125. Structure of Cohomology Groups Whose Coefficients are Microfunction Solution Sheaves of Systems of Pseudo-Differential Equations with Multiple Characteristics. II

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(Comm. by Kôsaku YOSIDA, M. J. A., Oct. 12, 1974)

This is a continuation of our preceding note Kashiwara-Kawai-Oshima [1], hereafter referred to as K-K-O [1]. The purpose of this note is to investigate the structure of cohomology groups whose coefficients are microfunction solution sheaf of a system $\mathcal{M}$ of pseudo-differential equations which satisfies conditions (2)$\sim$(8) in K-K-O [1], but does not necessarily satisfy condition (9) in general. The details of this note will appear elsewhere.

In this note we use the same notations as in K-K-O [1]. For example, W denotes the real locus of $V_1\cap V_1^c=V_2\cap V_2^c$. Since W acquires canonically the structure of a purely imaginary contact manifold by condition (6) in K-K-O [1], sheaf $\mathcal{C}_W$ of microfunctions and sheaf $\mathcal{P}_W$ of pseudo-differential operators can be defined on W.

When $\kappa=\left.\dfrac{\sigma(Q)}{\{\sigma(P_2),\sigma(P_1)\}}\right|_{V_1\cap V_2}$ takes an integral value, the structure of κ plays an important role in calculating the cohomology groups. So we give the following preparatory consideration concerning lower order terms.

Let R be a pseudo-differential operator on W whose principal symbol is κ. Such a pseudo-differential operator R is uniquely determined up to inner automorphism of $\mathcal{P}_W$ by condition (5) in K-K-O [1]. (See Theorem 2.1.2 in Chap. II of Sato-Kawai-Kashiwara [2].) Taking account of this fact, we denote by $\mathcal{L}_l$ the pseudo-differential equation $(R-l)u=0$ on W for $l\in \boldsymbol{Z}$.

In order to calculate the cohomology groups when κ takes an integral value, we should study in the following four cases classified according to the signatures of the generalized Levi forms of V_1, V_2 and $T^*_{V_1}X^C\cap T^*_{V_2}X^C$. We denote by L_j the generalized Levi form of V_j ($j=1,2$, respectively) and by L the hermitian form $\{\xi,\bar{\eta}\}$ on $(T^*_{V_1}X^C)_{x^*}\cap(T^*_{V_2}X^C)_{x^*}$.

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Case A_1. The signature of L_1 is $(d-q-1, q+1)$, that of L_2 is $(d-q, q)$ and that of L is $(d-q-1, q)$.

Case A_2. The signature of L_2 is $(d-q-1, q+1)$, that of L_1 is $(d-q, q)$ and that of L is $(d-q-1, q)$.

Case B_1. Both the signature of L_1 and L_2 are $(d-q, q)$ and the signature of L is $(d-q, q-1)$. $(1 \leqq q \leqq d)$

Case B_2. Both the signature of L_1 and L_2 are $(d-q, q)$ and the signature of L is $(d-q-1, q)$. $(0 \leqq q \leqq d-1)$

Theorem 1. *In Case* A_1, *we have*

$$\mathcal{E}xt^j_{\mathcal{P}}(\mathcal{M}, \mathcal{C}) \cong \bigoplus_{l=0,-1,-2,\cdots} \mathcal{E}xt^{j-q}_{\mathcal{P}_W}(\mathcal{L}_l, \mathcal{C}_W)$$

for every j, *and, in Case* A_2, *we have*

$$\mathcal{E}xt^j_{\mathcal{P}}(\mathcal{M}, \mathcal{C}) \cong \bigoplus_{l=1,2,3,\cdots} \mathcal{E}xt^{j-q}_{\mathcal{P}_W}(\mathcal{L}_l, \mathcal{C}_W)$$

for every j.

Theorem 2. *In Case* B_1, *we have*

$$\mathcal{E}xt^j_{\mathcal{P}}(\mathcal{M}, \mathcal{C}) = 0 \qquad \textit{for } j \neq q-1, q$$

and the following exact sequence holds:

$$0 \to \mathcal{E}xt^{q-1}_{\mathcal{P}}(\mathcal{M}, \mathcal{C}) \to \bigoplus_{l=0,-1,-2,\cdots} \mathcal{H}om_{\mathcal{P}_W}(\mathcal{L}_l, \mathcal{C}_W) \to \mathcal{C}^2_W$$
$$\to \mathcal{E}xt^q_{\mathcal{P}}(\mathcal{M}, \mathcal{C}) \to \bigoplus_{l=0,-1,-2,\cdots} \mathcal{E}xt^1_{\mathcal{P}_W}(\mathcal{L}_l, \mathcal{C}_W) \to 0.$$

Remark. We conjecture that $\mathcal{E}xt^{q-1}_{\mathcal{P}}(\mathcal{M}, \mathcal{C}) = 0$ holds in this case.

Theorem 3. *In Case* B_2, *we have*

$$\mathcal{E}xt^j_{\mathcal{P}}(\mathcal{M}, \mathcal{C}) = 0 \qquad \textit{for } j \neq q,$$

and the following exact sequence holds:

$$0 \to \bigoplus_{l=0,-1,-2,\cdots} \mathcal{H}om_{\mathcal{P}_W}(\mathcal{L}_l, \mathcal{C}_W) \to \mathcal{C}^2_W \to \mathcal{E}xt^q_{\mathcal{P}}(\mathcal{M}, \mathcal{C})$$
$$\to \bigoplus_{l=0,-1,-2,\cdots} \mathcal{E}xt^1_{\mathcal{P}_W}(\mathcal{L}_l, \mathcal{C}_W) \to 0.$$

References

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[2] Sato, M., T. Kawai, and M. Kashiwara: Microfunctions and Pseudo-Differential Equations. Lecture Note in Mathematics No. 287, Springer, Berlin-Heidelberg-New York, pp. 265–529 (1973).