general criteria, for Pringsheim shows that for every Σa_ν, *practical* criteria of the first and second kind do exist; but his proof of this fact yields no method of finding them, except when he knows beforehand the very thing to be determined, namely, whether Σa_ν be convergent or not!

In addition to the criteria of the first kind and second kind, Pringsheim establishes an entirely new criterion of a *third kind* and also *generalized criteria of the second kind*, which apply, however, only to series with never increasing terms. Those of the third kind rest mainly upon the consideration of the limit of the difference, either of consecutive terms or of their reciprocals. In the generalized criteria of the second kind he does not consider the ratio $\frac{a_{\nu+1}}{a_\nu}$ of two consecutive terms, but the ratio of any two terms, however far apart, and deduces, among others, two criteria previously given by Kohn * and Ermakoff,† respectively.

COLORADO COLLEGE, *April* 12, 1892.

NOTE ON AN ERROR IN BALL'S HISTORY OF MATHEMATICS.

BY DR. ARTEMAS MARTIN.

I DESIRE to call attention to what seems to me to be an error in Ball's "Short History of Mathematics," page 102, concluding clause of last paragraph, where the author ascribes to Diophantus the statement "that the sum of three square integers can never be expressed as the sum of two squares."

That the above statement is not in accordance with the facts is evident, since

$$(q^2+r^2-s^2-u^2)^2+(2qu)^2+(2ru)^2=(q^2+r^2-s^2+u^2)^2+(2su)^2$$

identically, no matter what values be assigned to q, r, s, u.

If we take $q = 1$, $r = 2$, $s = 3$, $u = 4$, then, after dividing all the numbers by 4^2, we have

$$2^2 + 4^2 + 5^2 = 3^2 + 6^2 = 45.$$

Let $q = 1$, $r = 2$, $s = 4$, $u = 3$, and we find, after dividing by 2^2,

$$3^2 + 6^2 + 10^2 = 1^2 + 12^2 = 145, = 8^2 + 9^2.$$

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* *Grunert's Archiv*, vol. 67, pp. 63–95.
† *Darboux's Bulletin*, vol. 2, p. 250; vol. 17, p. 142.

By varying the values of q, r, s, u, an infinite number of sets of three square integers can be found that can also be expressed as the sum of two squares; but the first given above is probably the *smallest* set in which the numbers are all different.

Since writing the foregoing note I have been informed by Mr. Ball that the above error probably arose from a confused memorandum of Diophantus's impossible forms for the sums of two and three squares.

WASHINGTON, D. C., *September* 15, 1892.

THE THEORY OF EQUATIONS.

An Elementary Course in the Theory of Equations. By C. H. CHAPMAN, Ph.D., Associate in Mathematics in Johns Hopkins University. New York, John Wiley & Sons, 1892. 12mo, pp. viii + 90.

IN the past no American text-book has treated exclusively of the theory of equations, but the subject has been presented briefly in the closing chapters of the more complete works on algebra. The marked tendency at the present time to prefer a sort of parallel study of several correlated branches of mathematics to carrying on successively each subject as far as possible before taking up another, has led to the introduction in many cases of short courses in trigonometry, analytical geometry, and calculus between the more elementary algebra and the theory of equations. Under this system it becomes at once desirable to treat the latter subject in a separate text-book, into which it is proper to introduce the notation and principles of the infinitesimal calculus, although logically they may be altogether unnecessary.

The first impression conveyed by the little book which comes from the pen of Dr. Chapman is that it has been unduly condensed, but upon a more careful examination one perceives that to a student possessing a very slight knowledge of analytical geometry and calculus the treatment of the subject is unusually intelligible. The book is worthy of praise both on scientific and on pedagogical grounds. The definitions are accurate, the demonstrations rigorous, and the choice of material excellent; while the questions addressed to the student at various points, the problems, and suggested demonstrations are sure to be effective.

The first part relates to determinants. It occupies but twenty one pages, and leads from first principles through the most elementary propositions to the solution of linear equa-