171. *On the Asymptotic Behaviour of Brauer-Siegel Type of Class Numbers of Positive Definite Quadratic Forms*

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For natural numbers n and D, $H_n(D)$ denotes the class number of positive definite integral matrices of degree n and determinant D, where two matrices A and B are in the same class if and only if $A={}^tTBT$ holds for some $T\in GL(n,Z)$. $W(n,D)$ denotes $\sum E(S)^{-1}$ with $E(S)=\#\{T\in GL(n,Z)\,|\,{}^tTST=S\}$, where S runs over representatives of classes of positive definite integral matrices of degree n and determinant D.

In [1] we have proved

Lemma. *For any fixed natural number n, we have*

$$H_n(D)\sim 2W(n,D) \qquad \text{as } D\to\infty.$$

From this lemma we see easily

Theorem 1. *There exists a sequence of natural numbers $\{D(n)\}_{n=1}^{\infty}$ satisfying*

$$H_{n_k}(D_k)\sim 2W(n_k,D_k) \qquad \text{as } \max(n_k,D_k)\to\infty$$

with, for any sequence $(n_k,D_k)_{k=1}^{\infty}$, $D_k>D(n_k)$ for all k.

If moreover n_k is odd and D_k is odd and square-free, then we have

$$(*)\qquad H_{n_k}(D_k)\sim\pi^{-(n_k(n_k+1))/4}\prod_{l=1}^{n_k}\Gamma\left(\frac{l}{2}\right)\prod_{l=1}^{(n_k-1)/2}\zeta(2l)D_k^{(n_k-1)/2}.$$

Our aim in this note is to announce an explicit value of $D(n)$ for odd n;

Theorem 2. *If n_k is odd and $n_k^2/\log\log D_k\to 0$ as $k\to\infty$, then*

$$H_{n_k}(D_k)\sim 2W(n_k,D_k) \qquad \text{as } k\to\infty.$$

If moreover D_k is odd and square-free, then we have $(*)$ *in Theorem* 1.

This theorem is obtained by giving an explicit value of constants c_i and $c_i(\varepsilon)$ except c_{22} in [1]. If c_{22} is explicitly given, then we have an explicit value of $D(n)$ for even n.

Remark 1. There is no essential difficulty to generalize Theorems 1 and 2 to the cases of algebraic number fields.

Remark 2. In our method we can not avoid that $D(n)$ tends to the infinity if $n\to\infty$. But the author does not know whether $\sup_n D(n)$ can be bounded or not. For example, let us consider cases of even unimodular positive definite quadratic forms; then the Siegel formula implies

$$\frac{1}{2}H_{8n}+M'_{8n}\sim M_{8n} \qquad \text{as } n\to\infty,$$

where firstly H_{8n} is the class number of even unimodular positive definite quadratic forms of degree $8n$ which have no non-trivial units, secondly $M'_{8n}=2\sum E(S)^{-1}$ where S runs over representatives of classes of even unimodular positive definite quadratic forms of degree $8n$ which do not represent 2 but have a non-trivial unit, and finally $M_{8n}=2^{1-8n}\dfrac{B_{2n}}{(4n)!}\prod_{j=1}^{4n-1}B_j$ (=the weight).

For $8n=24$, the quadratic form concerning M'_{24} is only one and it is a so-called Leech lattice. It seems natural to expect $H_{8n}\sim 2M_{8n}$ or more strongly [the class number of even unimodular positive definite quadratic forms of degree $8n$] $\sim 2M_{8n}$. To answer this question, however, detailed studies on unit groups will be required.

Reference

[1] Y. Kitaoka: Two theorems on the class number of positive definite quadratic forms. Nagoya Math. J., **51**, 79–89 (1973).