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CORRECTION

CENTRAL LIMIT THEOREMS FOR THE WASSERSTEIN DISTANCE BETWEEN THE EMPIRICAL AND THE TRUE DISTRIBUTIONS

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There is an error in Proposition 6.4 of our paper. It comes from a wrong expression for the covariance, $K(\rho)$, between $|Z_1|$ and $|Z_2|$, where (Z_1, Z_2) is a centered random vector with bivariate normal distribution such that $\mathrm{Var}(Z_1) = \mathrm{Var}(Z_2) = 1$ and $\mathrm{Cov}(Z_1, Z_2) = \rho$. The formula for $K(\rho)$ should read

$$K(\rho) = \frac{2}{\pi}\bigl(\rho \arcsin \rho + \sqrt{1-\rho^2} - 1\bigr), \qquad \rho \in [-1, 1],$$

[see, e.g., Nabeya (1951) or Wellner and Smythe (2002), Proposition 2]. Hence, using the notation in the proof of Proposition 6.4, $K(\rho) = K_1(\rho) + K_3(\rho)$. The core of that proof remains valid, except that we should drop the contribution of K_2 to the limit in (6.14). Consequently, Proposition 6.4 should be restated as follows.

PROPOSITION 6.4. *Let Q be the quantile function of a random variable X in $DA_2(b_n)$. Assume X has regularly varying tails with exponent -2 and $\mathbb{E}X^2 = \infty$. Let B be a Brownian bridge and let G_n be as defined by* (6.9). *Then*

$$\lim_{n\to\infty} \mathbb{E}G_n^2 = 1 - \frac{2}{\pi}(2 - \log 2). \tag{6.14}$$

This amendment carries over to Theorems 6.7 and 6.8. The limiting distribution (6.42) in Theorem 6.7 should be

$$\frac{Z_n - \gamma_n}{b_n} \xrightarrow{d} \sqrt{1 - \frac{2}{\pi}(2 - \log 2)}\,g. \tag{6.42}$$

Also, in this theorem, the dividing term $\alpha + 1$ in the expression for the constants b_n, case $\alpha = -1$, should be replaced by 1. This also occurs in Theorem 6.8, where, moreover, there is a factor of 2 missing [see (6.25) and (6.26)]. The correct statement for Theorem 6.8 is:

THEOREM 6.8. *Let $V(t)$, $t \in \mathbb{R}$, be a stationary Ornstein–Uhlenbeck process and let $\alpha \in [-2, \infty)$. Then*:

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(a) *if* $\alpha > -1$,

$$\frac{2}{\sqrt{8 \cdot 2^{-\alpha}/(\alpha+1)}\, s^{(\alpha+1)/2}} \int_{-s/2}^{s/2} \big(|V(t)| - \mathbb{E}|V(t)|\big)|t|^{\alpha/2}\, dt \xrightarrow{d} \sqrt{1 - \frac{2}{\pi}(2 - \log 2)}\, g;$$

(b) *for* $\alpha = -1$,

$$\frac{1}{2(\log s)^{1/2}} \int_{-s/2}^{s/2} \big(|V(t)| - \mathbb{E}|V(t)|\big)|t|^{-1/2}\, dt \xrightarrow{d} \sqrt{1 - \frac{2}{\pi}(2 - \log 2)}\, g;$$

(c) *the integrals*

$$\int_{-\infty}^{\infty} \big(|V(t)| - \mathbb{E}|V(t)|\big)|t|^{\alpha/2}\, dt \qquad \textit{for } -2 < \alpha < -1$$

as well as

$$\int_{-\infty}^{\infty} \big(|V(t)| - \mathbb{E}|V(t)|\big)(|t|^{-1} \wedge 1)\, dt$$

exist in the sense of convergence of all moments as the limits of integration expand to $+\infty$ *and to* $-\infty$.

Finally, the definition of d_n in Proposition A.1 is missing a factor of $\sqrt{n}$.

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