JGSP **6** (2006) 118–126

JOURNAL OF
Geometry and Symmetry in Physics

NONLINEAR QUANTUM GRAVITY

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Communicated by Gerald A. Goldin

Abstract. Nonlinear quantum mechanics at the Planck scale can produce nonlocal effects contributing to resolution of singularities, to cosmic acceleration, and modified black-hole dynamics, while avoiding the usual causality issues.

1. Introduction

We explore here some possible consequences for quantum gravity if quantum mechanics becomes nonlinear at the Planck scale, and speculate that black hole dynamics may be linked to the accelerated expansion of the universe through a connection between short- and long-wavelength modes. Space limitations only allow for limited discussion and references on nonlinear quantum mechanics for which see [8, 9]. One outstanding characteristic of nonlinearity is: generically, entangled systems become **causal channels**. Consider a composite system described by a product Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$ and a Schrödinger time evolution $\mathrm{i}\partial_t\Psi = H\Psi$ where H is a not necessarily linear operator and $\Psi \in \mathcal{H}_1 \otimes \mathcal{H}_2$. Among such evolutions there are those known as **separating**, meaning that product vectors evolve as product vectors (a nonlinear expression of interaction-free), that is, if $\Psi(0) = \Psi_1(0) \otimes \Psi_2(0)$ then $\Psi(t) = \Psi_1(t) \otimes \Psi_2(t)$ where $\mathrm{i}\partial_t\Psi_j = H_j\Psi_j$, $j = 1, 2$ are independent evolutions. Separable systems for distinguishable parts have been fully classified by Goldin and Svetlichny [4]. One has then $H = H_1 + H_2 + K$ where K is an operator that vanishes on product states. Now if H is linear and separable, then $K = 0$ and even if Ψ is not a product state, its partial trace in $\mathcal{H}_1$, $\rho_1 = \mathrm{Tr}_2|\Psi\rangle\langle\Psi|$ satisfies the von Neumann evolution equation $\mathrm{i}\partial_t\rho_1 = [H_1, \rho_1]$, independently of what the operator H_2 is and of the further details of entanglement of the two parts represented by $\Psi(0)$. The same of course for ρ_2. If the H_i are nonlinear, then even if $K = 0$, the partial traces generically do not have independent evolution. This means that entanglement in $\Psi(0)$ leads to a causal connection between the two parts even though the evolution is ostensibly noninteractive. This has been used as an argument against quantum nonlinearity since now one can show that EPR-type correlations along with the usual hypothesis of state collapse due to measurements can be used to send superluminal signals, calling into question relativistic causality. Since the idea of measurements being performed during the Planck epoch of the evolution of the universe is somewhat bizarre, the consequence of this situation for quantum gravity has not been properly appreciated. Decoherence however shares many properties of measurement and so if we consider each $\mathcal{H}_i$ as describing a system and an environment and in which decoherence occurs through some nonlinear quantum process, then if $\Psi(0)$ is entangled, generically, the decoherence process in one part will causally influence that in the other part. One cannot deny the importance of decoherence in the early evolution of the universe, and so this type of causal channel is quite relevant. There have been many attempts to circumvent the causality issue by a deeper analysis of the measurement process and its relation to evolution, introducing appropriate modifications or reinterpretations of both. On the one hand it does seem ironic that by this one is attempting to eliminate precisely one of the main distinguishing characteristic of nonlinearity. On the other hand the ubiquity of entangled systems means that such causal channels must proliferate in wide variety of circumstances (for example, entanglement of spin and orbital angular momentum states in atomic physics) which may or may not be causally problematic. Ironically, again, there seems to be no considerations of these other channels in the literature. With nonlinear quantum mechanics causal channels abound beyond anything conceived in linear theory. This situation turns nonlinear quantum systems radical, introducing effects which to many are unwelcome and often leads to a rejection, practically off-hand, of nonlinear theories.

There are however cogent reasons for considering nonlinear quantum mechanics. Surprisingly enough, non-linear quantum mechanics appears through **linear** representations of the diffeomorphism group [3]. As a consequence one can expect nonlinear quantum processes to unexpectedly show up in any theory that says it is both quantum and geometric (see [5] for an example which independently finds equations of the same type as in [3]). Nonlinear quantum mechanics is intrinsically associated with quantum geometry and ignoring this may not be wise. Another reason is more conceptual. Take the usual dictum of general relativity: Space-time tells matter how to move; matter tells space-time how to curve, and perform a verbal quantization of general relativity introducing the adjective "quantum" for space-time and matter: Quantum space-time tells matter how to move; quantum matter tells space-time how to curve*. This seemingly natural,

*Maybe "move" and "curve" are no longer appropriate words, but we can remove this "anomaly" with some words such as "behave" and "be".

relational, viewpoint leads to a nonlinear quantum theory as there is a back reaction of matter on its own dynamics. The prevailing **absolutist** position is that **linear** quantum mechanics tells both space-time and matter how to be. Of course only experiment can decide who is right.

2. Nonlinear Quantum Effects

Entanglement causality channels abound, eliminate effects in some, others are still there. Our concern here is not whether such channels exist or not (they are ubiquitous) but asking which ones modify linear behavior substantially, and which modify it below practical levels. Experiments suggest that at low energies nonlinear effects are $\leq 10^{-20}$ times smaller than linear effects. Pushing this further we can speculate that if effects only appear at Planck energies they may not cause problems as then space-time itself becomes quantum with an ill-defined causal structure making it nonsensical to talk about its violation. The next question is: can effects be large at the Plank scale and still be suppressed at low energies?

A hint comes from considering the Doebner-Goldin (DG) equation, which is the nonlinear evolution connected to representations of the diffeomorphism group. Explicitly the one-particle equation is

$$\mathrm{i}\hbar\partial_t\psi_s = F_s\psi_s = -\frac{\hbar^2}{2m_s}\nabla^2\psi_s + \mathrm{i}D_s\hbar\left(\nabla^2\psi_s + \frac{|\nabla\psi_s|^2}{|\psi_s|^2}\psi_s\right) + R_s(\psi)\psi$$

where s labels the particle's species, D_s is a physical constant and $R_s(\psi)$ is **real** and complex homogeneous of degree zero: $R_s(z\psi) = R_s(\psi)$.

Using a zero-momentum two-particle (a and b) EPR state ϕ, one finds that the difference, to first order in t, of the matrix element $(\psi(t), B\psi(t)) = t\Delta_1(B|p,q) + O(t^2)$ of an observable B on particle b between post-position (q) and post-momentum (p) measurement (at $t = 0$) upon particle a is, asymptotically (as $s \to \infty$)

$$\Delta_1(B|p,q) = 4snD_b(\phi, B\phi) + O(1)$$

where we use gaussian position states of width $1/s$ and n is the dimension of space.

One sees then that even if D_b is extremely small, under extreme localization the effect can be large.

The effect is relatively larger for longer wavelength modes. Roughly the ratio of nonlinear effects to linear ones is of order $(2mD_b/\hbar)sL^2$, where L is the wavelength of the affected mode.

Thus the DG equation suggests a nonlinear quantum gravity effect coupling short (Planck) wavelengths to long (possibly Hubble) wavelengths. The resulting nonlinear effects could be large while others still be suppressed at normal energies [10].

Is there any hope though of seeing a signature of nonlinear quantum mechanics? If one needs Planck energies, it does seem discouraging. However it has been recently recognized that some form of "quantum gravity phenomenology" (cosmic rays propagation, tests of violations of Lorentz invariance, CTP, or unitarity) is possible. Such effects are proposed within linear quantum mechanics, but can also, alternatively, be constructed as tests of nonlinear quantum theories which through other mechanisms leads to the same sort of effects. Unfortunately these tests do not distinguish between the two types of theories. Just as elliptical planetary orbits can be explained by enough epicycles, effects arising from nonlinear quantum mechanics can most likely be explained by more elaborate linear theories. Because of its radical nature, a nonlinear theory would only be accepted if it bring greater simplicity (such as ellipses over epicycles) or if there is some observation for which a linear quantum theory explanation is not readily forthcoming. Such may be the case in cosmology.

3. Nonlinear Quantum Gravity

One possible nonlinear effect has to do with the accelerated expansion of the universe and the problem of space-time singularities, which are generic in many situations. The singularity theorems depend on the satisfaction of so-called energy conditions on the stress-energy tensor $T_{\mu\nu}$. One such is the **dominant** energy condition: for U time-like, $T_{\mu\nu}U^{\mu}U^{\nu} \geq 0$ and $T^{\mu}{}_{\nu}U^{\nu}$ not space-like.

Since quantum gravity should resolve problems of singularities, a semiclassical theory of quantum gravity should violate some energy conditions. Now ordinary quantum field theory violates energy conditions, but such violations are limited by so-called quantum inequalities and so far it has not been shown that one can avoid singularities this way. On the empirical side, the accelerated expansion of the universe possibly does violate the dominant energy condition as the observational data concerning the so-called "dark energy" component of the universe is consistent with an equation of state $p = w\rho$ with $w < -1$, where $\rho > 0$ is energy density and $p < 0$ is pressure. Such a dark energy has been dubbed "phantom energy" and has a series of remarkable properties, one of which is that black holes accreting phantom energy can lose mass instead of gaining it [1].

The space-time region outside a black hole event horizon is about as causally remote from the region inside as can be imagined. The metric has a central singularity. According to present ideas, near the singularity one enters a Planck regime and only some form of quantum gravity can give account of the physics, in particular avoiding a true singularity. Near such a would-be singularity a semi-classical theory will violate some of the usual energy conditions. Since phantom energy violates the dominant energy condition and since phantom energy can also diminish the central singularity of a black hole through a reduction of its mass, one can speculate that quantum gravity uses phantom energy to resolve singularities. Phantom energy in our universe seems to be tied to Hubble-scale processes, but singularities are Planck-scale. From what was said above it seems possible that nonlinear quantum mechanics can by relating these two scales be responsible for the presence of phantom energy and at the same time resolve (at least some) space-time singularities. Under such a strange hypothesis nonlinear quantum mechanics, which can relate short- and long-wavelengths processes, would entangle them, this entanglement becomes a causal channel between the interior and exterior regions of the event horizon, the would-be singularity region would act as a decoherence environment for short-wavelength and excite the long-wavelength partner modes into Hubble-size phantom energy states which by accretion diminish the black-hole mass and in time remove the would-be singularity. This scenario is explained in greater detail in [11]. Space limitation here allows us only to address a few relevant issues.

There is one cosmological observational consequence of such a hypothesis. The universe today is dominated by dark energy and dark matter. Assume the energy is phantom, and that it is produced by nonlinear quantum processes within black holes as they accrete matter (or not), then at some time in the cosmic epoch there would be energy flow from the matter sector to the dark energy sector. Such a two-sector situation would be modelled by the following FRW equations valid during a certain time period of cosmic evolution, including the current one

$$(\dot{a}/a)^2 = (\kappa_0^2/3)(\rho_m + \rho_{\rm ph})$$

$$\dot{\rho}_m + 3H\rho_m = -b\rho_m, \qquad \dot{\rho}_{\rm ph} + 3\gamma H\rho_{\rm ph} = b\rho_m.$$

Here ρ_m and $\rho_{\rm ph}$ are the matter and phantom energy densities, $\kappa_0 = 8\pi G$, $H = \dot{a}/a$ is the Hubble parameter, $\gamma = w + 1 < 0$, and b represents the coupling of matter to phantom energy mediated by black-hole singularities.

The above equations fall under the broad category of interacting dark energy models. The prevailing hypothesis seems to be that energy flows from the dark energy sector to the dark matter one. Our hypothesis that at some epoch the flow is in the other directions is a distinguishing feature. Introducing $\rho = \rho_m + \rho_{\text{ph}}$ and $\Omega_m = \rho_m/\rho$ one can deduce

$$b = 3wH(1 - \Omega_m) - \dot{\Omega}_m/\Omega_m \tag{1}$$

the right hand side of of which can in principle be evaluated by empirical data.

The only definite prediction we can make about the b function is that it must be positive at some time. It is positive if energy flows from matter to phantom, and negative otherwise. If our scenario is true, the sign of the right hand side of (1) should be positive during some part of the universe's evolution after the radiation dominated era. Whether it should still be positive at the present moment is not a-priori clear as phantom energy could also be transferring energy to dark matter by some mechanism making the net energy flow from phantom to matter. Presently considered interacting dark energy theories apparently all consider negative b, hence positivity could very well be a true signature of nonlinear quantum mechanics which can naturally accommodate it. A failure will impose non-trivial constraints on any nonlinear version of quantum gravity eliminating some of the more striking aspects the theory might have otherwise. The right-hand side of (1) is an important empirical datum in our search for a better understanding of quantum features of space-time.

Though there is data concerning the evolution of dark matter [2], it does not seem to be sufficiently precise to determine the derivative $\dot{\Omega}_m$ with any accuracy. Szydłowski [12] argues for a negative sign for b_0 (the value now) based on SNIa supernova observations, but again, given the uncertainties in the data, a positive sign cannot be entirely ruled out.

A vision of quantum gravity that would allow for the above type of effect can be formulated in analogy with thermodynamics. Think of space-time as a ferromagnet. The metric would correspond to magnetization, Planck energy to critical energy (temperature), and the region near a singularity as the disordered phase. This phase would not be metrically related to the ordered phase (since order **is** metricity) where a metric structure exists, allowing thus for non-local effects to be mediated by would-be classical singularities. In our view, nonlinear quantum processes would be relevant near Planck energy ($\approx 10^{19}$ Gev), and because of short-long wavelength entanglement, also at Hubble energy ($\approx 10^{-35}$ ev). Between these energies ordinary linear quantum mechanics holds sway, and beyond in either direction is the disordered phase, completing thus a full circle.

We now address the causality issue of nonlinear quantum mechanics. A causal link through entangled Planck and Hubble wavelengths does not create the usual causality problems since to detect a mode of wavelength L requires an apparatus acting on a time scale of order L/c. Any Hubble size mode created non-locally by a Planck-scale process could only be detected at a time that is already future time-like to the creation event, avoiding, in a new way, the usual causality violation problem claimed of nonlinear quantum mechanics. Nonlinear quantum gravity could thus escape the problem of an abundance of causal channels due to entanglement, limiting them only to the two far ends of the energy spectrum (Planck and Hubble) while at the same time resolve in a novel way the singularity issues of classical general relativity.

4. Things to Do

The lack of a clear mathematical theory and empirical data makes the above exposition highly speculative. Certain avenues of future research naturally present themselves:

1) Improve the cosmological model and compare to observations, especially concerning the direction of energy flow between matter and dark energy. The direction of this flow during the epoch when these two sectors dominate the dynamics of the universe is an important datum for nonlinear quantum theory.

2) Develop the decoherence theory for nonlinear evolution, especially for the DG equation. This would allow the causality questions that have so far been limited to measurement situations to be adequately addressed in contexts where measurement activity does not make sense.

3) Investigate to what extent nonlinear quantum mechanics violates quantum inequalities obeyed by linear quantum mechanics. Violation of the energy conditions by ordinary quantum field theory is limited by quantum inequalities to such an extent that space-time singularities (such as wormhole collapse) are not avoided. A nonlinear theory could be more effective in this regard.

4) Investigate further the apparent connection of nonlinear quantum mechanics and noncommutative spaces as suggested by Singh *et al.* [6], and Singh [7]. This would further our understanding of the connection between nonlinear quantum mechanics and geometry, already implied by diffeomorphism group representations, and give further support to the idea that nonlinear quantum mechanics may have something to do with our world.

Acknowledgements

This research was partially supported by the Conselho Nacional de Desenvolvimento Cienífico e Tecnológico (CNPq).

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