

gathers in its meshes all types of subgroups and lets none escape. My list of groups should be verified or corrected by the application of Lie's methods.

I append here a list of the group types discussed in the foregoing pages with a brief characterization of each.

- (1) The six-parameter group G_6 of all circular transformations.
- (2) The four-parameter group G_4 leaving a single point invariant.
- (3) The two-parameter group G_2 of type T leaving a pair of points invariant.
- (4) The two-parameter group G_2' of type T' leaving a single point invariant.
- (5) The one-parameter group G_{1c} of type T and constant c in $k = e^{(c+i)\theta}$ leaving two points invariant. (a) The one-parameter group of elliptic transformations for which $c = 0$. (b) The one-parameter group of hyperbolic transformations for which $c = \infty$ and $\theta = 0$.
- (6) The one-parameter parabolic group G_1' with constant θ in $a = re^{i\theta}$ leaving a single point invariant.
- (7) The three-parameter group G_{3c} of type T and constant c in $k = e^{(c+i)\theta}$ leaving a single point invariant.
- (8) The three-parameter group G_c of elliptic, hyperbolic and parabolic transformations leaving a circle invariant.
- (9) The two-parameter group G_{2c} of hyperbolic transformations leaving invariant a circle and a point on it.
- (10) The three-parameter group of elliptic transformations G_{1c} .

The real projective transformations of the plane and of space may be treated in the same spirit and by the same methods here employed for the circular transformations. The writer hopes to be able in the near future to publish the full results of his investigations in these fields.

UNIVERSITY OF KANSAS,
March 6, 1897.

PLÜCKER'S COLLECTED PAPERS.

Julius Plücker's gesammelte mathematische Abhandlungen. Herausgegeben von A. SCHOENFLIES. Mit einem Bildniss Plücker's, und 73 in den text gedruckten Figuren. Leipzig, Druck und Verlag von B. G. Teubner. 1895. 8vo, pp. xxxvi + 620.

The Kgl. Gesellschaft der Wissenschaften zu Göttingen, of which Plücker was a corresponding member, recently under-

took the publication of his scientific papers, and appointed as editors Professors Schoenflies and Pockels. The volume whose separate title is given above is the first of the two which together form this collected edition. The only point in the arrangement that could be wished changed is the natural decision, referred to in the preface, not to reprint any of Plücker's longer works, inasmuch as they are still to be bought in the original editions. But interesting as these original editions may be, they are not satisfactory, for type and paper alike leave much to be desired; it is hardly possible, for instance, to read the *Theorie der algebraischen Curven* with any enjoyment; and so it is perhaps permissible to express the hope that at some future time, not too far distant, these also may appear in as readable a form as the *Collected Papers*.

This volume contains the thirty-nine memoirs by Plücker, and also Clebsch's "Gedächtnissrede", from the 16th volume* of the *Göttinger Abhandlungen*, a paper whose intrinsic value as the account given by one great geometer of the life-work of another can only increase with the lapse of time. It comes in here with special appropriateness, supplying the links between Plücker's work and that of his contemporaries, putting it into organic connection with the rest of the mathematical work of the period. This is a rôle for which Clebsch seems to have been peculiarly adapted by temperament, as witness one of his pupils, "von ihm lernten wir Anderen die Tendenz, auch fremde Untersuchungen umfassend in Betracht zu ziehen und mit den eigenen zu verweben"; and in the case of Plücker this sympathetic interpretation was specially needed, for his work produces an impression of aloofness—there is very little apparent consciousness of what the rest of the world might be doing. This intellectual isolation makes it easy to trace the workings of his mind all through the years in which he devoted himself to pure mathematics; we can see him generalizing his conceptions, as a preparation for a new stage in his work, making a clearance of unused materials after the pub-

* In the preface the reference for this notice is given as vol. 15, 1872; as a matter of fact it was read on the 2d of December, 1871, and is to be found in vol. 16, printed in 1872, but nevertheless the 1871 volume. The mistake is evidently due to the misprint on the title page of the reprint of the "Gedächtnissrede", and is to be found in various works, *e. g.*, in the text of the notice of Clebsch in vol. 7 of the *Mathematische Annalen*—though the correct reference is given in the bibliographical table at the end of this same article—and also on p. 35 of the second edition of Professor Gino Loria's "Il passato ed il presente delle principali teorie geometriche."

lication of a book, reaching out after a new geometry of space from the very first. He was not an erudite mathematician, for his mathematics came from within; he followed out his own train of thought, undistracted by the pressure of any ideas but his own, driven on by the "aufdrängende Fluth" of these, to which of deliberate purpose he yielded himself. A great part of the interest of the memoirs is in the view they give us of Plücker, possessed in turn by his various ideas, abridged notation, homogeneous coördinates, etc. This self-containedness early plunged him into difficulties in the matter of "plagiarism"; the first paper that he published after his Marburg dissertation, "Théorèmes et problèmes sur les contacts des sections coniques", *Annales de Mathématiques* (Montpellier), 1826, provoked a savage attack from Poncelet, due mainly to a misunderstanding arising from Gergonne's editorial alterations in and additions to the original memoir. These not unnaturally led Poncelet to believe that Plücker had worked by the unacknowledged light of his own *Propriétés projectives*, published in 1822; this misapprehension accounts for his irritation, though it does not justify the expression he gave to it. This protest, and Plücker's temperate rejoinder,* are here given by the editor, and with good reason, for apart from their historical interest they help us to understand Plücker's attitude towards his contemporaries. The accusation of plagiarism might, with a little ingenuity, be brought against many creative mathematicians; when the same field is occupied simultaneously by several, even though they be of different nationalities, and totally different intellectual bias and equipment, coincidences of results are certain to occur, the more frequently in proportion to the actual independence of the workers; for the plagiarist of ideas does not openly label his work by plagiarizing results also. Different as was Plücker's line of thought from that of his contemporaries, he evidently determined to be on the safe side; so he inserted, even impatiently, references to Poncelet wherever he detected any likeness in the results.

But the march of his ideas is his best defence. Every one of his books develops some particular conception, original with him, even though it may have occurred also to his

*The editor of the *Bulletin des Sciences de Férussac*, in vol. 9 of which Plücker's reply appeared, printed it with the remark that he had suppressed "quelques épithètes inutiles au succès de la discussion," so possibly the rejoinder was not really so temperate as it now appears.

Plücker refers to the matter again in the preface to the second volume of the *Entwicklungen*, p. vi.

contemporaries; and the first germs and applications of each conception are to be found in the memoirs immediately preceding the issue of the volume. On account of this intimate connection between the memoirs and the longer works, one is inclined to wish that the names of the successive volumes had been inserted in their proper places in the table of contents, and that all through this table the dates had been supplied.

In memoirs Nos. 4 and 5, 1827, 1828, abridged notation is used; to the explanation of this, and its application to conics, vol. 1 of the *Analytisch-geometrische Entwicklungen*, 1828, is devoted. The 5th paper, "Recherches sur les courbes algébriques de tous les degrés", is fundamental in the theory of the intersections of curves; in the 6th the argument is applied to surfaces; Nos. 7 and 8 also relate to solid geometry. Nos. 9, 10 and 11 deal respectively with homogeneous coördinates, abridged notation, and line coördinates, all of which are expounded in vol. 2 of the *Entwicklungen*, 1831; in the closing words of the preface we see Plücker already under the influence of new ideas, destined to find their expression in his next volume, the *System der analytischen Geometrie*, 1835. Of the intervening papers, No. 12 contains a number of enunciations of theorems and problems; Nos. 13-19 seem to have been an emptying of his note-books, whose miscellaneous contents he cast into the form of "Analytisch-geometrische Aphorismen"; Nos. 20 and 21 treat of the foci of curves, and of the relations connecting the order and class of a curve with the numbers of its point and line singularities. These two were written shortly after the time, the summer of 1832, at which, by Plücker's own account, the principal ideas of the *System* came into his mind; the kernel of the second of these papers is the precise description of the two-fold generation of a curve. In the preface to the *System*, after referring to his "Vorliebe" for line coördinates, he insists at some length on this dualistic view, and its expression by means of point and line coördinates, and points out that the passage from one mode of representation to the other involves a discontinuity. This statement is of course exact in the very special sense in which it was first intended, as referring to any singularity on the curve, and it is a part of Plücker's renown to have discriminated so clearly between point singularities and line singularities. But though the theories of point and line coördinates depend on two different conventions as to the nature of a geometrical figure—conventions which primarily are mutually exclusive, whose

combination appears at first sight to result in arguing in a circle—yet it seems an unnecessary restriction on the generality and elasticity of the symbolism to employ exclusively one set of coördinates or the other in any given investigation. The statement in the first chapter of Salmon's *Higher Plane Curves* (Cayley) "there is little occasion for any explicit use of line coördinates" seems distinctly unfortunate; for the full advantage of the duality between point and line coördinates is obtained only when we hold ourselves at liberty to use both systems at once, representing points and lines indifferently by coördinates or equations. The objection that this is using the conventions in an unjustifiable manner, relying on two different ones simultaneously, loses its force when we reflect that neither convention alone expresses the whole truth, and that a judicious misuse of conventions is frequently essential to progress.

The *System* contains also Plücker's discussion of cubic curves, analyzed by Cayley in his memoir "On the classification of cubic curves", 1864 (Collected Mathematical Papers, V, No. 350); additional theorems on cubics are given in No. 27, 1847. Practically continuous with the *System* is the 22d paper, 1836, "Énumération des courbes du quatrième ordre, d'après la nature différente de leurs branches infinies"; this and the next two were published during the preparation of the *Theorie der algebraischen Curven*, 1839, in which Plücker applied his method of abridged notation to the discussion of asymptotes and of singular points and lines, with respect to quartics, making use of the method of counting constants. With the *Theorie* he regarded his work in plane analytical geometry as finished, "liegt der Cyclus meiner Arbeiten im Gebiete der analytischen Geometrie vollständig vor", and he turned to the geometry of space, a subject frequently referred to in his earlier writings; in 1846 he published the *System der Geometrie des Raumes*, in which is already to be found the germ of his later line-geometry. Only one paper appeared in this interval, No. 26, "Aphorismen aus der Geometrie des Raumes", 1842.

He now resolved to give up his mathematical researches, to which he had devoted more than twenty years, feeling that his ideas, as to whose value he himself had no misgivings, did not meet with due recognition from his contemporaries; but before renouncing geometry, as he supposed for ever, he arranged for publication the investigations he had in hand, so that Nos. 27–32 all appeared in 1847.

For nearly twenty years Plücker devoted himself to physics, and accordingly this volume contains no memoirs

between 1847 and 1865. In 1865, however, encouraged by the interest in his researches manifested by many English mathematicians, in the forefront of whom must be named Cayley, he returned to the subject of the geometry of space. Referring to his abandonment of his geometrical work twenty years before, he said that while the details had escaped from his memory, two leading questions had remained dormant in his mind; the first was that of introducing right lines as elements of space, instead of points and planes; and in close connection with this was the second, a question in mechanics. To these he devoted the last three years of his life, developing his ideas in a series of papers, Nos. 33–38, published in England, France and Italy, all leading up to his *Neue Geometrie des Raumes*, 1868–9.

The last paper in the volume, No. 39, published in 1833, is a preliminary account of the *System der analytischen Geometrie*, on which he was then engaged. It is sometimes remarked that the eminently proper person to review a book is the writer thereof, and in this announcement, as in his prefaces, Plücker appears as his own sympathetic reviewer.

The concluding thirty pages of the volume are devoted to the editorial comments; by his interesting historical notes and helpful criticisms here contained, as well as by the care bestowed in making good the original carelessness in the matter of proof-reading, Professor Schoenflies has made all lovers of Plücker his debtors.

CHARLOTTE ANGAS SCOTT.

BRYN MAWR COLLEGE,
October, 1897.

NOTES.

PROFESSOR NEWCOMB has chosen the philosophy of hyper-space as the subject of his presidential address to be delivered at the annual meeting of the AMERICAN MATHEMATICAL SOCIETY on December 29th.

A NEW list of members of the AMERICAN MATHEMATICAL SOCIETY will be published in January. Blank forms for furnishing necessary information have been sent to each member, and a full and prompt response is requested.

At the annual meeting of the London Mathematical Society, held on November 11, the following officers and members of the Council were elected for the ensuing year: President, Professor E. B. Elliott; Vice-Presidents, Major