rational fraction is decomposed into partial fractions—these and other points are worthy of mention.

Taking the two volumes together, they form a valuable work of reference to the college teacher of calculus. To an unusual degree they give just what the student should know. The French sparkle is perhaps missing, but we are well satisfied with the German accuracy, thoughtfulness and thoroughness.

EDWARD B. VAN VLECK.

WESLEYAN UNIVERSITY, *May*, 1895.

TRIANGULAR NUMBERS. *

In a review † of Blater's Table of Quarter Squares, Mr. J. W. L. Glaisher, in 1889, referred to the convenience that would be afforded by a table of triangular numbers, and said that the only extensive published table of such numbers known to him was that by E. de Joncourt, published at the Hague in 1762, giving the values of $\frac{1}{2}n(n+1)$ from $n = 1$ to $n = 20{,}000$. Letting S_n represent the n^{th} triangular number, the sum of the n numbers up to and including n, or $\frac{1}{2}n(n+1)$, the application of a table of triangular numbers to facilitate multiplication is seen from the formula, $ab = S_{a-1} + S_b - S_{a-b-1}$. This formula shows the chief advantage claimed for the use of triangular numbers in preference to quarter squares—that the table need only extend as far as the highest number it is to be used to multiply. Thus, such a table need be only half as large as a table of quarter squares that may be used to multiply the same numbers by taking the difference between the quarter squares of the sum and difference of the factors. On the other hand the method of quarter squares requires but two instead of three tabular entries, and may be modified ‡ so as not to use an argument exceeding the larger factor; but in that case three tabular entries are required. A modification of the use of triangular numbers is also applicable to reduce the number of entries to two, but then we may need to use an argument greater than either factor.

* Projet de Table de Triangulaires de 1 à 100,000, etc.; A. ARNAUDEAU (Paris: Gauthier-Villars et Fils, 1896).

† Reprinted in the *Journal of the Institute of Actuaries*, London, Jan. 1890.

‡ $ab = 2\left[\frac{a^2}{4} + \frac{b^2}{4} - \frac{(a-b)^2}{4}\right]$.

The explanatory text, with sample pages of a table of triangular numbers, by M. Arnaudeau, has recently been published as a prospectus of a complete table that has been calculated by him for all numbers up to 100,000 and that is to be published in about 400 octavo pages. Its scope will therefore be similar to that of Blater's Table of Quarter Squares, which extends to 200,000 and requires 200 quarto pages. M. Arnaudeau's table will be more readily used on account of the more convenient size of the pages and for not having the device, used by Mr. Blater to make his table more compact, of printing the last figures of the function at the foot of the columns.

Exact multiplication may be performed by a number of different formulas involving triangular numbers. It may be of interest to repeat here some of the formulas given by M. Arnaudeau.

$$\begin{aligned} ab &= S_a + S_{b-1} - S_{a-b} \\ &= S_{a-1} + S_b - S_{a-b-1} \\ &= S_{a-n} + S_{b+n-1} - S_{a-b-n} - S_{n-1} \\ &= S_{\frac{a-1}{2}+b} - S_{\frac{a-1}{2}-b} \\ &= S_{\frac{a}{2}+b-1} - S_{\frac{a}{2}-b-1} + b \\ &= S_{a+b} - S_a - S_b \end{aligned}$$

For several formulas the author gives geometrical demonstrations, using points arranged in triangles, so that the formulas are at once evident to the eye and may be readily remembered. He designs the text and tables of his work for the use of pupils in primary and secondary schools, and of surveyors and of those who may not be familiar with logarithms and trigonometry, as well as for the use of experienced computers. Five figure tables of reciprocals and of natural sines and tangents are added. The text explains the application of these tables in connection with the table of triangular numbers for division* and the solution of triangles. These methods, of course, give only approximate

* The only example of division given in the author's text (p. 15) is of 3,186,415,611 by 37,154. By means of his table of reciprocals and two multiplications of partial factors, 85,762.376,170,065 is obtained as the quotient; but the author does not call attention to the fact that not all the 14 figures are equally reliable. As a matter of fact, although the divisor must have been specially chosen so as to involve an exceptionally small error in the reciprocal, only the first six figures in the quotient are correct, the true quotient (to 15 places of decimals) being 85,762.383, 888,679,549,981.

results and are less convenient than the use of logarithms. In division closer approximations might be obtained, if desired, by the use of Col. W. H. Oakes' table of reciprocals (to seven figures) published by C. & E. Layton, London.

The text of the author is preceded by very favorable reports made by members of the French Academy of Sciences, and the French Institute of Actuaries. It does not, however, seem likely that a table of triangular numbers will be much used (as M. Arnaudeau intends) by persons who are not familiar with logarithms, or that it will, to any serious extent, be adopted as a substitute for logarithms. Tables of quarter squares or of triangular numbers may, however, be of great service in the comparatively rare cases where it is necessary to have more than seven correct figures in the product. Without the use of these tables such multiplications are probably most readily performed by using Crelle's Tables, or the arithometer. M. Arnaudeau's work should be gladly welcomed as a valuable and interesting addition to the tables now in existence for facilitating multiplication and as a great advance over any previously published table of triangular numbers. The author's perseverance in computing so extensive a table should be highly commended, and it is to be hoped that he will have no difficulty in obtaining, from learned societies and others, the pecuniary assistance needed for the publication of the tables.

EDWARD L. STABLER.

NEW YORK, *June*, 1897.

NOTES.

THE Committee of Arrangements of the International Congress of Mathematicians at Zürich has announced the following preliminary programme:—Monday, August 9: Opening of the Congress; Address by H. POINCARÉ, "Sur les rapports de l'analyse pure et de la physique mathematique;" Report of the committee on the functions and organization of international mathematical congresses; Address by A. HURWITZ, "Modern development of the general theory of analytical functions." Tuesday, August 10: Sectional Meetings. The following sectional divisions are contemplated: Arithmetic and Algebra, Analysis and Theory of Functions, Geometry, Mechanics and Mathematical Physics, Astronomy and Geodesy, History and Bibli-