANN. Erster Band: *Binäres.* B. G. Teubner, 1909. 8vo. xii + 360 pp.

MODERN projective geometry is two-sided. Either use is made of algebraic analysis in its development or it is developed from the fundamental concepts of point, line, plane by means of certain axioms and postulates. In the one case it is analytic, in the other synthetic. Usually the two methods of presentation are more or less combined, with the emphasis laid upon the one or the other. If the analytic method is adopted, operations are usually carried out in cartesian space with the aid of a system of coordinates. The synthetic method makes no use o coordinate systems.

Professor Grassmann's work is analytic in character in that use is made of algebraic analysis. It is unique in discarding the usual coordinate systems and adopting ideas due to Möbius and to the elder Grassmann. These ideas found expression in the Baryzentrische Calcul and in the Ausdehnungslehre.

In the last quarter century a number of writers have made use of these ideas; notably, Stéphanos, H. Wiener, Segre, Peano, Aschieri, Study, Burali-Forti. It is the author's purpose to bring the results of these writers and of others together into a connected course covering the fields of binary and ternary linear transformations. This is certainly a most worthy purpose and mathematicians will be grateful to the author for the evident care and devotion with which he has set about the performance of his task.

* Maxime Bôcher, An introduction to the study of integral equations. Cambridge, Eng., 1909.