76. *Poisson Transformations on Affine Symmetric Spaces*[*)]

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1. Introduction. Let G be a connected real semisimple Lie group with finite center, σ any involutive analytic automorphism of G, and H any closed subgroup which lies between the totality G^σ of fixed points of σ and the identity component of G^σ. Then the homogeneous space G/H is an *affine symmetric space*. It is known that any eigenfunction of all invariant differential operators on a Riemannian symmetric space of the noncompact type can be represented by the Poisson integral of a hyperfunction on its maximal boundary, which was conjectured by Helgason [2] and completely solved by [3]. In this note we define a generalization of the Poisson integral on G/H and extend the result in [3] to the case of G/H. For example, by the involution $(g, g')\mapsto(g', g)$ of $G\times G$, the group G itself can be regarded as an affine symmetric space and then our result gives integral representations of simultaneous eigenfunctions of biinvariant differential operators on G. If G/H satisfies some conditions, this problem was studied by [6] (cf. also [5]). An extended version of this note is to appear later.

2. Preliminary results. We fix a Cartan involution θ of G commuting with σ (cf. [1] for the existence of θ) and also denote by σ and θ the corresponding involutions of the Lie algebra $\mathfrak{g}$ of G. Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ (resp. $\mathfrak{g}=\mathfrak{h}+\mathfrak{q}$) be the decomposition of $\mathfrak{g}$ into $+1$ and -1 eigenspaces for θ (resp. σ). Let $\mathfrak{a}$ be a maximal abelian subspace of $\mathfrak{p}\cap\mathfrak{q}$, $\mathfrak{a}_\mathfrak{p}$ a maximal abelian subspace of $\mathfrak{p}$ containing $\mathfrak{a}$, and $\tilde{\mathfrak{j}}$ a Cartan subalgebra of $\mathfrak{g}$ containing both $\mathfrak{a}_\mathfrak{p}$ and a maximal abelian subspace of $\mathfrak{m}\cap\mathfrak{q}$, where $\mathfrak{m}$ denotes the centralizer of $\mathfrak{a}_\mathfrak{p}$ in $\mathfrak{k}$. Furthermore we put $\mathfrak{j}=\tilde{\mathfrak{j}}\cap\mathfrak{q}$ and $\mathfrak{t}=\mathfrak{j}\cap\mathfrak{k}$. For a linear subspace $\mathfrak{b}$ of $\mathfrak{g}$, $\mathfrak{b}_c$ denotes the complexification of $\mathfrak{b}$. If $\mathfrak{b}$ is a subalgebra, $U(\mathfrak{b})$ denotes the universal enveloping algebra of $\mathfrak{b}_c$. Let Ad (resp. ad) denote the adjoint representation of G (resp. $\mathfrak{g}_c$) on $\mathfrak{g}_c$ or $U(\mathfrak{g})$. For a linear subspace $\tilde{\mathfrak{a}}$ of $\tilde{\mathfrak{j}}$, $\tilde{\mathfrak{a}}^*$ denotes the dual space of $\tilde{\mathfrak{a}}$ and $\tilde{\mathfrak{a}}_c^*$ the complexification of $\tilde{\mathfrak{a}}^*$. Then we put $\mathfrak{g}_c(\tilde{\mathfrak{a}};\lambda)=\{X\in\mathfrak{g}_c;\ ad(Y)X=\lambda(X)$ for all $Y\in\tilde{\mathfrak{a}}\}$ for any λ in $\tilde{\mathfrak{a}}_c^*$ and moreover $\Sigma(\tilde{\mathfrak{a}})=\{\lambda\in\tilde{\mathfrak{a}}_c^*-\{0\};\ \mathfrak{g}_c(\tilde{\mathfrak{a}};\lambda)\neq\{0\}\}$. By the Killing form $\langle\ ,\ \rangle$ of the complex Lie algebra $\mathfrak{g}_c$, we identify $\mathfrak{j}_c^*$ and $\mathfrak{j}_c$, and therefore $\tilde{\mathfrak{a}}_c^*$ is identified with a subspace of $\mathfrak{j}_c^*$. Let K denote the analytic subgroup of G cor-

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responding to $\mathfrak{k}$, and M (resp. M^*) the centralizer (resp. the normalizer) of $\mathfrak{a}_\mathfrak{p}$ in K. The quotient group M^*/M is the Weyl group of the restricted root system $\Sigma(\mathfrak{a}_\mathfrak{p})$, which we denote by $W(\mathfrak{a}_\mathfrak{p})$.

Lemma 1. i) *$\mathfrak{j}$ is a maximal abelian subspace of $\mathfrak{q}$.*

ii) *$\Sigma(\mathfrak{j})$ and $\Sigma(\mathfrak{a})$ satisfy the axiom of root systems. Let $W(\mathfrak{j})$ and $W(\mathfrak{a})$ denote the corresponding Weyl groups.*

iii) *Put $W(\mathfrak{j})_\theta=\{w\in W(\mathfrak{j})\,;\, w|_\mathfrak{a}=id\}$, $W^\theta(\mathfrak{j})=\{w\in W(\mathfrak{j})\,;\, w(\mathfrak{a})=\mathfrak{a}\}$, $W(\mathfrak{a}_\mathfrak{p})_\sigma=\{w\in W(\mathfrak{a}_\mathfrak{p})\,;\, w|_\mathfrak{a}=id\}$, $W^\sigma(\mathfrak{a}_\mathfrak{p})=\{w\in W(\mathfrak{a}_\mathfrak{p})\,;\, w(\mathfrak{a})=\mathfrak{a}\}$ and $W(\mathfrak{a}_\mathfrak{p}\,;H)=(M^*\cap H)/(M\cap H)$. Then $W(\mathfrak{a}_\mathfrak{p})_\sigma\subset W(\mathfrak{a}_\mathfrak{p}\,;H)\subset W^\sigma(\mathfrak{a}_\mathfrak{p})$ and the quotient groups $W^\theta(\mathfrak{j})/W(\mathfrak{j})_\theta$ and $W^\sigma(\mathfrak{a}_\mathfrak{p})/W(\mathfrak{a}_\mathfrak{p})_\sigma$ are naturally identified with $W(\mathfrak{a})$.*

iv) *We can define a system of compatible orderings of the root systems $\Sigma(\tilde{\mathfrak{j}})$, $\Sigma(\mathfrak{a}_\mathfrak{p})$, $\Sigma(\mathfrak{j})$ and $\Sigma(\mathfrak{a})$. Let $\Sigma(\tilde{\mathfrak{j}})^+$, $\Sigma(\mathfrak{a}_\mathfrak{p})^+$, $\Sigma(\mathfrak{j})^+$ and $\Sigma(\mathfrak{a})^+$ denote the corresponding sets of all positive roots.*

Let ρ denote half the sum of the elements of $\Sigma(\tilde{\mathfrak{j}})^+$ and $\boldsymbol{D}(G/H)$ the $\boldsymbol{C}$-algebra of all G-invariant differential operators on G/H. For any $D\in U(\mathfrak{g})$, we define $D_\mathfrak{j}\in U(\mathfrak{j})$ so that $D-D_\mathfrak{j}\in\sum_{\alpha\in\Sigma(\mathfrak{j})^+}\mathfrak{g}_c(\mathfrak{j}\,;-\alpha)U(\mathfrak{g})+U(\mathfrak{g})\mathfrak{h}$ and we denote by $\bar{\iota}(D)$ the image of $D_\mathfrak{j}$ under the automorphism of $U(\mathfrak{j})$ which maps X to $X-\rho(X)$ for every $X\in\mathfrak{j}$. Putting $U(\mathfrak{g})^H=\{D\in U(\mathfrak{g})\,;\, Ad(h)D=D$ for any $h\in H\}$, we have

Lemma 2. *The map $\bar{\iota}$ and the natural identification $\boldsymbol{D}(G/H)\simeq U(\mathfrak{g})^H/U(\mathfrak{g})\mathfrak{h}\cap U(\mathfrak{g})^H$ induce the surjective $\boldsymbol{C}$-algebra isomorphism*

$$\iota:\boldsymbol{D}(G/H)\xrightarrow{\sim}I(\mathfrak{j}),$$

where $I(\mathfrak{j})$ denotes the set of all $W(\mathfrak{j})$-invariants in $U(\mathfrak{j})$.

Extending any $\nu\in\mathfrak{j}_c^*$ to an algebra homomorphism of $U(\mathfrak{j})$ onto $\boldsymbol{C}$, we define a system of differential equations on G/H:

$$\mathcal{M}_\nu:(D-\nu(\iota(D)))u=0 \qquad \text{for all } D\in\boldsymbol{D}(G/H).$$

3. Eigenspaces and principal series. For a real analytic manifold U we denote by $\mathcal{B}(U)$ (resp. $\mathcal{D}'(U)$, $C^\infty(U)$, $\mathcal{A}(U)$) the space of Sato's hyperfunctions (resp. Schwartz' distributions, indefinitely differentiable functions, real analytic functions) on U. Then $\mathcal{B}(U)\supset\mathcal{D}'(U)\supset C^\infty(U)\supset\mathcal{A}(U)$. Each $x\in G$ acts on the linear space $\mathcal{B}(G)$ by the left translation

$$\tau_x: f(g)\mapsto(\tau_x f)(g)=f(x^{-1}g) \qquad \text{for } f\in\mathcal{B}(G)$$

and $\mathcal{B}(G/H)$ is identified with the G-submodule $\{f\in\mathcal{B}(G)\,;\, f(gh)=f(g)$ for all $h\in H\}$ of $\mathcal{B}(G)$. We define a Fréchet space

$$C_*(G/H)=\{f\in C^\infty(G/H)\,;\, \|f\|_{D,j}<\infty \text{ for any } (D,j)\in U(\mathfrak{g})\times\boldsymbol{Z}\}$$

with the seminorms $\|\ \|_{D,j}$ and the dual space $C'_*(G/H)$ of $C_*(G/H)\otimes d\bar{g}$, where any $D\in U(\mathfrak{g})$ is regarded as a left-invariant differential operator on G, $\|f\|_{D,j}=\sup_{(k,X)\in K\times\mathfrak{a}}|(Df)(k\exp X)|\exp(j\langle X,X\rangle^{1/2})$ and $d\bar{g}$ is an invariant measure on G/H. We denote by $\mathcal{F}(G/H\,;\mathcal{M}_\nu)$ the space of all solutions of $\mathcal{M}_\nu$ in $\mathcal{F}(G/H)$, where $\mathcal{F}=\mathcal{B}$, $\mathcal{D}'$, C'_*, C^∞ or $\mathcal{A}$.

We put $\mathfrak{n}=\mathfrak{g}\cap\sum_{\alpha\in\Sigma(\mathfrak{a}_\mathfrak{p})^+}\mathfrak{g}_c(\mathfrak{a}_\mathfrak{p}\,;\alpha)$. Let N, A and $A_\mathfrak{p}$ denote the analytic subgroups of G corresponding to $\mathfrak{n}, \mathfrak{a}$ and $\mathfrak{a}_{\mathfrak{p}}$, respectively. We define a parabolic subgroup $P_\sigma=\bigcup_{w\in W(\mathfrak{a}_{\mathfrak{p}})_\sigma} MA_{\mathfrak{p}}NwN$. Let $P_\sigma=M_\sigma A_\sigma N_\sigma$ be the Langlands decomposition of P_σ such that $M_\sigma A_\sigma$ centralizes $\mathfrak{a}$. Let $\mathfrak{m}_\sigma$ denote the Lie algebra of M_σ and let $\mathfrak{m}_\sigma=\mathfrak{m}(\sigma)+\mathfrak{g}(\sigma)$ be the decomposition of $\mathfrak{m}_\sigma$ into the direct sum of a compact reductive Lie algebra $\mathfrak{m}(\sigma)$ and a noncompact semisimple Lie algebra $\mathfrak{g}(\sigma)$. Let $M(\sigma)_0$ and $G(\sigma)$ denote the analytic subgroups of G corresponding to $\mathfrak{m}(\sigma)$ and $\mathfrak{g}(\sigma)$, respectively. Putting $M(\sigma)=M(\sigma)_0 Ad^{-1}(Ad(K)\cap \exp ad(\sqrt{-1}\mathfrak{a}_{\mathfrak{p}}))$, we have

Lemma 3. *$M(\sigma)\subset M$, $G(\sigma)\subset H$ and $M_\sigma=M(\sigma)G(\sigma)$.*

Let $w_1, w_2, \cdots, w_r$ be representatives of the factor set
$$W(\mathfrak{a}_{\mathfrak{p}}; H)\backslash W^\sigma(\mathfrak{a}_{\mathfrak{p}}),$$
where $r=[W^\sigma(\mathfrak{a}_{\mathfrak{p}}): W(\mathfrak{a}_{\mathfrak{p}}; H)]$. We choose representatives $\bar{w}_i$ of w_i in M^* so that $\mathfrak{m}_\sigma\cap Ad(\bar{w}_i^{-1})\mathfrak{h}=\mathfrak{m}_\sigma\cap\mathfrak{h}$ $(i=1, \cdots, r)$. We put $M(\sigma)_i=M(\sigma)\cap \bar{w}_i^{-1}H\bar{w}_i$, $Q_\sigma=G(\sigma)A_\sigma N_\sigma$ and $P_{\sigma,i}=M(\sigma)_i Q_\sigma$. Let $\widehat{M(\sigma)}^i$ denote the set of all equivalence classes of irreducible unitary representations of $M(\sigma)$ with non-zero $M(\sigma)_i$-fixed vectors. For a $\delta\in\widehat{M(\sigma)}^i$, let χ_δ be the corresponding character and let $\bar{\delta}\in(\mathfrak{m}(\sigma)\cap\tilde{\mathfrak{j}})_c^*$ be the dominant weight of the corresponding representation of $\mathfrak{m}(\sigma)$ which is compatible with the orderings in Lemma 1. Then $\bar{\delta}\in\mathfrak{t}_c^*$. Conversely, for a $\Lambda\in\mathfrak{t}_c^*$, we put $\mathfrak{W}(\Lambda)=\{(\delta, i);\ i=1, \cdots, r, \delta\in\widehat{M(\sigma)}^i$ and $\bar{\delta}+\rho|_{\mathfrak{t}}\in W(\mathfrak{j})_\theta(-\Lambda)\}$. Last in this section we define idempotent maps
$$p_\delta^i: \mathcal{F}(G)\longrightarrow\mathcal{F}(G)$$
$$f(g)\longmapsto(p_\delta^i f)(g)=\chi_\delta(e)\int_{M(\sigma)_i}\int_{M(\sigma)}\chi_\delta(m_i m)f(gm)dm_i dm$$
and G-modules
$$\mathcal{F}(G/Q_\sigma; L_\lambda)=\{f\in\mathcal{F}(G);\ f(gxan)=f(g)\exp((\lambda-\rho)(\log a))$$
$$\text{for all } g\in G,\ x\in G(\sigma),\ a\in A_\sigma \text{ and } n\in N_\sigma\},$$
$$\mathcal{F}(G/P_{\sigma,i}; L_\lambda)_\delta=\{f\in\mathcal{F}(G/Q_\sigma; L_\lambda);\ p_\delta^i f=f\},$$
where $i\in\{1, \cdots, r\}$, $\delta\in\widehat{M(\sigma)}^i$, $\mathcal{F}\in\{\mathcal{B}, \mathcal{D}', C^\infty$ and $\mathcal{A}\}$ and $\lambda\in\mathfrak{a}_c^*$. In this note any measure on any compact group is the Haar measure so normalized that the total measure equals one. We call every $\mathcal{F}(G/P_{\sigma,i}; L_\lambda)_\delta$ a *function space of principal series* for G/H.

4. Poisson transformations. To define Poisson kernels we prepare the following lemma which easily follows from [4].

Lemma 4. i) *$\bigcup_{i=1}^r H\bar{w}_i P_\sigma$ is a disjoint union of the open subsets $H\bar{w}_i P_\sigma$ of G and the union is dense in G.*

ii) *$(\bar{w}_i^{-1}H\bar{w}_i)\cap(M(\sigma)AN_\sigma)=M(\sigma)_i$ for $i=1, \cdots, r$.*

For $i=1, \cdots, r$, $\delta\in\widehat{M(\sigma)}^i$, $\lambda\in\mathfrak{a}_c^*$ and $g\in G$, we put
$$h_\sigma^i(\delta, \lambda; g)=\begin{cases}\displaystyle\int_{M(\sigma)_i}\chi_\delta(m_i m)\exp((\lambda-\rho)(X))dm_i \\ \qquad \text{if } g\in H\bar{w}_i m(\exp X)N_\sigma \text{ with } m\in M(\sigma) \text{ and } X\in\mathfrak{a}, \\ 0 \qquad \text{if } g\notin H\bar{w}_i P_\sigma.\end{cases}$$

Lemma 5. *The functions $h^i_\sigma(\delta, \lambda; g)$ on G are continuous when* $\mathrm{Re}\,\langle\lambda-\rho, \alpha\rangle>0$ *for all $\alpha \in \Sigma(\mathfrak{a})^+$. They can be meromorphically extended for all $\lambda \in \mathfrak{a}^*_c$ as distributions on G. The extensions will be denoted by the same notation and called Poisson kernels.*

Then *partial Poisson transformations* $\mathcal{P}^i_{\sigma,\delta,\lambda}$ of $\mathcal{B}(G/Q_\sigma; L_\lambda)$ are defined by

$$(\mathcal{P}^i_{\sigma,\delta,\lambda}f)(g)=\int_K f(k)h^i_\sigma(\delta, -\lambda; g^{-1}k)dk \qquad \text{for } f \in \mathcal{B}(G/Q_\sigma; L_\lambda).$$

5. The main theorem. We put $\mathfrak{A}=\{\Sigma n_\alpha\alpha; \alpha \in \Sigma(\mathfrak{j}) \text{ and } n_\alpha \in \boldsymbol{Z}\}$, $\mathfrak{B}=\{\beta \in \Sigma(\mathfrak{j}); \beta|_{\mathfrak{t}}=0 \text{ and } \mathfrak{g}_c(\mathfrak{j}; \beta)\cap((\mathfrak{k}\cap\mathfrak{q})+(\mathfrak{h}\cap\mathfrak{p}))\neq\{0\}\}$ and for each root β in $\mathfrak{B}$ we choose a root vector X_β in $\mathfrak{g}_c(\mathfrak{j}; \beta)\cap((\mathfrak{k}\cap\mathfrak{q})+(\mathfrak{h}\cap\mathfrak{p}))$ so that $2\langle X_\beta, \theta(X_\beta)\rangle=-\langle\beta, \beta\rangle$. For a complex abelian vector space $\mathfrak{c}$ of $\mathfrak{k}_c$, we put $\mathfrak{C}(\mathfrak{c})=\{\mu|_{\mathfrak{c}}; \mu$ is a weight of an irreducible unitary representation of K with non-zero $(K\cap H\cap M_\sigma)$-fixed vectors, where μ belongs to the dual of a maximal complex abelian vector space of $\mathfrak{k}_c$ containing $\mathfrak{c}\}$. Then we can state our theorem.

Theorem 6. i) *The following conditions for ν in $\mathfrak{j}^*_c$ are equivalent:*

(1) $$\mathcal{A}(G/H; \mathcal{M}_\nu)=\{0\},$$

(2) $$\mathcal{B}(G/H; \mathcal{M}_\nu)=\{0\},$$

(3) $$\mathfrak{W}(w\nu|_{\mathfrak{t}})=\phi \qquad \text{for any } w \in W(\mathfrak{j}).$$

ii) *Assume an element $\nu=(\Lambda, \lambda) \in \mathfrak{j}^*_c$ ($\Lambda \in \mathfrak{t}^*_c, \lambda \in \mathfrak{a}^*_c$) satisfies*

(A. 0) *λ is not a pole of $h^i_\sigma(\delta, -\lambda; g)$ for any $(\delta, i) \in \mathfrak{W}(\Lambda)$,*

(A. 1) *$-2\langle\lambda, \alpha\rangle/\langle\alpha, \alpha\rangle \notin \boldsymbol{N}\equiv\{1, 2, 3, \cdots\}$ for any $\alpha \in \Sigma(\mathfrak{a})^+$,*

(A. 2) *$w(\nu-\rho)|_{\mathfrak{t}}\neq\alpha|_{\mathfrak{t}}+\mu|_{\mathfrak{t}}$ for any $(\alpha, \mu, w) \in \mathfrak{A}\times\mathfrak{C}(\mathfrak{t}_c)\times(W(\mathfrak{j})-W^\theta(\mathfrak{j}))$,*

(A. 3) *$\langle w\lambda, \beta\rangle\neq\langle\alpha+\rho, \beta\rangle+\sqrt{-1}\langle\mu, X_\beta-\sigma(X_\beta)\rangle$ for any $(\alpha, \beta, w) \in \mathfrak{A}\times\mathfrak{B}\times W^\theta(\mathfrak{j})$ and any $\mu \in \mathfrak{C}(\boldsymbol{C}(X_\beta-\sigma(X_\beta)))$.*

Then the Poisson transformation

$$\mathcal{P}_{\sigma,\nu}: \bigoplus_{(\delta,i)\in\mathfrak{W}(\Lambda)} \mathcal{B}(G/P_{\sigma,i}; L_\lambda)_\delta \longrightarrow \mathcal{B}(G/H)$$

defined by $\mathcal{P}_{\sigma,\nu}((f_{\delta,i}))=\sum\mathcal{P}^i_{\sigma,\delta,\lambda}f_{\delta,i}$ ($f_{\delta,i} \in \mathcal{B}(G/P_{\sigma,i}; L_\lambda)_\delta$) is a G-isomorphism of the direct sum of function spaces of principal series for G/H onto the eigenspace $\mathcal{B}(G/H; \mathcal{M}_\nu)$ and induces a homeomorphism of $\bigoplus_{(\delta,i)\in\mathfrak{W}(\Lambda)}\mathcal{D}'(G/P_{\sigma,i}; L_\lambda)_\delta$ onto $C'_*(G/H; \mathcal{M}_\nu)$.

Here we remark that $\mathcal{M}_\nu=\mathcal{M}_{w\nu}$ for any $w \in W(\mathfrak{j})$ and that there exists an open dense subset $(\mathfrak{a}^*_c)'$ of $\mathfrak{a}^*_c$ such that every condition in Theorem 6 ii) holds if $\mathfrak{W}(\Lambda)\neq\phi$ and $\lambda \in (\mathfrak{a}^*_c)'$. Moreover, defining Poisson kernels by a suitable analytic continuation of linear combinations of $h^i_\sigma(\delta, \lambda; g)$, we can omit the condition (A. 0).

The proof of Theorem 6 is based on the construction of the inverse of $\mathcal{P}_{\sigma,\nu}$, which is the map of taking the boundary values of eigenfunctions in $\mathcal{B}(G/H; \mathcal{M}_\nu)$. This method of the proof is also used in [3] and [6]. By the way, if all the boundary values of an eigenfunction in

$\mathcal{B}(G/H;\mathcal{M}_\nu)$ vanish, the function must be zero. This implies the following

Remark 7. For any ν in $\mathfrak{j}_c^*$ there exist closed G-invariant linear subspaces E_j ($j \in N$) of $C^\infty(G/H;\mathcal{M}_\nu)$ and G-equivariant maps Φ_j of E_j to $\mathcal{B}(G/P_{\sigma,i_j};L_{\lambda_j})_{\delta_j}$ with the kernel E_{j+1}, respectively, such that $E_1 = C^\infty(G/H;\mathcal{M}_\nu) \supset \cdots \supset E_j \supset E_{j+1} \supset \cdots$, $E_j=\{0\}$ for a sufficiently large j and $\lambda_j - \rho|_{\mathfrak{t}} - \bar{\delta}_j \in \{w\nu; w \in W(\mathfrak{j})\}$.

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