

Now since C is convex, $\lambda x_n + (1-\lambda)x_m$ is in C , so that

$$\|x_n - x_m\|^2 < \frac{(\rho + \varepsilon)^2}{\lambda(1-\lambda)} - \frac{\rho^2}{\lambda(1-\lambda)} < \frac{(2\rho + \varepsilon)\varepsilon}{\alpha^2}.$$

Let $x_0 = \lim_{n \rightarrow \infty} x_n$, then x_0 is in C since C is closed, and it follows from the continuity of the norm that $\|x_0\| = \rho$. It is an immediate consequence of Lemma 1 and the condition (*) that the element x_0 is unique.

We shall now proceed to prove the above-mentioned statement. Let x_0 be an element of E which does not belong to M ; then the set $\{y - x_0 \mid y \in M\}$ is clearly convex and closed, so by Lemma 2 there is a unique element y_0 such that $\|y_0 - x_0\| \leq \|y - x_0\|$ for all $y \in M$.

It is easy to see that for all $y \in M$, we have

$$\|y - y_0\| \leq \|y - x_0\|.$$

In fact, if $\|y - y_0\|$ is greater than $\|y - x_0\|$ for some $y \in M$, then in virtue of Lemma 1 there exists a λ , $0 < \lambda < 1 - \alpha$, such that

$$\|\lambda y + (1-\lambda)y_0 - x_0\| < \|y_0 - x_0\|$$

which is a contradiction since $\lambda y + (1-\lambda)y_0$ is in M .

Now we define $I^*(x) = I(y) + \lambda y_0 = y + \lambda y_0$ for any $x = y + \lambda x_0$, $y \in M$, $\lambda \in R$.

Then it is clear that I^* is linear and an extension of I to $M + Rx_0$, and hence it remains only to prove the continuity of I^* and that the norm is 1. For that matter the relation

$$\|y + \lambda y_0\| = |\lambda| \cdot \|\lambda^{-1}y + y_0\|$$

holds for $\lambda \neq 0$.

On the other hand, $\|\lambda^{-1}y + y_0\| \leq \|-\lambda^{-1}y - x_0\|$, and so

$$\|y + \lambda y_0\| \leq \|y + \lambda x_0\|,$$

which guarantees the continuity of I^* and shows the norm is 1. Thus we have reached the desired conclusion.

Additions and Corrections to Shouro Kasahara:

“A Note on f -completeness”

(Proc. Japan Acad., 30, No. 7, 572-575 (1954))

Pages 572-573, delete “Proposition 2”.

Page 574, delete “Proposition 6”.

Page 574, line 19 from foot, for “mapping of W , we have $p(I^*(x)) \leq p(x)$ for any $p \in (p_\alpha)$ and $x \in E$.” read “mapping of W , concerning to $p \in (p_\alpha)$, we have $p^*(I(x)) \leq p(x)$ for any $x \in E$.”

Page 574, lines 26-29, delete “Now, since...inequality (*) for u^* .”

Page 574, line 10 from foot, for “for any $p \in (p_\alpha)$ there is” read “there exist a $p \in (p_\alpha)$ and”.

Page 574, line 2 from foot, for “same a ” read “same p and a ”.