Introduction to the Theory of Fourier's Series and Integrals and the Mathematical Theory of the Conduction of Heat. By H. S. CARSLAW. London, Macmillan and Company, 1906. 8vo. 17 + 434 pp.

THIS book is an attempt to place the mathematical theory of heat on a rigorous basis. On the one hand there are in English many books dealing with such pure mathematical subjects as the theory of infinite series, the theory of definite integrals, and the theory of functions of a real variable, which give rigorous developments of their subjects, and on the other there are books on applied mathematical subjects which in many cases take as axiomatic many of the pure mathematical theorems on which the applied subject is based. Mr. Carslaw is to be congratulated on the success of his endeavor to deal rigorously with the theory of heat. His book divides naturally into two parts. The first deals with such matters as the theory of series and of definite integrals in general, and in particular with the Fourier series and integrals. The second part consists of an application of this machinery to the standard problems of the conduction of heat.

In the chapters on theory of numbers and series attention should be drawn to the clearness of explanation of Dedekind's axiom and of the sum of a series. The student usually finds so much difficulty in understanding what is meant by the sum of an infinite series, that it is worth while to repeat the statement given by Carslaw and due to Baker.

"When we speak of the sum of an infinite series $u_1(x) + u_2(x) + u_3(x) + \cdots$ it is to be understood (1) that we settle for what value of x we wish the sum of the series; (2) that we insert this value of x in the different terms of the series; (3) that we find the sum $S_n(x)$ of the first n terms of the series; and (4) that we find the value of the limit of this sum as n increases indefinitely, keeping x all the time at the value settled upon.

"With this understanding, the series is said to be convergent for the value x, and to have $f(x)$ for its sum, when, this value of x having first been inserted in the different terms of the series, and any positive quantity ϵ having been chosen as small as we please, there exists a finite integer ν such that $|f(x) - S_n(x)| < \epsilon$ for $n > \nu$."

The author gives a good account of uniform convergence. He states and gives a proof of the statement that uniform convergence implies continuity. This is of course true if the terms of the series are all continuous, and the author makes this assumption. It is clear however that a series may satisfy the condition of uniform convergence and yet have a finite number of terms which are not continuous in the interval considered, provided that each of these terms has a definite value at every point in the interval, and in this case the continuity breaks down. Since the author aims so definitely at completeness, he might have included in this chapter (Series) the definition of continuity.

Chapter IV contains a treatment of the theory of infinite integrals. It is noteworthy that though until a year ago there was no detailed treatment of this subject in English, there have appeared almost simultaneously three books which contain full accounts of it, those of Pierpont, Carslaw, and Bromwich.

Chapters V and VI deal with Fourier's series. The discussion of convergence of these series is admirably illustrated by a series of graphs, which give the successive curves of approximation as one, two, three, etc., of the terms of some particular series are taken. These graphs illustrate very well the way in which discontinuities arise in a series of continuous functions. It is interesting to note that as n increases indefinitely the curve $y = S_n(x)$ does not tend precisely to the discontinuous curve $y = S(x)$ and the vertical straight lines joining the points of discontinuity. The vertical lines in fact extend through and beyond the points of discontinuity. This was first pointed out by Gibbs, and has been generalized for Fourier series by Bôcher.

The second part of the book, after a chapter on the general theory, deals with particular classical problems, such as Fourier's ring, the straight rod, the sphere, etc.

The book is not free from errors which should not have escaped the proof reader. For example, the graph on page 59 for the curve $y = nx/(1 + n^2x^2)$ is stated on the figure itself to be drawn for the two cases $n = 10$, $n = 100$. Beneath the figure these numbers are said to be $n = 5$, $n = 50$, whilst in the text they are said to be 5 and 100. In a second edition it might be well to express differently such phrases as "the rapidity of the convergence becomes slower and slower" (page 55), and "By choosing q large enough we may make this as small as we please for that and all greater values of q" (page 182).

The book contains a number of exercises and concludes with a full bibliography and a fairly good index.

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