African Diaspora Journal of Mathematics
Volume 10, Number 1, pp. 39–48 (2010)
ISSN 1539-854X
www.african-j-math.org

# TORSE-FORMING PROJECTIVE $N$-CURVATURE COLLINEATION IN NP-$F_n$

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**Abstract**

In this paper we have defined torse-forming projective $N-$ curvature collineation and discuss the existence of torse-forming projective $N-$ curvature collineation in $NP-F_n$ (normal projective Finsler space) and study the corresponding results for contra, concurrent and special concircular transformations.

**AMS Subject Classification:** 53B40; 53C60.

**Keywords**: Finsler spaces, $NP-F_n$, projective motion, torse-forming, contra, concurrent and special concircular transformations.

## 1 Introduction

Let $Fn$ be an n-dimensional Finsler space equipped with 2n line elements $(x^i,\, \dot{x}^i)$ and positively homogeneous metric function $F(x,\, \dot{x})$ of degree one in directional arguments $\dot{x}^i$.
The normal projective covariant derivative of a vector field $X^i(x,\, \dot{x})$ with respect to $\dot{x}^k$ is given by [1]

$$\nabla_k X^i = \partial_k X^i - (\dot{\partial}_j X^i)\Pi^j_{kh}\dot{x}^h + X^j\Pi^i_{jk,}$$

where

$$\Pi^i_{kh} = G^i_{kh} - \frac{\dot{x}^i}{n+1}G^r_{khr},$$

which form a connection called the normal projective connection and $\partial_k = \frac{\partial}{\partial_{x^k}}$, $\dot{\partial}_k = \frac{\partial}{\dot{\partial}_{x^k}}$ preserve the vector character of $X^i$.

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The functions $\Pi^i_{kh}$, $G^i_{kh}$ and $G^r_{khr}$ are symmetric in their lower indices and are positively homogeneous of degree 0, 0 and -1 respectively in their directional arguments. The functions $G^i_{kh}$ are the Berwald's connection parameters [2]. The derivatives $\dot{\partial}_j\Pi^i_{kh}$ denoted by $\Pi^i_{jkh}$ is given by

$$\Pi^i_{jkh} = G^i_{jkh} - \frac{1}{n+1}(\delta^i_j G^r_{khr} + \dot{x}^i G^r_{jkhr}),$$

are symmetric in their lower indices and positively homogeneous of degree -1 in directional arguments and satisfy the following relations

$$\begin{cases} (a) & \Pi^i_{kh}\dot{x}^k = \Pi^i_{hk}\dot{x}^k = G^i_h, \\ (b) & \Pi^i_{ki} = G^i_{ki,} \\ (c) & \dot{x}^j\Pi^i_{jhk} = 0, \\ (d) & \Pi^i_{jki} = \Pi^i_{jik} = G^i_{jki,} \\ (e) & \Pi^i_{ikh} = \frac{2}{n+1}\, G^i_{ikh.} \end{cases} \tag{1.1}$$

Let us consider a point transformation

$$\bar{x}^i = x^i + \varepsilon v^i(x), \tag{1.2}$$

where $v^i$ is a contravariant vector field. Then Lie-derivative of a tensor $T^i_j$ and the connection coefficient $\Pi^i_{jk}$ are characterized by [1]

$$\pounds T^i_j = v^h(\nabla_h T^i_j) - T^h_j(\nabla_h v^i) + T^i_h(\nabla_j v^h) + (\dot{\partial}_h T^i_j)(\nabla_s v^h)\dot{x}^s$$

and

$$\pounds\Pi^i_{jk} = \nabla_j\nabla_k v^i + N^i_{hjk}\, v^h + \Pi^i_{hjk}(\nabla_l v^h)\dot{x}^l \tag{1.3}$$

respectively. The commutation formulae with respect to Lie-derivative and other for any tensor $T^i_{jk}$ are given by

$$\begin{cases} (a) \quad \pounds(\nabla_l T^i_{jk}) - \nabla_l\pounds(T^i_{jk}) = (\pounds\Pi^i_{lh})T^h_{jk} - (\pounds\Pi^i_{jl})T^i_{rk} - (\pounds\Pi^r_{kl})T^i_{jr}, \\ (b) \quad \dot{\partial}_l(\pounds T^i_{jk}) - \pounds(\dot{\partial}_l T^i_{jk}) = 0. \end{cases}$$

The Lie-derivative of the normal projective curvature tensor $N^i_{kjh}$ expressed in the form

$$\nabla_k(\pounds\Pi^i_{jh}) - \nabla_j(\pounds\Pi^i_{kh}) = \pounds N^i_{kjh} + (\pounds\Pi^r_{km})\dot{x}^m\Pi^i_{rjh} - (\pounds\Pi^r_{jm})\dot{x}^m\Pi^i_{rkh}. \tag{1.4}$$

The corresponding curvature $N^i_{jkh}(x,\dot{x})$ as called by[1], the normal projective curvature tensor, is given by

$$N^i_{jkh} = 2\{\partial_{[j}\Pi^i_{k]h} + \Pi^i_{lh[j}\Pi^l_{k]m}\dot{x}^m + \Pi^i_{l[j}\Pi^l_{k]h}\},$$

is skew-symmetric in $j$ and $k$ indices and satisfied the following relations

$$\begin{cases} (a) & N^i_{jkh} = -N^i_{kjh}, \\ (b) & \dot{\partial}_l N^i_{jkh}\dot{x}_l = 0, \\ (c) & N^i_{jki} = 2N_{[kj],} \\ (d) & N^i_{ikh} = N_{kh}, \\ (e) & N^i_{jih} = -N^i_{ijh}, \\ (f) & N^i_{jkh}\dot{x}_h = H^i_{jk}. \end{cases} \tag{1.5}$$

where $H^i_{jk}$ is Berwald curvature tensor deviation. It is connected to Berwald curvature tensor $H^i_{jkh}$ by

$$\begin{cases} (a) \quad H^i_{jkh} = \dot{\partial}_j H^i_{kh} = \tfrac{2}{3}\dot{\partial}_j \dot{\partial}_{[k} H^i_{h]}, \\ (b) \qquad H^i_{jkh}\dot{x}^j = H^i_{kh}. \end{cases} \tag{1.6}$$

The Berwald's curvature tensor satisfies the Bianchi identity

$$H^i_{[jkh]} = H^i_{jkh} + H^i_{khj} + H^i_{hjk} = 0. \tag{1.7}$$

The commutation formulae for any general tensor, involving the curvature tensor are given as follows

$$2\nabla_{[k}\nabla_{h]}T^i_j = N^i_{khl}T^l_j - N^l_{khj}T^i_l - (\dot{\partial}_l T^i_j)N^l_{khm}\dot{x}^m, \tag{1.8}$$

and

$$(\dot{\partial}_j\nabla_k - \nabla_k\dot{\partial}_j)T^i_h = \Pi^i_{jkl}T^l_h - \Pi^l_{jkh}T^i_l - \Pi^l_{jkm}\dot{x}^m(\dot{\partial}_l T^i_h). \tag{1.9}$$

**Definition 1.1.[4]** *The space $F_n$ with normal projective connection parameter $\Pi^i_{hk}$ and normal projective curvature tensor $N^i_{jkh,}$ is termed as normal projective Finsler space and usually denoted by $NP-F_n$.*

## 2 Preliminaries

Torse-forming infinitesimal transformations in a Finsler space were discussed by R. B. Misra and C. K. Mishra [7]. Special concircular projective curvature collineation in recurrent Finsler space was introduced by S. P. Singh[6].

**Definition 2.1.[5]** *A Finsler space $Fn$ is said to admit N-curvature collineation, if there exist a vector field $v^i$ such that*

$$\pounds N^i_{jkh} = 0,$$

We also consider an infinitesimal transformation of the form

$$\bar{x}^i = x^i + \varepsilon v^i(x), \qquad \nabla_k v^i = v^i \mu_k + \lambda\delta^i_k. \tag{2.1}$$

where $\lambda$ is a scalar function and $\mu_k$ being any non null vector field. Such a transformation is called a torse-forming transformation[3].

In view of infinitesimal transformation (1.2) in [1], defined a projective motion, if there a homogeneous scalar function $p$ of degree one in $\dot{x}'s$ satisfying

$$\pounds\Pi^i_{jk} = 2\delta^i_{(j}p_{k)}, \tag{2.2}$$

where

$$p_j = \dot{\partial}_j p, \tag{2.3}$$

and satisfy the conditions

$$\begin{cases} (a) \qquad \dot{x}^k p_k = p\ , \\ (b) \qquad \dot{x}^k p_{kj} = 0. \end{cases} \tag{2.4}$$

Theorem 4.1 [3], have proved that the scalar function $\lambda$ appearing in (2.1) is a point function.

$$\dot{\partial}_l \lambda = 0. \tag{2.5}$$

We have the next particular cases:
A torse-forming transformation becomes
(1) a contra transformation, If $\lambda = 0$ and $\mu_j = 0$ in (2.1), such that

$$\bar{x}^i = x^i + \varepsilon v^i(x), \qquad \nabla_k v^i = 0. \tag{2.6}$$

(2) a concurrent transformation, If $\mu_j = 0$ and $\lambda = c$ ($c$ being a constant) in (2.1), such that

$$\bar{x}^i = x^i + \varepsilon v^i(x), \qquad \nabla_k v^i = c\delta^i_k. \tag{2.7}$$

(3) a special concircular transformation, If $\mu_j = 0$ and $\lambda \neq$constant in (2.1), such that

$$\bar{x}^i = x^i + \varepsilon v^i(x), \qquad \nabla_k v^i = \lambda\delta^i_k. \tag{2.8}$$

## 3 Torse-forming projective $N-$curvature collineation in $NP-F_n$

**Definition 3.1.** *In $NP-F_n$, if the normal projective curvature tensor field $N^i_{jkh}$ satisfies the relation*

$$\pounds N^i_{jkh} = 0, \tag{3.1}$$

*where £ represents Lie-derivative defined by the transformation (2.1), which defines a projective motion, then the transformation (2.1) is called the torse-forming projective $N-$curvature collineation.*

Differentiating (2.1) partially with respect to $\dot{x}^i$ and applying the commutation formula (1.9), we have

$$(\dot{\partial}_j \nabla_k - \nabla_k \dot{\partial}_j)v^i = (\dot{\partial}_j \mu_k - \dot{\partial}_k \mu_j)v^i = \Pi^i_{jkl} v^l. \tag{3.2}$$

Transvecting (3.2) by $\dot{x}^j$ and using (1.1)(c), we get

$$(\dot{\partial}_j \nabla_k - \nabla_k \dot{\partial}_j)\dot{x}^j v^i = 0 \qquad or \qquad (\dot{\partial}_j \mu_k - \dot{\partial}_k \mu_j)\dot{x}^j v^i = 0.$$

Since $\dot{x}^j$ and $v^i$ is non zero, it implies

$$\begin{cases} (a) \quad \dot{\partial}_j \nabla_k v^i = \nabla_k \dot{\partial}_j v^i, \\ (b) \quad \dot{\partial}_j \mu_k = \dot{\partial}_k \mu_j. \end{cases} \tag{3.3}$$

The normal projective covariant differentiation of (2.1), we have

$$\nabla_j \nabla_k v^i = v^i \mu_j \mu_k + \lambda \delta^i_j \mu_k + v^i(\nabla_j \mu_k) + \lambda_j \delta^i_k, \tag{3.4}$$

where

$$\nabla_j \lambda = \lambda_j.$$

If $\mu_k$ follows the invariance property with respect to normal projective covariant differentiation, such that

$$\nabla_j \mu_k = 0. \tag{3.5}$$

In view of (3.5), the equation (3.4) reduces to

$$\nabla_j\nabla_k v^i = v^i\mu_j\mu_k + \lambda\delta^i_j\mu_k + \lambda_j\delta^i_k. \tag{3.6}$$

Interchanging the indices $j$ and $k$ in (3.6) and subtracting the equation thus obtained to (3.6), we have

$$2\nabla_{[j}\nabla_{k]}v^i = \lambda_j\delta^i_k + \lambda\delta^i_j\mu_k - \lambda_k\delta^i_j - \lambda\delta^i_k\mu_j. \tag{3.7}$$

Using equation (3.7) in commutation formula (1.8), we get

$$N^i_{jkh}v^h = \lambda_j\delta^i_k + \lambda\delta^i_j\mu_k - \lambda_k\delta^i_j - \lambda\delta^i_k\mu_j. \tag{3.8}$$

Transvecting (3.8) by $\dot{x}^h$, we get

$$H^i_{jk}v^h = (\lambda_j\delta^i_k + \lambda\delta^i_j\mu_k - \lambda_k\delta^i_j - \lambda\delta^i_k\mu_j)\dot{x}^h, \tag{3.9}$$

in view of (1.5)(f).
Differentiating (3.9) partially with respect to $\dot{x}^l$ and using (1.6)(a), we have

$$H^i_{ljk}v^h = \lambda\delta^i_j\mu_k\delta^h_l + \lambda\dot{\partial}_l\mu_k\delta^i_j\dot{x}^h - \lambda\delta^i_k\mu_{lj}\dot{x}^h - \lambda\delta^i_k\mu_j\delta^h_l + \lambda_j\delta^i_k\delta^h_l - \lambda_k\delta^i_j\delta^h_l. \tag{3.10}$$

Adding the expressions obtained by cyclic change of (3.10) with respect to indices $l$, $j$ and $k$ in cyclic order, we have

$$\begin{aligned} 0 \;=\; & \lambda\delta^i_j\mu_k\delta^h_l + \lambda\dot{\partial}_l\mu_k\delta^i_j\dot{x}^h - \lambda\delta^i_k\dot{\partial}_l\mu_j\dot{x}^h - \lambda\delta^i_k\mu_j\delta^h_l \\ & +\lambda_j\delta^i_k\delta^h_l - \lambda_k\delta^i_j\delta^h_l + \lambda\delta^i_k\mu_l\delta^h_j + \lambda\dot{\partial}_j\mu_l\delta^i_k\dot{x}^h \\ & -\lambda\delta^i_l\dot{\partial}_j\mu_k\dot{x}^h - \lambda\delta^i_l\mu_k\delta^h_j + \lambda_k\delta^i_l\delta^h_j - \lambda_l\delta^i_k\delta^h_j \\ & +\lambda\delta^i_l\mu_j\delta^h_k + \lambda\dot{\partial}_k\mu_j\delta^i_l\dot{x}^h - \lambda\delta^i_j\dot{\partial}_k\mu_l\dot{x}^h - \lambda\delta^i_j\mu_l\delta^h_k \\ & +\lambda_l\delta^i_j\delta^h_k - \lambda_j\delta^i_l\delta^h_k. \end{aligned} \tag{3.11}$$

in view of (1.7).
Using (3.3)(b) in (3.11), we obtain

$$\begin{aligned} 0 \;=\; & \lambda\delta^i_j\mu_k\delta^h_l - \lambda\delta^i_k\mu_j\delta^h_l + \lambda_j\delta^i_k\delta^h_l - \lambda_k\delta^i_j\delta^h_l \\ & +\lambda\delta^i_k\mu_l\delta^h_j - \lambda\delta^i_l\mu_k\delta^h_j + \lambda_k\delta^i_l\delta^h_j - \lambda_l\delta^i_k\delta^h_j \\ & +\lambda\delta^i_l\mu_j\delta^h_k - \lambda\delta^i_j\mu_l\delta^h_k + \lambda_l\delta^i_j\delta^h_k - \lambda_j\delta^i_l\delta^h_k. \end{aligned} \tag{3.12}$$

Contracting indices $h$ and $l$ in (3.12), we derive

$$(n-2)(\lambda\delta^i_j\mu_k - \lambda\delta^i_k\mu_j + \lambda_j\delta^i_k - \lambda_k\delta^i_j) = 0, \tag{3.13}$$

Contracting indices $i$ and $k$ in (3.13), we drive

$$(n-2)(n-1)(\lambda_j - \lambda\mu_j) = 0. \tag{3.14}$$

for $n > 2$, the equation (3.14) yields

$$\lambda_j = \lambda\mu_j. \tag{3.15}$$

Differentiating (3.15) partially with respect to $\dot{x}^l$ and using (2.5), we find

$$\dot{\partial}_l \mu_j = 0. \tag{3.16}$$

In view of (3.16), the equation (3.8) immediately reduces to

$$N^i_{jkh} v^h = 0. \tag{3.17}$$

Applying (2.1), (2.2), (1.1)(c), (3.6) and (3.15) in (1.3), we obtain

$$\delta^i_j p_k + \delta^i_k p_j = v^i \mu_j \mu_k + \lambda \delta^i_j \mu_k + \lambda \mu_j \delta^i_k + N^i_{hjk} v^h + \Pi^i_{hjk} v^h \mu, \tag{3.18}$$

where $\mu_l \dot{x}^l = \mu$.
Transvecting (3.18) by $\dot{x}^h$ and using (1.1)(c), we get

$$(\delta^i_j p_k + \delta^i_k p_j)\dot{x}^h = (v^i \mu_j \mu_k + \lambda \delta^i_j \mu_k + \lambda \mu_j \delta^i_k)\dot{x}^h + N^i_{hjk} v^h \dot{x}^h. \tag{3.19}$$

Differentiating (3.19) partially with respect to $\dot{x}^l$, we have

$$\begin{aligned}(\delta^i_j p_{lk} + \delta^i_k p_{lj})\dot{x}^h + (\delta^i_j p_k + \delta^i_k p_j)\delta^h_l \quad &= \quad (v^i \mu_j \mu_k + \lambda \delta^i_j \mu_k + \lambda \mu_j \delta^i_k)\delta^h_l \\ &\quad\ (\dot{\partial}_l N^i_{hjk})\dot{x}^h v^h + N^i_{ljk} v^h.\end{aligned} \tag{3.20}$$

Transvecting (3.20) by $\dot{x}^l$ and using (1.5)(b) and (2.4)(b), we derive

$$(\delta^i_j p_k + \delta^i_k p_j)\dot{x}^h = (v^i \mu_j \mu_k + \lambda \delta^i_j \mu_k + \lambda \mu_j \delta^i_k)\dot{x}^h + N^i_{ljk} v^h \dot{x}^l. \tag{3.21}$$

Contracting the indices $h$ and $k$ in (3.21), we get

$$\delta^i_j p + \dot{x}^i p_j = v^i \mu_j \mu + \lambda \delta^i_j \mu + \dot{x}^i \lambda \mu_j, \tag{3.22}$$

in view of (3.17).
Differentiating (3.22) partially with respect to $\dot{x}^k$, we obtain

$$\delta^i_j p_k + \delta^i_k p_j + \dot{x}^i p_{kj} = v^i \mu_j \mu_k + \lambda \delta^i_j \mu_k + \lambda \mu_j \delta^i_k. \tag{3.23}$$

Contracting $i$ and $k$ in equation (3.23) and using (2.4)(b), we have

$$(n+1)(p_j) = v^h \mu_j \mu_h + (n+1)\lambda \mu_j. \tag{3.24}$$

Differentiating (3.24) covariantly with respect to indices $x^k$ and using equations (2.5) and (3.16), we get

$$(n+1)(p_{kj}) = 0,$$

which implies

$$p_{kj} = 0. \tag{3.25}$$

In view of equation (3.25), the equation (3.23) may be written as

$$\delta^i_j p_k + \delta^i_k p_j = v^i \mu_j \mu_k + \lambda \delta^i_j \mu_k + \lambda \mu_j \delta^i_k. \tag{3.26}$$

Equations (3.18) and (3.26) gives

$$(N^i_{hjk} = \Pi^i_{hjk}\mu)v^h = 0. \tag{3.27}$$

Since $v^h$ is non zero, therefore the equation (3.27) implies

$$N^i_{hjk} = \Pi^i_{hjk}\mu. \tag{3.28}$$

Interchanging the indices $j$ and $k$ in equation (3.28) and subtracting the equation thus obtained to (3.28), we obtain

$$(N^i_{hjk} - N^i_{hkj}) = 0. \tag{3.29}$$

Transvecting (3.29) by $v^j$ and using (3.17), we get

$$v^j N^i_{hjk} = 0. \tag{3.30}$$

Since $v^h$ is a non zero Lie-invariant vector for infinitesimal transformation (1.2), we have

$$\pounds v^j = 0. \tag{3.31}$$

Taking the Lie derivative of (3.30) and noting (3.31), we get

$$v^j \pounds N^i_{hjk} = 0. \tag{3.32}$$

which implies

$$\pounds N^i_{hjk} = 0. \tag{3.33}$$

Thus we state:

**Theorem 3.1.** *In $NP-F_n(n>2)$, the torse-forming transformation (2.1), which admits projective motion, is the torse-forming projective $N-$curvature collineation.*

# 4 The study of some other transformations

**Case 1** *In $NP-F_n$, the contra transformation (2.6), which defines projective motion and admits the relation (3.1), is called contra projective $N-$curvature collineation.*

In view of (2.6) the commutation formula (1.8) gives

$$N^i_{jkh}v^h = 0. \tag{4.1}$$

Contracting (4.1) with respect to indices $i$ and $j$ and using (1.5)(d), we get

$$N_{kh}v^h = 0. \tag{4.2}$$

Using equations (2.2) and (2.6) in (1.3), we obtain

$$\delta^i_j p_k + \delta^i_k p_j = N^i_{hjk}v^h. \tag{4.3}$$

Contracting indices $i$ and $j$ in (4.3) and using (1.5)(e) and (1.5)(d), we get

$$(n+1)p_k = N_{hk}v^h. \tag{4.4}$$

Transvecting (4.4) by $v^k$ and using (4.2), we get

$$(n+1)p_k v^k = 0. \tag{4.5}$$

Since $v^k$ is non zero, therefore the equation (4.5) implies

$$(n+1)p_k = 0, \tag{4.6}$$

for $n \geq 1$, (4.6) gives

$$p_k = 0. \tag{4.7}$$

In view of (2.2) and (4.7), the equation (2.3) reduces to

$$\pounds N^i_{kjh} = 0. \tag{4.8}$$

Accordingly we state:

**Theorem 4.1.** *In $NP - F_n(n \geq 1)$, the contra transformation (2.6), which admits projective motion, is the contra projective $N-$curvature collineation.*

**Case 2** *In $NP - F_n$, the concurrent transformation (2.7), which defines projective motion and admits the relation (3.1), is called concurrent projective $N-$curvature collineation.*

**Theorem 4.2.** *In $NP - F_n(n \geq 1)$, the concurrent transformation (2.7), which admits projective motion, is the concurrent projective $N-$curvature collineation.*

*Proof.* The proof is analogous to theorem (4.1). □

**Case 3** *In $NP - F_n$, the special concircular transformation (2.8), which defines projective motion and admits the relation (3.1), is called special concircular projective $N-$curvature collineation.*

In view of (2.8) the commutation formula (1.8) gives

$$\lambda_j \delta^i_k - \lambda_k \delta^i_j = N^i_{jkh} v^h. \tag{4.9}$$

Transvecting (4.9) by $\dot{x}^h$ and using (1.5)(f), we get

$$(\lambda_j \delta^i_k - \lambda_k \delta^i_j)\dot{x}^h = H^i_{jk} v^h. \tag{4.10}$$

Differentiating (4.10) Partially with respect to $\dot{x}^l$, we have

$$(\lambda_j \delta^i_k - \lambda_k \delta^i_j)\delta^h_l = H^i_{ljk} v^h, \tag{4.11}$$

in view of (1.6)(a) and (2.5).
Adding the expressions obtained by cyclic change of (4.11) with respect to indices $l$, $j$ and $k$ in cyclic order and using equation (1.7), we have

$$(\lambda_j \delta^i_k - \lambda_k \delta^i_j)\delta^h_l + (\lambda_k \delta^i_l - \lambda_l \delta^i_k)\delta^h_j + (\lambda_l \delta^i_j - \lambda_j \delta^i_l)\delta^h_k = 0. \tag{4.12}$$

Contracting (4.12) with respect to indices $h$ and $l$ , we drive

$$(n-2)(\lambda_j \delta^i_k - \lambda_k \delta^i_j) = 0, \tag{4.13}$$

for $n>2$, the equation (4.13) gives

$$\lambda_j\delta^i_k=\lambda_k\delta^i_j. \tag{4.14}$$

From (4.9) and (4.14), we get

$$N^i_{jkh}v^h=0. \tag{4.15}$$

Using (2.2) and (2.8) in (1.3), we obtain

$$\delta^i_j p_k+\delta^i_k p_j=\lambda_j\delta^i_k+N^i_{hjk}v^h. \tag{4.16}$$

Transvecting (4.16) by $v^k$ and using (4.15), we get

$$\delta^i_j p_k v^k+p_j v^i=\lambda_j v^i. \tag{4.17}$$

Transvecting (4.17) by $\dot{x}^k$ and using (2.4)(a), we obtain

$$\delta^i_j p v^k+p_j v^i\dot{x}^k=\lambda_j\dot{x}^k v^i. \tag{4.18}$$

Contracting indices $j$ and $k$ in (4.18) and using (2.4)(a), we get

$$(2p-\lambda_j\dot{x}^j)v^i=0,$$

which implies

$$2p=\lambda_j\dot{x}^j. \tag{4.19}$$

In view of (4.19) and (4.14) the equation (4.16) reduces to

$$N^i_{hjk}v^h=0. \tag{4.20}$$

Contracting indices $i$ and $k$ in equation (4.14), we get

$$(n-1)\lambda_j=0, \tag{4.21}$$

for $n>1$, equation (4.21) give

$$\lambda_j=0. \tag{4.22}$$

From equations (4.16), (4.20), (4.22)and (2.2), we get

$$£\Pi^i_{jk}=\delta^i_j p_k+\delta^i_k p_j=0. \tag{4.23}$$

In view of equation (4.23), the equation (1.4) immediately reduces to

$$£N^i_{kjh}=0. \tag{4.24}$$

Accordingly we have:

**Theorem 4.3.** *In $NP-F_n(n>2)$, the special concircular transformation (2.8), which admits projective motion, is the special concircular projective $N-$curvature collineation.*

**Acknowledgement** The authors would like to thank Prof. Dr. V. Balan and to the referees for their useful suggestions and comments.

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