SHORTER NOTICE.

Lehrbuch der Differenzenrechnung. Von D. SELIWANOFF. Leipzig, B. G. Teubner, 1903. 92 pp.

AT the request of the well-known Leipzig publisher, B. G. Teubner, the author has elaborated his article on finite differences in the Encyklopädie der Mathematischen Wissenschaften, volume 1, pages 918–937, to the dimensions of a book. Thus, while the encyclopædia gives but one page to the approximate evaluation of definite integrals and four pages to the subject of difference equations, the book devotes six pages to the former and twenty-nine pages to the latter. The gain in lucidity over the encyclopædia article is therefore considerable, and, indeed, although the author omits certain questions which might well be taken up, those that he does treat are presented in a delightfully clear and simple manner.

The books of Boole and Markoff are more complete, but this work of Seliwanoff should be regarded, not as a handbook for one who is familiar with the subject, but as a textbook for the beginner who desires to learn the theory and technique of computation, such as the methods of interpolation, constructing tables, estimation of unavoidable errors.

Part I devotes thirty-two pages to the subject of differences. After developing some of the most important general theorems in chapter 1, the question of interpolation is taken up in chapter 2, where the author considers exact and approximate interpolation, computation of the roots of numerical equations, and the computation of logarithms and antilogarithms. The methods of evaluation of definite integrals in chapter 3 are all elementary, culminating with Simpson's formula.

In Part II, chapter 1 treats the subject of indefinite and definite summation. According to the conventional usage, the symbol $\sum$ is employed as the calculus analogue of the sign of integration $\int$. In the opinion of the reviewer the symbol S would serve the purpose somewhat better. First, the use of the letter $\sum$ as a functional symbol where the (finite) integration is not possible is a departure from its recognized significance in various domains of analysis and consequently a little confusing; second, the letter S corresponds closer to the long s of the integral calculus; and third, the $\sum$ would remain available to indicate a summation of integrals. Chapter 2 develops in an elementary manner the Jakob Bernoulli function

$$\phi_n(x) = \sum_0^x \frac{x^{n-1}}{(n-1)!}$$

and some of its properties. Chapters 3, 4 are devoted to the Euler formula

$$F(a) = \frac{1}{h}\int_a^{a+h} F(u)du + A_1[F(a+h) - F(a)] + A_2 h^2 F''(a + \theta h)$$

and to some of its important applications, such as the development of Stirling's formula

$$\log(1 \cdot 2 \cdot 3 \cdots x) = \log \sqrt{2\pi} + (x + \tfrac{1}{2}) \log x - x - A_2 \frac{1}{x} + A_4 \frac{2!}{x^3} + \cdots.$$

Part III takes up the subject of difference equations. In chapter 1 the general equation $y_{x+n} = f(x, y_x, y_{x+1}, \cdots, y_{x+n-1})$ and the linear equation $y_{x+n} + p_1 y_{x+n-1} + \cdots + p_n y_x = Q_x$ are discussed very briefly. The second chapter is devoted to the linear equation of the first order and to the interesting application of expanding $\cos xt$ according to powers of $\cos t$. Chapter 3 treats in somewhat greater detail the linear difference equation with constant coefficients.

Although many questions of interest from the theoretical standpoint as well as from that of the higher applications are not touched upon, we are indebted to the author for an excellent elementary textbook on a fascinating and important subject.

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