substitutions have the same composition formula as linear fractional substitutions. Hence, according as -1 is a square or a not-square, H' is simply isomorphic to the "real" or the "imaginary" form* of the group of linear fractional substitutions of determinant unity. Thus, for $p^n > 3$, H' is simple.

15. Observing that the squares of the substitutions

$$O_{1,2}^{\alpha,\beta}, \quad O_{1,2}^{\alpha,\beta}\, T_{13} C_1 C_2 C_3, \quad O_{1,2}^{\alpha,\beta}\, T_{13} T_{24}$$

are respectively $Q_{1,2}^{\alpha,-\beta}$, $O_{1,2}^{\alpha,\beta}\, O_{3,2}^{\alpha,\beta}$, $O_{1,2}^{\alpha,\beta}\, O_{3,4}^{\alpha,\beta}$, we may unite our results into the following

THEOREM: *The squares of the linear substitutions on m indices in the $GF[p^n]$, $p \neq 2$, which leave invariant the sum of the squares of the m indices, generate a group, which for $m = 2k+1$ has the order*

$$\tfrac{1}{2}(p^{2nk} - 1)\, p^{2nk-n}\, (p^{2nk-2n} - 1)\, p^{2nk-3n} \cdots (p^{2n} - 1)\, p^n$$

and is simple except when $p^n = 3$, $m = 3$; while for $m = 2k > 4$ it has the factors of composition 2 and

$$\tfrac{1}{4}[p^{nk} - (\pm 1)^k]\, p^{nk-n}\, (p^{2nk-2n} - 1)\, p^{2nk-3n} \cdots (p^{2n} - 1)\, p^n,$$

the sign $\pm$ depending upon the form $4l \pm 1$ of p^n.

UNIVERSITY OF CALIFORNIA,
February 10, 1898.

A PROOF OF THE THEOREM: $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$

BY MR. J. K. WHITTEMORE.

(Read before the American Mathematical Society at the Meeting of April 30, 1898.)

THEOREM: *Let $u = f(x, y)$ denote a function of the two independent variables x and y which, together with its first derivatives and the two second derivatives in question, is continuous in the neighborhood of the point (x, y); then* $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$

Let $\dfrac{\partial^2 f(x, y)}{\partial x \partial y}$ denote $\dfrac{\partial}{\partial x}\left(\dfrac{\partial f(x, y)}{\partial y}\right)$

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*Moore: Mathematical Papers of the Chicago Congress (1893), "A doubly-infinite system of simple groups," §§ 5–6.

and let $\frac{\partial^2 f(x, y)}{\partial y \partial x}$ denote $\frac{\partial}{\partial y}\left(\frac{\partial f(x, y)}{\partial x}\right)$.

Let (x_0, y_0) be any point for which the conditions of the theorem are fulfilled and let the lines $x = a$, $x = b$, $y = c$, $y = d$ bound a region of the plane enclosing the point (x_0, y_0) and so small that the conditions stated are satisfied throughout the interior of the rectangle and on its boundary. Under these conditions we have

$$\int_c^d dy \int_a^b dx \frac{\partial^2 f(x, y)}{\partial x \partial y} = f(b, d) - f(b, c) - f(a, d) + f(a, c),$$

$$\int_a^b dx \int_c^d dy \frac{\partial^2 f(x, y)}{\partial y \partial x} = f(b, d) - f(a, d) - f(b, c) + f(a, c).$$

But, under the conditions of the theorem,

$$\int_a^b dx \int_c^d dy \frac{\partial^2 f(x, y)}{\partial y \partial x} = \int_c^d dy \int_a^b dx \frac{\partial^2 f(x, y)}{\partial y \partial x}.$$

Hence $$\int_c^d dy \int_a^b dx \left(\frac{\partial^2 f(x, y)}{\partial x \partial y} - \frac{\partial^2 f(x, y)}{\partial y \partial x}\right) = 0.$$

Now, if a function, continuous in the neighborhood of a point (x_0, y_0), is such that its integral, extended over *any* rectangle enclosing this point , is zero, it is readily seen that the function cannot be positive or negative at the point (x_0, y_0). Hence

$$\frac{\partial^2 f(x, y)}{\partial x \partial y} - \frac{\partial^2 f(x, y)}{\partial y \partial x} = 0$$

at the point (x_0, y_0). But this was any point, and the theorem is proved.

SOME OBSERVATIONS ON THE MODERN THEORY OF POINT GROUPS.

BY MISS FRANCES HARDCASTLE.

THE origins of the theory of point groups are to be found in Brill and Noether's classic memoir (see infra) published nearly twenty-five years ago, but it is only within the last fifteen years that systematic attention has been given to the subject by the Italian mathematicians, Segre, Bertini,