ON $(\in_\alpha, \in_\alpha \vee q_\beta)$-FUZZY SOFT BCI-ALGEBRAS

CHIRANJIBE JANA AND MADHUMANGAL PAL

Abstract. Molodtsov initiated soft set theory which has provided a general mathematical framework for handling uncertainties that occur in various real life problems. The aim of this paper is to provide fuzzy soft algebraic tools in considering many problems that contain uncertainties. In this article, the notion of $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra of BCI-algebra is introduced. Some operational properties on $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebras are discussed as well as lattice structures of this kind of fuzzy soft set on BCI-subalgebras are derived.

1. Introduction

Uncertainties may occur in many real life applications like economics, engineering, sociology, medical science, and environmental science. Classical methods cannot always successfully overcome uncertainties, because the uncertainties appearing in these domains may be of various types. In order to overcome this problem, Molodtsov [40] first introduced the concept of a soft set theory as a new mathematical tool for dealing with uncertainties and also pointed out several directions of study for the applications of soft sets. Decision making [7, 8, 33], theory of soft sets [3, 32] and fuzzy soft sets [4, 25] are some of these fields. Recently, soft set theory has been applied to different algebraic structures (see [29, 34, 35, 36, 37]). After the introduction of fuzzy sets by Zadeh [45], there have been a number of generalizations of this fundamental concept. A new type of fuzzy subgroups, called $(\in, \in \vee q)$-fuzzy subgroups, was introduced in a paper of Bhakat et al. [6] using the combined notions of "belongingness" and "quasi-coincidence" of fuzzy points and fuzzy sets, introduced by Pu and Liu [42]. Bej and Pal [5], Muhiuddin and others [38, 39], and Jana et al. [12–22] have done much work on BCK/BCI-algebras and its related algebras.

In fact, the $(\in, \in \vee q)$-fuzzy subgroup is an important generalization of Rosenfeld's fuzzy subgroup [43]. Subsequently, Narayanan and Manikantan [41] extended these results to near-rings. With this objective in mind, Jun and Meng [23, 28] introduced the concept of (α, β)-fuzzy subalgebras and ideals of BCK/BCI-algebras and investigated related results. Further, Ma et al. [30] discussed the properties of some kinds of $(\in, \in \vee q)$-interval-valued fuzzy ideals of BCI-algebras. Also, Ma et al. [31] introduced the notion of some kinds of $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy ideals in BCI-algebras. In addition, much research work has been done by combining fuzzy set and soft set theory applied in soft algebraic structures [1, 9] on soft groups, soft semiring. Jun et al. [24, 26, 27] applied the notion of soft sets to the theory of BCK/BCI-algebras, soft subalgebras, and described their basic properties. Recently, Yin and Zhan [44] introduced the concepts of $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy soft h-ideals (respectively, h-bi-ideals, h-quasi-ideals) in hemiring, which are a further generalization of all kinds of fuzzy soft h-ideals. To the best of our knowledge, there is no work done on $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebras. For this reason, we are motivated to develop the theory on $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebras.

In this paper, the notion of $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebras are introduced and their properties are investigated. We provide conditions for a soft set to become $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebras. We prove that the extended intersection of two $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebras is also a $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra. We show that the family of $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebras is a completely distributive lattice.

2. Preliminaries

In this section, some elementary aspects that are necessary for this paper are included.

An algebra $(X, *, 0)$ of type $(2, 0)$ is called BCI-algebra if it satisfies the following conditions, if for all $x, y, z \in X$.

(1) $(((x * y) * (x * z)) * (z * y)) = 0$.
(2) $(x * (x * y)) * y = 0$.
(3) $x * x = 0$.
(4) $x * y = 0$ and $y * x = 0 \Rightarrow x = y$.

We can define a partial ordering " $\leq$ " by $x \leq y$ if and only if $x * y = 0$. If a BCI-algebra X satisfies $0 * x = 0$ for all $x \in X$, then we say that X is a BCK-algebra.

Proposition 2.1. [11] *Let X be a BCI-algebra. The following conditions are held on BCI-algebra over X, then for all $x, y, z \in X$.*

(1) $x \leq y \Rightarrow x * z \leq y * z$ *and* $z * y \leq z * y$.
(2) $(x * y) * z = (x * z) * y$.
(3) $(x * z) * (y * z) \leq x * y$.
(4) $x * 0 = x$.

A nonempty subset S of a BCK/BCI-algebra X is called a BCK/BCI-subalgebra of X if $x * y \in S$ for all $x, y \in S$. A fuzzy set μ in a BCK/BCI-algebra X is called a fuzzy BCK/BCI-subalgebra if it satisfies $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

We refer the reader to the book [10] for further information regarding BCK/BCI-algebras. A fuzzy set μ in a set X is of the form

$$\mu(y) = \begin{cases} t \in (0,1], & \text{if } y = x; \\ 0, & \text{if } y \neq x. \end{cases}$$

A fuzzy point with support x and value t is denoted by x_t. For a fuzzy point x_t and a fuzzy set μ of a set X, Pu and Liu [42] gave meaning to the symbol $x_t\Phi\mu$, where $\Phi \in \{\in, q, \in \vee q, \wedge q\}$. To say that $x_t \in \mu$ (respectively, $x_t q\mu$) means that $\mu(x) \geq t$ (respectively, $\mu(x) + t > 1$), and in this case, x_t is said to belong to (respectively, be quasi-coincident with) a fuzzy set μ. To say that $x_t \in \vee q\mu$ (respectively, $x_t \in \wedge q\mu$) means that $x_t \in \mu$ or $x_t q\mu$ (respectively $x_t \in \mu$ and $x_t q\mu$). To say that $x_t\overline{\phi}\mu$ means that $x_t\Phi\mu$ does not hold, where $\Phi \in \{\in, q, \in \vee q, \in \wedge q\}$.

Let $\alpha, \beta \in [0,1]$ be such that $\alpha < \beta$. For any fuzzy point x_t and μ of X, we say [44].

(1) $x_t \in_\alpha \mu$ if $\mu(x) \geq t > \alpha$.
(2) $x_t q_\beta \mu$ if $\mu(x) + t > 2\beta$.
(3) $x_t \in_\alpha \vee q_\beta \mu$ if $x_t \in_\alpha \mu$ or $x_t q_\beta \mu$.
(4) $x_t \overline{\in_\alpha} \vee \overline{q_\beta} \mu$ if $x_t \overline{\in_\alpha} \mu$ and $x_t \overline{q_\beta} \mu$.

Molodstov [40] defined the soft set in the following way. Let U be an initial universe and E be a set of parameters. Let $\mathcal{P}(U)$ be power set of U and $A \subset E$.

Definition 2.2. [40] *A pair $(\mathcal{F}, A)$ is called a soft set over U, where $\mathcal{F}$ is a mapping given by*

$$\mathcal{F} : A \to \mathcal{P}(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\theta \in A$, $\mathcal{F}(\theta)$ may be considered as the set of θ-approximate elements of the soft set $(\mathcal{F}, A)$. Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [40].

Definition 2.3. [33] *For two soft sets $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ over a common universe U, we say that $(\mathcal{F}, A)$ is a soft subset of $(\mathcal{G}, B)$, denoted by $(\mathcal{F}, A)\tilde{\subset}(\mathcal{G}, B)$, if it satisfies*

(1) *$A \subset B$.*
(2) *For every $\alpha \in A$, $\mathcal{F}[\theta]$ and $\mathcal{G}[\theta]$ are identical approximations.*

Definition 2.4. [33] *Let U be an initial universe set and E be a set of parameters and $A \subseteq E$. Then $(\mathcal{F}, A)$ is called a fuzzy soft set over U,* *where $\mathcal{F}$ is a mapping given by $\mathcal{F} : A \to \mathcal{F}(U)$ with $\mathcal{F}(U)$ being the family of all fuzzy sets of U.*

Lemma 2.5. [44] *Let $\mu, \nu \in F(X)$. Then $\mu \subseteq \vee q_{(\alpha,\beta)}\nu$ if and only if $\max\{\nu(x), \alpha\} \geq \min\{\mu(x), \beta\}$ for all $x \in X$.*

Lemma 2.6. [44] *Let $\mu, \nu, \omega \in F(X)$. If $\mu \subseteq \vee q_{(\alpha,\beta)}\nu$ and $\nu \subseteq \vee q_{(\alpha,\beta)}\omega$, then $\mu \subseteq \vee q_{(\alpha,\beta)}\omega$.*

Lemmas 2.5 and 2.6 give that "$=_{(\alpha,\beta)}$" is an equivalence relation on $F(X)$. It is also worth noting that $\mu =_{(\alpha,\beta)} \nu$ if and only if

$$\max\{\min\{\mu(x), \beta\}, \alpha\} = \max\{\min\{\nu(x), \beta\}, \alpha\}$$

for all $x \in X$ by Lemmas 2.5.

Now, we introduce some operations of two fuzzy soft sets over X as follows.

Definition 2.7. [32] *Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be two fuzzy soft sets over the universe U, we say that $(\mathcal{F}, A)$ is a fuzzy soft subset of $(\mathcal{G}, B)$ and $(\mathcal{F}, A)\tilde{\Subset}(\mathcal{G}, B)$ if*

1. *$A \subseteq B$.*
2. *For any $\theta \in A$, $\mathcal{F}[\theta] \subseteq \mathcal{G}[\theta]$. $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ are said to be fuzzy soft equal and write $(\mathcal{F}, A) = (\mathcal{G}, B)$ if $(\mathcal{F}, A) \Subset (\mathcal{G}, B)$ and $(\mathcal{G}, B) \Subset (\mathcal{F}, A)$.*

Definition 2.8. [26] *The extended intersection of two fuzzy soft sets $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ over X is a fuzzy soft set denoted by $(\mathcal{H}, C)$, where $C = A \cup B$) and*

$$\mathcal{H}(\theta) = \begin{cases} \mathcal{F}(\theta), & \text{if } \theta \in A \backslash B; \\ \mathcal{G}(\theta), & \text{if } \theta \in B \backslash A; \\ \mathcal{F}(\theta) \cap \mathcal{G}(\theta), & \text{if } \theta \in A \cap B, \end{cases}$$

for all $\theta \in C$. It is denoted by $(\mathcal{H}, C) = (\mathcal{F}, A)\tilde{\cap}(\mathcal{G}, B)$.

Definition 2.9. *The extended union of two fuzzy soft sets $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ over X is a fuzzy soft set denoted by $(\mathcal{H}, C)$, where $C = A \cup B$ and*

$$\mathcal{H}(\theta) = \begin{cases} \mathcal{F}(\theta), & \text{If } \theta \in A \backslash B; \\ \mathcal{G}(\theta), & \text{if } \theta \in B \backslash A; \\ \mathcal{F}(\theta) \cup \mathcal{G}(\theta), & \text{if } \theta \in A \cap B, \end{cases}$$

for all $\theta \in C$. It is denoted by $(\mathcal{H}, C) = (\mathcal{F}, A)\tilde{\cup}(\mathcal{G}, B)$.

Definition 2.10. [26] *Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be two fuzzy soft sets over X such that $A \cap B \neq \emptyset$. The restricted intersection of $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ is a fuzzy soft set $(\mathcal{H}, C)$, where $C = A \cap B$ and $\mathcal{H}(\theta) = \mathcal{F}(\theta) \cap \mathcal{G}(\theta)$ for all $\theta \in C$. It is denoted by $(\mathcal{H}, C) = (\mathcal{F}, A) \cap (\mathcal{G}, B)$.*

TABLE 1

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Definition 2.11. [3] *Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be two fuzzy soft sets over X such that $A \cap B \neq \emptyset$. The restricted union of $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ is a fuzzy soft set $(\mathcal{H}, C)$, where $C = A \cap B$ and $\mathcal{H}(\theta) = \mathcal{F}(\theta) \cup \mathcal{G}(\theta)$ for all $\theta \in C$. It is denoted by $(\mathcal{H}, C) = (\mathcal{F}, A) \cup (\mathcal{G}, B)$.*

Definition 2.12. [26] *Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be two fuzzy soft sets over X. Then the AND operation of two fuzzy soft sets $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ is a fuzzy soft set over X, denoted by $(\mathcal{F}, A) \wedge (\mathcal{G}, B)$, and defined by $(\mathcal{F}, A) \wedge (\mathcal{G}, B) = (\mathcal{H}, A \times B)$, where $\mathcal{H}(\theta_1, \theta_2) = \mathcal{F}[\theta_1] \tilde{\cap} \mathcal{G}[\theta_2]$ for all $(\theta_1, \theta_2) \in A \times B$.*

Definition 2.13. [3] *The product of two fuzzy soft sets $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ over common universe U is a fuzzy soft sets is denoted by $(\mathcal{F} * \mathcal{G}, C)$, where $C = A \cup B$ and*

$$(\mathcal{F} * \mathcal{G})(\theta) = \begin{cases} \mathcal{F}(\theta), & \text{if } \theta \in A \backslash B; \\ \mathcal{G}(\theta), & \text{if } \theta \in B \backslash A; \\ \mathcal{F}(\theta) * \mathcal{G}(\theta), & \text{if } \theta \in A \cap B, \end{cases}$$

for all $\theta \in C$. It is denoted by $(\mathcal{F} * \mathcal{G}, C) = (\mathcal{F}, A) \diamond (\mathcal{G}, B)$.

3. $(\in_\alpha, \in_\alpha \vee q_\beta)$-FUZZY SOFT BCI-SUBALGEBRAS

In this section, the characterizations of $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebras of BCI-algebras are described. In what follows, let X denote a BCI-algebra unless otherwise specified.

Definition 3.1. *A fuzzy soft set $(\mathcal{F}, A)$ over a BCI-algebra X is called an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra of X if for all $\theta \in A$ and $t, s \in (\alpha, 1]$ such that*

(T1) *$x_t \in_\alpha \mu_{\mathcal{F}[\theta]}$ and $y_s \in_\alpha \mu_{\mathcal{F}[\theta]}$ implies $(x * y)_{\min\{t,s\}} \in_\alpha \vee q_\beta \mu_{\mathcal{F}[\theta]}$.*

Example 3.2. *Let $X = \{0, a, b, c\}$ be a proper BCI-algebra with the following Caley Table 1 as follows. Define a fuzzy soft set $\mu_{\mathcal{F}[\theta]}$ of X such that $\mu_{\mathcal{F}[\theta]}(0) = 0.7$, $\mu_{\mathcal{F}[\theta]}(a) = 0.8$ and $\mu_{\mathcal{F}[\theta]}(b) = \mu_{\mathcal{F}[\theta]}(c) = 0.3$, and take $U = X$ and $A = [0, 1]$ and $\theta \in A$. Then it is routine to show that $(\mathcal{F}, A)$ is a $(\in_{0.7}, \in_{0.7} \vee q_{0.8})$-fuzzy soft BCI-subalgebra of X, but it is not a fuzzy soft subalgebra of X.*

The above definition is equivalent to the following lemma.

Lemma 3.3. *Let $(\mathcal{F}, A)$ be a fuzzy soft set over X. Then $(T1)$ holds if and only if the following conditions hold, for all $\theta \in A$ and $x, y \in X$*

(T2) $\max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\}$.
(T3) $(\mathcal{F}, A) * (\mathcal{F}, A) \subseteq \vee q_{(\alpha,\beta)}(\mathcal{F}, A)$.

Proof. (T1) $\Rightarrow$ (T2). Let there be any $x, y \in X$ and $\theta \in A$ such that $\max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} < t < \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\}$. Then $\mu_{\mathcal{F}[\theta]}(x) \geq t$, $\mu_{\mathcal{F}[\theta]}(y) \geq t$ and $\mu_{\mathcal{F}[\theta]}(x * y) < t \leq \beta$. Thus, $x_t \in \mu_{\mathcal{F}[\theta]}$, $y_t \in \mu_{\mathcal{F}[\theta]}$ but $(x * y)\overline{\in}_\alpha \vee \overline{q}_\beta \mu_{\mathcal{F}[\theta]}$. This contradicts (T1). Hence, (T2) holds.
(T2) $\Rightarrow$ (T3). If $(\mathcal{F}, A) * (\mathcal{F}, A)\overline{\subseteq \vee q_{(\alpha,\beta)}}(\mathcal{F}, A)$, then there exist $\theta \in A$ and $x_t \in_\alpha \mu_{(F*F)[\theta]}$ such that $x_t \overline{\in}_\alpha \vee \overline{q}_\beta \mu_{\mathcal{F}[\theta]}$, where $t \in (\alpha, 1]$. Hence, $\mu_{(\mathcal{F}*\mathcal{F})[\theta]}(x) \geq t > \alpha$, $\mu_{\mathcal{F}[\theta]}(x) < t$ and $\mu_{\mathcal{F}[\theta]}(x) + t \leq 2\beta$, which imply that $\mu_{\mathcal{F}[\theta]}(x) < \beta$. If $x = m * n$ for some $m, n \in X$, then by (T2), we have

$$\max\{\mu_{\mathcal{F}[\theta]}(x), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(y), \mu_{\mathcal{F}[\theta]}(z), \beta\}.$$

Again, $\mu_{\mathcal{F}[\theta]}(x) < \beta$ and $\alpha < \beta$, which follows that

$$\max\{\mu_{\mathcal{F}[\theta]}(x), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(y), \mu_{\mathcal{F}[\theta]}(z), \beta\}.$$

Hence,

$$\begin{aligned} t \leq \mu_{(\mathcal{F}*\mathcal{F})[\theta]}(x) = \mu_{F[\theta]*F[\theta]}(x) &= \sup_{x=m*n} \min\{\mu_{\mathcal{F}[\theta]}(m), \mu_{\mathcal{F}[\theta]}(n)\} \\ &\leq \sup_{x=m*n} \max\{\mu_{\mathcal{F}[\theta]}(x), \alpha\} = \max\{\mu_{\mathcal{F}[\theta]}(x), \alpha\}, \end{aligned}$$

a contradiction. Therefore, condition (T3) holds.
(T3) $\Rightarrow$ (T1). Let $t, s \in (\alpha, 1]$, $\theta \in A$ and $x, y \in X$ such that $x_t \in_\alpha \mu_{\mathcal{F}[\theta]}$ and $y_s \in_\alpha \mu_{\mathcal{F}[\theta]}$, then we get $\mu_{\mathcal{F}[\theta]}(x) \geq t > \alpha$, $\mu_{\mathcal{F}[\theta]}(y) \geq s > \alpha$ and for all $z = x * y$, we have

$$\begin{aligned} \mu_{(\mathcal{F}*\mathcal{F})[\theta]}(z) = \mu_{(\mathcal{F}[\theta]*\mathcal{F}[\theta])}(z) &= \sup_{z=x*y} \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y)\} \\ &\geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y)\} \geq \min\{t, s\} > \alpha. \end{aligned}$$

It follows that $(x * y)_{\min\{t,s\}} \in_\alpha \mu_{(\mathcal{F}*\mathcal{F})[\theta]}$. By (T3), $(x * y)_{\min\{t,s\}} \in_\alpha \vee q_\beta \mu_{\mathcal{F}[\theta]}$. Therefore, condition (T1) is valid. □

Remark 3.4. *For any $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra $(\mathcal{F}, A)$ of X, we can conclude that*

1. *$\mu_{\mathcal{F}[\theta]}$ is a fuzzy soft BCI-subalgebra of X, for all $\theta \in A$ when $\alpha = 0$ and $\beta = 1$.*
2. *$(\mathcal{F}, A)$ is an $(\in, \in \vee q)$-fuzzy soft BCI-subalgebra of X when $\alpha = 0$ and $\beta = 0.5$.*

For any fuzzy soft set $(\mathcal{F}, A)$ over X, $\theta \in A$ and $t \in [0, 1]$, we define $\mathcal{F}_t^{(\alpha,\beta)}[\theta] = \{x \in X | \ x_t \in_\alpha \mu_{\mathcal{F}[\theta]}\}$, $\langle \mathcal{F} \rangle_t^{(\alpha,\beta)}[\theta] = \{x \in X | \ x_t q_\beta \mu_{\mathcal{F}[\theta]}\}$, and $[\mathcal{F}]_t^{(\alpha,\beta)}[\theta] = \{x \in X |\ x_t \in_\alpha \vee q_\beta \mu_{\mathcal{F}[\theta]}\}$. It is clear that $[\mathcal{F}]_t^{(\alpha,\beta)} = \mathcal{F}_t^{(\alpha,\beta)} \cup \langle \mathcal{F} \rangle_t^{(\alpha,\beta)}$.

Theorem 3.5. *Let $(\mathcal{F}, A)$ be a fuzzy soft set over X. Then*

(1) *$(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra of X if and only if the non-empty set $\mathcal{F}_t^{(\alpha,\beta)}[\theta]$ is a BCI-subalgebra of X for all $\theta \in A$ and $t \in (\alpha, \beta]$.*
(2) *If $2\beta = 1 + \alpha$, then $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra of X if and only if non-empty set $\langle \mathcal{F} \rangle_t^{(\alpha,\beta)}[\theta]$ is a BCI-subalgebra of X for all $\theta \in A$ and $t \in (\beta, 1]$.*
(3) *If $2\beta = 1 + \alpha$, then $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra of X if and only if non-empty set $[\mathcal{F}]_t^{(\alpha,\beta)}[\theta]$ is a BCI-subalgebra of X for all $\theta \in A$ and $t \in (\alpha, 1]$.*

Proof. (1) Let $(\mathcal{F}, A)$ be an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra of X and assume that $\mathcal{F}_t^{(\alpha,\beta)}[\theta] \neq \emptyset$ for some $\theta \in A$ and $t \in (\alpha, \beta]$. Let $x, y \in \mathcal{F}_t^{(\alpha,\beta)}[\theta]$. Then $x_t \in_\alpha \mu_{\mathcal{F}[\theta]}$ and $y_t \in_\alpha \mu_{\mathcal{F}[\theta]}$. Since $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra of X, then $\max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\}$. Hence, by $t > \alpha$

$$\max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\} \geq \min\{t, \beta\} = t.$$

That is, $\mu_{\mathcal{F}[\theta]}(x * y) \geq t$, which indicates that $x \in \mathcal{F}_t^{(\alpha,\beta)}[\theta]$. Thus, $\mathcal{F}_t^{(\alpha,\beta)}[\theta]$ is a BCI-subalgebra of X.

Conversely, assume that the given condition holds. Suppose that there exist $\theta \in A$ and $x, y \in X$ such that

$$\max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} < t < \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\}.$$

Then $\mu_{\mathcal{F}[\theta]}(x) \geq t > \alpha$, $\mu_{\mathcal{F}[\theta]}(y) \geq t > \alpha$, and $\mu_{\mathcal{F}[\theta]}(x * y) < t < \beta$. Thus, $x_t \in_\alpha \mu_{\mathcal{F}[\theta]}$ and $y_t \in_\alpha \mu_{\mathcal{F}[\theta]}$, but $(x * y)_t \overline{\in_\alpha} \mu_{\mathcal{F}[\theta]}$. This is a contradiction. Therefore, $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra of X.

(2) Assume that $2\beta = 1 + \alpha$. Let $(\mathcal{F}, A)$ be an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra of X and assume that $\langle \mathcal{F} \rangle_t^{(\alpha,\beta)}[\theta] \neq \emptyset$ for some $\theta \in A$ and $t \in (\beta, 1]$. Let $x, y \in \langle \mathcal{F} \rangle_t^{(\alpha,\beta)}[\theta]$. Then $x_t q_\beta \mu_{\mathcal{F}[\theta]}$ and $y_t q_\beta \mu_{\mathcal{F}[\theta]}$, i.e., $\mu_{\mathcal{F}[\theta]}(x) + t \geq 2\beta$ and $\mu_{\mathcal{F}[\theta]} + t \geq 2\beta$. Since $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft algebra of X, then $\max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\}$.

Hence by, $t > \beta$,

$$\begin{aligned}
&\max\{\mu_{\mathcal{F}[\theta]}(x * y) + t, \alpha + t\}\\
&= \max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} + t\\
&\geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\} + t\\
&= \min\{\mu_{\mathcal{F}[\theta]}(x) + t, \mu_{\mathcal{F}[\theta]}(y) + t, \beta + t\}\\
&> 2\beta.
\end{aligned}$$

For $t \leq 1 = 2\beta - \alpha$, we have $t + \alpha \leq 2\beta$. It follows that $\mu_{\mathcal{F}[\theta]}(x * y) + t \geq 2\beta$ and so, $(x * y) \in \langle\mathcal{F}\rangle_t^{(\alpha,\beta)}[\theta]$. Hence, $\langle\mathcal{F}\rangle_t^{(\alpha,\beta)}[\theta]$ is a fuzzy BCI-subalgebra of X.

Conversely, assume that the given condition holds. Suppose that there exist $\theta \in A$ and $x, y \in X$ such that

$$\max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} < t < \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\}.$$

Let $t = 2\beta - \max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\}$. Then $t \in (\beta, 1]$, $\mu_{\mathcal{F}[\theta]}(x * y) \leq 2\beta$, $\mu_{\mathcal{F}[\theta]}(x) > \max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} = 2\beta - t$, and $\mu_{\mathcal{F}[\theta]}(y) > \max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} = 2\beta - t$, i.e., $x, y \in \langle\mathcal{F}\rangle_t^{(\alpha,\beta)}[\theta]$, but $(x * y)\overline{\in}_\alpha\langle\mathcal{F}\rangle_t^{(\alpha,\beta)}[\theta]$, a contradiction. Therefore, $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra of X.

(3) Let $(\mathcal{F}, A)$ be an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra of X and $x, y \in [\mathcal{F}]_t^{(\alpha,\beta)}[\theta] \neq \emptyset$ for some $\theta \in A$ and $t \in (\alpha, 1]$. Then $x_t \in_\alpha \vee q_\beta \mu_{\mathcal{F}[\theta]}$ and $y_t \in_\alpha \vee q_\beta \mu_{\mathcal{F}[\theta]}$, i.e., $\mu_{\mathcal{F}[\theta]}(x) \geq 2\beta - t > 2\beta - 1 = \alpha$ and $\mu_{\mathcal{F}[\theta]}(y) \geq 2\beta - t > 2\beta - 1 = \alpha$. Since $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra of X and also, $\min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\} > \alpha$, we have $\mu_{\mathcal{F}[\theta]}(x * y) \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\}$. Now we consider the cases.
Case 1. $t \in (\alpha, \beta]$. Then we have $2\beta - \alpha \geq \beta \geq t$.

Case 1(i). $\mu_{\mathcal{F}[\theta]}(x) \geq t$ or $\mu_{\mathcal{F}[\theta]}(y) \geq t$. Then,

$$\mu_{\mathcal{F}[\theta]}(x * y) \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\} \geq t.$$

Hence, $(x * y)_t \in_\alpha \mu_{\mathcal{F}[\theta]}$.

Case 1(ii). $\mu_{\mathcal{F}[\theta]}(x) > 2\beta - t$ and $\mu_{\mathcal{F}[\theta]}(y) > 2\beta - t$. Then,

$$\mu_{\mathcal{F}[\theta]}(x * y) \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\} = \beta \geq t.$$

Hence, $(x * y) \in_\alpha \mu_{\mathcal{F}[\theta]}$.

Case 2. $t \in (\beta, 1]$. Then we have $2\beta - t < \beta < t$.

Case 2(i). $\mu_{\mathcal{F}[\theta]}(x) \geq t$ and $\mu_{\mathcal{F}[\theta]}(y) \geq t$. Then we have

$$\mu_{\mathcal{F}[\theta]}(x * y) \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\} = \beta > 2\beta - t.$$

Hence, $(x * y)_t q_\beta \mu_{\mathcal{F}[\theta]}$.

Case 2(ii). $\mu_{\mathcal{F}[\theta]}(x) > 2\beta - t$ or $\mu_{\mathcal{F}[\theta]}(y) > 2\beta - t$. Then

$$\mu_{\mathcal{F}[\theta]}(x * y) \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\} > 2\beta - t.$$

Hence, $(x * y)_t q_\beta \mu_{\mathcal{F}[\theta]}$.

Therefore, in any case, $(x * y)_t \in_\alpha \vee q_\beta \mu_{\mathcal{F}[\theta]}$, i.e., $(x * y) \in [\mathcal{F}]_t^{(\alpha,\beta)}[\theta]$.

Conversely, assume that the given condition holds. Let $\theta \in A$ and $x, y \in X$ such that $\max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} < t = \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\}$. Then $\mu_{\mathcal{F}[\theta]}(x) \geq t > \alpha$, $\mu_{\mathcal{F}[\theta]}(y) \geq t > \alpha$, and $\mu_{\mathcal{F}[\theta]}(x * y) < t \leq \beta$, i.e., $x_t, y_t \in_\alpha \mu_{\mathcal{F}[\theta]}$ but $(x * y)_t \overline{\in}_\alpha \vee \overline{q}_\beta \mu_{\mathcal{F}[\theta]}$. Hence, $x, y \in [\mathcal{F}]_t^{(\alpha,\beta)}[\theta]$ but $(x * y) \overline{\in}_\alpha [\mathcal{F}]_t^{(\alpha,\beta)}[\theta]$, a contradiction. Therefore,

$$\max\{\mu_{\mathcal{F}[\theta]}(x * y), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \mu_{\mathcal{F}[\theta]}(y), \beta\}.$$

This proves that condition (T3) holds. Therefore, $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra of X. $\square$

If we take $\alpha = 0$ and $\beta = 0.5$ in Theorem 3.5, we conclude the following results.

Remark 3.6.

(1) *$(\mathcal{F}, A)$ is an $(\in, \in \vee q)$-fuzzy soft BCI-subalgebra of X if and only if the non-empty set $\mathcal{F}_t[\theta]$ is a BCI-subalgebra of X for all $\theta \in A$ and for all $t \in (0, 0.5]$.*

(2) *$(\mathcal{F}, A)$ is an $(\in, \in \vee q)$-fuzzy soft BCI-subalgebra of X if and only if non-empty set $\langle \mathcal{F} \rangle_t[\theta]$ is a BCI-subalgebra of X for all $\theta \in A$ and for all $t \in (0.5, 1]$.*

(3) *$(\mathcal{F}, A)$ is an $(\in, \in \vee q)$-fuzzy soft BCI-subalgebra of X if and only if non-empty set $[\mathcal{F}]_t[\theta]$ is a BCI-subalgebra of X for all $\theta \in A$ and for all $t \in (0, 1]$.*

Definition 3.7. *Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra over X. We say that $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft subset of $(\mathcal{G}, B)$ and $(\mathcal{F}, A) \Subset \vee q_{(\alpha,\beta)} (\mathcal{G}, B)$ if*

(1) *$A \subseteq B$.*

(2) *For any $\theta \in A$, $\mathcal{F}[\theta] \subseteq \mathcal{G}[\theta]$. $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ are an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft equal, i.e., $(\mathcal{F}, A) =_{(\alpha,\beta)} (\mathcal{G}, B)$ if $(\mathcal{F}, A) \Subset \vee q_{(\alpha,\beta)} (\mathcal{G}, B)$ and $(\mathcal{G}, B) \Subset \vee q_{(\alpha,\beta)} (\mathcal{F}, A)$.*

For any $t \in [0,1]$ and $\theta \in A$, $\left(\mathcal{F}_t^{(\alpha,\beta)}, A\right)$, $\left(\langle\mathcal{F}\rangle_t^{(\alpha,\beta)}, A\right)$ and $\left([\mathcal{F}]_t^{(\alpha,\beta)}, A\right)$ are fuzzy soft set over X.

Theorem 3.8. *Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X. If $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft subset of $(\mathcal{G}, B)$, then*

(1) *fuzzy soft set $\left(\mathcal{F}_t^{(\alpha,\beta)}, A\right)$ is a fuzzy soft subset of $\left(\mathcal{G}_t^{(\alpha,\beta)}, B\right)$ over X for all $t \in (\alpha, \beta]$.*
(2) *fuzzy soft set $\left(\langle\mathcal{F}\rangle_t^{(\alpha,\beta)}, A\right)$ is a fuzzy soft subset of $\left(\langle\mathcal{G}\rangle_t^{(\alpha,\beta)}, B\right)$ over X for all $t \in (\beta, \min\{2\beta - \alpha, 1\}]$.*
(3) *fuzzy soft set $\left([\mathcal{F}]_t^{(\alpha,\beta)}, A\right)$ is a fuzzy soft subset of $\left([\mathcal{G}]_t^{(\alpha,\beta)}, B\right)$ over X for all $t \in (\alpha, \min\{2\beta - \alpha, 1\}]$.*

Proof. (1) Let X be a BCI-algebra and $\theta \in A$, $t \in (\alpha, \beta]$, and $x \in \mathcal{F}_t^{(\alpha,\beta)}[\theta]$. Then $x_t \in_\alpha \mu_{\mathcal{F}[\theta]}$, i.e., $\mu_{\mathcal{F}[\theta]}(x) \geq t > \alpha$. Since $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft subset of $(\mathcal{G}, B)$ we have $\max\{\mu_{\mathcal{G}[\theta]}(x), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \beta\} \geq \{t, \beta\} = t$, and so $\mu_{\mathcal{G}[\theta]}(x) \geq t > \alpha$, since $t > \alpha$, i.e., $x \in \mathcal{G}_t^{(\alpha,\beta)}[\theta]$. Therefore, (1) holds.

(2) Let $\theta \in A$, $t \in (\beta, \min\{2\beta - \alpha, 1\}]$ and $x \in \langle\mathcal{F}\rangle_t^{(\alpha,\beta)}[\theta]$. Then $x_t q_\beta \mu_{\mathcal{F}[\theta]}$, i.e., $\mu_{\mathcal{F}[\theta]}(x) + t > 2\beta$. Since $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft subset of $(\mathcal{G}, B)$ over X, then we have $\max\{\mu_{\mathcal{G}[\theta]}(x), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \beta\}$. Hence, for $t > \beta$,

$$\begin{aligned}\max\{\mu_{\mathcal{G}[\theta]}(x) + t, \alpha + t\} &= \max\{\mu_{\mathcal{G}[\theta]}, \alpha\} + t \\ &\geq \min\{\mu_{\mathcal{F}[\theta]}(x), \beta\} + t \\ &= \min\{\mu_{\mathcal{F}[\theta]} + t, \beta + t\} > 2\beta,\end{aligned}$$

$t \leq 2\beta - \alpha$, i.e, $t + \alpha \leq 2\beta$. We have $\mu_{\mathcal{G}[\theta]}(x) + t > 2\beta$ and so $x \in \langle\mathcal{G}\rangle_t^{(\alpha,\beta)}[\theta]$. Therefore, (2) holds.

(3) $\theta \in A$, $t \in (\alpha, \min\{2\beta - \alpha, 1]$, and let $x \in [\mathcal{F}]_t^{(\alpha,\beta)}[\theta]$. Then $x_t \in_\alpha \vee q_\beta \mu_{\mathcal{F}[\theta]}$ and $\mu_{\mathcal{F}[\theta]}(x) \geq t > \alpha$ or $\mu_{\mathcal{F}[\theta]}(x) > 2\beta - \alpha > 2\beta - (2\beta - \alpha) = \alpha$. Since $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft subset of $(\mathcal{G}, B)$ over X we have $\max\{\mu_{\mathcal{G}[\theta]}(x), \alpha\} \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \beta\}$, and so $\mu_{\mathcal{G}[\theta]}(x) \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \beta\}$. Since $\alpha < \min\{\mu_{\mathcal{F}[\theta]}(x), \beta\}$, we consider the following cases.

<u>Case 1</u>. $t \in (\alpha, \beta]$. As $t \in (\alpha, \beta]$, we have $2\beta - \alpha \geq \beta \leq t$. Then from $\mu_{\mathcal{F}[\theta]}(x) \geq t$ or $\mu_{\mathcal{F}[\theta]}(x) \geq 2\beta - t$, we get $\mu_{\mathcal{G}[\theta]}(x) \geq \min\{\mu_{\mathcal{F}[\theta]}, \beta\} \geq t$.

Hence, $x_t \in_\alpha \mu_{\mathcal{G}[\theta]}$.

Case 2. $t \in (\beta, 1]$, since $t \in (\beta, 1]$, then we have $2\beta - t < \beta < t$. Then from $\mu_{\mathcal{F}[\theta]}(x) \geq t$ or $\mu_{\mathcal{F}[\theta]}(x) \geq 2\beta - t$, we get $\mu_{\mathcal{G}[\theta]}(x) \geq \min\{\mu_{\mathcal{F}[\theta]}(x), \beta\} > 2\beta - t$. Hence, $x_t q_\beta \mu_{\mathcal{G}[\theta]}$.

Thus, in both cases, we have $x \in [\mathcal{G}]_t^{(\alpha,\beta)}[\theta]$. Therefore, (3) holds. □

If we take $\alpha = 0$ and $\beta = 0.5$ in Theorem 3.8, we conclude the following results.

Corollary 3.9. *Then $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ are $(\in, \in \vee q)$-fuzzy soft BCI-subalgebra over X.*

1. *Fuzzy soft set $(\mathcal{F}_t, A)$ is an $(\in, \in \vee q)$-fuzzy soft subset of $(\mathcal{G}_t, B)$ over X for all $t \in (0, 0.5]$.*
2. *Fuzzy soft set $(\langle\mathcal{F}\rangle_t, A)$ is an $(\in, \in \vee q)$-fuzzy soft subset of $(\langle\mathcal{G}\rangle_t, B)$ over X for all $t \in (0.5, 1]$.*
3. *Fuzzy soft set $([\mathcal{F}]_t, A)$ is an $(\in, \in \vee q)$-fuzzy soft subset of $([\mathcal{G}]_t, B)$ over X for all $t \in (0, 1]$.*

Proposition 3.10. *If $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra of X, then $\max\{\mathcal{F}[\theta](0), \alpha\} \geq \min\{\mathcal{F}[\theta](x), \beta\}$ for all $x \in X$ and any parameter θ in A.*

Proof. Let $x \in X$ and $\theta \in A$. Then $\max\{\mathcal{F}[\theta](0), \alpha\} = \max\{\mathcal{F}[\theta](x * x), \alpha\} \geq \min\{\mathcal{F}[\theta](x), \mathcal{F}\ [\alpha](x), \beta\} = \min\{\mathcal{F}[\theta](x), \beta\}$. Thus,

$$\max\{\mathcal{F}[\theta](0), \alpha\} \geq \min\{\mathcal{F}[\theta](x), \beta\}$$

for all $x \in X$ and $\theta \in A$. □

Theorem 3.11. *Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be fuzzy soft sets over X. If $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ are two $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X, then the extended intersection of $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X.*

Proof. Let $(\mathcal{F}, A)\widetilde{\cap}_e(\mathcal{G}, B) = (\mathcal{H}, C)$ be the extended intersection of $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$. Then $C = A \cup B$. For any $\theta \in C$, if $\theta \in A - B$ (respectively, $\theta \in B - A$), then $\mathcal{H}[\theta] = \mathcal{F}[\theta]$ (respectively, $\mathcal{H}[\theta] = \mathcal{F}[\theta]$) is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X. If $A \cap B \neq \emptyset$, then $\mathcal{H}[\theta] = \mathcal{F}[\theta] \cap \mathcal{G}[\theta]$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X for any $\theta \in A \cap B$, since the intersection of two $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy BCI-subalgebra is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy BCI-subalgebra over X. Therefore, $(\mathcal{H}, C)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X. □

The two corollaries are straightforward results of Theorem 3.11.

TABLE 2

$*$	0	a	b	c	d
0	0	0	b	c	d
a	a	0	b	c	d
b	b	b	0	d	c
c	c	c	d	0	b
d	d	d	c	b	0

Corollary 3.12. *Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be two $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebras X. Then their extended intersection $(\mathcal{F}, A)\widetilde{\cap}(\mathcal{G}, B)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X.*

Corollary 3.13. *The restricted intersection of two $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebras is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X.*

Theorem 3.14. *Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be two $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X. If $A \cap B = \emptyset$, then the union $(\mathcal{F}, A)\widetilde{\cup}(\mathcal{G}, B)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X.*

Proof. By Definition 2.9, $(\mathcal{F}, A)\widetilde{\cup}(\mathcal{G}, B) = (\mathcal{H}, C)$, where $C = A \cup B$ and for all $\theta \in C$,

$$\mathcal{H}(\theta) = \begin{cases} \mathcal{F}(\theta), & \text{If } \theta \in A \backslash B; \\ \mathcal{G}(\theta), & \text{if } \theta \in B \backslash A; \\ \mathcal{F}(\theta) \cup \mathcal{G}(\alpha), & \text{if } \theta \in A \cap B. \end{cases}$$

Since $A \cap B = \emptyset$, then either $\theta \in A - B$ or $\theta \in B - A$ for all $\alpha \in C$. If $\theta \in A - B$, then $\mathcal{H}[\theta] = \mathcal{F}[\theta]$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X because $(\mathcal{F}, A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X. If $\theta \in B - A$, then $\mathcal{H}[\theta] = \mathcal{G}[\theta]$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X because $(\mathcal{G}, B)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X. Hence, $(\mathcal{F}, A)\widetilde{\cup}(\mathcal{G}, B) = (\mathcal{H}, C)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X. □

The following example shows that the Theorem 3.14 is not valid if $A \cap B \neq \emptyset$.

Example 3.15. *Consider a BCI-algebra $U = \{0 = white, a = blackish, b = reddish, c = green, d = yellow\}$ with the following Caley Table 2. Let the set of parameters $E = \{\alpha_1 = Beautiful, \alpha_2 = Fine, \alpha_3 = Moderate, \alpha_4 = Smart, \alpha_5 = Chaste\}$, $A = \{\alpha_1 = Beautiful, \alpha_2 = Fine, \alpha_3 = Moderate, \alpha_4 = Smart\}$, and $B = \{\alpha_3 = Moderate, \alpha_4 = Smart, \alpha_5 = Chaste\}$. Then $A \cap B \neq \emptyset$. Let $(\mathcal{F}, A)$ be a fuzzy soft set over U. Then $\mathcal{F}[Beautiful]$, $\mathcal{F}[Fine]$, $\mathcal{F}[Moderate]$, and $\mathcal{F}[Smart]$ are fuzzy sets in U. We define as*

TABLE 3

$\mathcal{F}$	$white$	$blakish$	$reddish$	$green$	$yellow$
$beautiful$	0.7	0.6	0.3	0.3	0.3
$fine$	0.6	0.5	0.4	0.2	0.2
$modarate$	0.8	0.5	0.1	0.3	0.1
$smart$	0.5	0.5	0.2	0.2	0.4

TABLE 4

$\mathcal{G}$	$white$	$blakish$	$reddish$	$green$	$yellow$
$modarate$	0.8	0.6	0.3	0.1	0.1
$smart$	0.7	0.6	0.3	0.3	0.5
$chaste$	0.9	0.5	0.2	0.4	0.2

Then $(\mathcal{F}, A)$ is an $(\in_{0.6}, \in_{0.6} \vee q_{0.7})$-fuzzy soft BCI-subalgebra over U. Let $(\mathcal{G}, B)$ be fuzzy soft set over U. Then $\mathcal{G}[Moderate]$,$\mathcal{G}[Smart]$ and $\mathcal{G}[Chaste]$ are fuzzy sets in U. We define as in the following.

Then $(\mathcal{G}, A)$ is an $(\in_{0.6}, \in_{0.6} \vee q_{0.7})$-fuzzy soft BCI-subalgebra over U. But, the union $(\mathcal{F}, A)\widetilde{\cup}(\mathcal{G}, B)$ is not a fuzzy soft BCI-subalgebra over U, because

$$\begin{aligned}
&(\mathcal{F}[moderate] \cup \mathcal{G}[moderate])(green * reddish)\\
&= (\mathcal{F}[moderate] \cup \mathcal{G}[moderate](yellow)\\
&= \max\{\mathcal{F}[moderate](yellow), \mathcal{G}[moderate](yellow)\} = 0.1,
\end{aligned}$$

and

$$\begin{aligned}
&\min\{(\mathcal{F}[moderate] \cup \mathcal{G}[moderate])(green),\\
&(\mathcal{F}[moderate] \cup \mathcal{G}[moderate])(reddish)\}\\
&= \min\{\max\{\mathcal{F}[moderate](green), \mathcal{G}[moderate](green)\},\\
&\max\{\mathcal{F}[moderate](reddish), \mathcal{G}[moderate](reddish)\}\}\\
&= \min\{\max\{0.3, 0.1\}, \max\{0.1, 0.3\}\} = 0.3.
\end{aligned}$$

Therefore, $(\mathcal{F}, A)\widetilde{\cup}(\mathcal{G}, B)$ is not a fuzzy soft BCI-subalgebra over U. But, $(\mathcal{F}, A)\widetilde{\cup}(\mathcal{G}, B)$ is an $(\in_{0.6}, \in_{0.6} \vee q_{0.7})$-fuzzy soft BCI-subalgebra over U, because

$$\begin{aligned}
&\max\{(\mathcal{F}[moderate] \cup \mathcal{G}[moderate])(green * reddish), \alpha = 0.6\}\\
&= \max\{(\mathcal{F}[moderate] \cup \mathcal{G}[moderate](yellow), \alpha = 0.6\}\\
&= \max\{\max\{\mathcal{F}[moderate](yellow), \mathcal{G}[moderate](yellow)\}, \alpha = 0.6\}\\
&= \max\{0.1, 0.6\} = 0.6,
\end{aligned}$$

and

$$\min\{(\mathcal{F}[moderate]\cup\mathcal{G}[moderate])(green),$$
$$(\mathcal{F}[moderate]\cup\mathcal{G}[moderate])(reddish),\beta=0.7\}$$
$$=\min\{\max\{\mathcal{F}[moderate](green),\mathcal{G}[moderate](green),\beta=0.7\},$$
$$\max\{\mathcal{F}[moderate](reddish),\mathcal{G}[moderate](reddish),\beta=0.7\}\}$$
$$=\min\{\{\max\{0.3,0.1\},\max\{0.1,0.3\},0.7\}$$
$$=\min\{0.3,0.7\}=0.3.$$

Theorem 3.16. *Let $(\mathcal{F},A)$ and $(\mathcal{G},B)$ be two $(\in_\alpha,\in_\alpha\vee q_\beta)$-fuzzy soft BCI-subalgebra over X. Then $(\mathcal{F},A)\widetilde{\wedge}(\mathcal{G},B)$ is an $(\in_\alpha,\in_\alpha\vee q_\beta)$-fuzzy soft BCI-subalgebra over X.*

Proof. By means of Definition 2.12, we have $(\mathcal{F},A)\widetilde{\wedge}(\mathcal{G},B)=(\mathcal{H},A\times B)$, where $\mathcal{H}[\theta_1,\theta_2]=\mathcal{F}[\theta_1]\cap\mathcal{G}[\theta_2]$ for all $(\theta_1,\theta_2)\in A\times B$.

For any $x,y\in X$, we have

$$\max\{\mathcal{H}[\theta_1,\theta_2](x*y),\alpha\}=\max\{\mathcal{F}[\theta_1]\cap\mathcal{G}[\theta_2](x*y),\alpha\}$$
$$=\max\{\min\{\mathcal{F}[\theta_1](x*y),\alpha\},\min\{\mathcal{G}[\theta_2](x*y),\alpha\}\}$$
$$\geq\min\{\min\{\mathcal{F}[\theta_1](x),\mathcal{F}[\theta_1](y),\beta\},$$
$$\min\{\mathcal{G}[\theta_2](x),\mathcal{G}[\theta_2](y),\beta\}\}$$
$$=\min\{\min\{\mathcal{F}[\theta_1](x),\mathcal{G}[\theta_2](x),\beta\},$$
$$\min\{\mathcal{F}[\theta_1](y),\mathcal{G}[\theta_2](y),\beta\}\}$$
$$=\min\{(\mathcal{F}[\theta_1]\cap\mathcal{G}[\theta_2])(x),(\mathcal{F}[\theta_1]\cap\mathcal{G}[\theta_2])(y),\beta\}.$$

Hence, $(\mathcal{H},A\times B)=(\mathcal{F},A)\widetilde{\cap}(\mathcal{G},B)$ is an $(\in_\alpha,\in_\alpha\vee q_\beta)$-fuzzy soft BCI-subalgebra based on (α,β) over X. Since (θ_1,θ_2) is arbitrary, we know that $(\mathcal{H},A\times B)=(\mathcal{F},A)\widetilde{\cap}(\mathcal{G},B)$ is an $(\in_\alpha,\in_\alpha\vee q_\beta)$-fuzzy soft BCI-subalgebra over X. □

Proposition 3.17. *Let $(\mathcal{F},A)$ and $(\mathcal{G},B)$ be two $(\in_\alpha,\in_\alpha\vee q_\beta)$-fuzzy soft BCI-subalgebras over X. Then $(\mathcal{F},A)\vee(\mathcal{G},B)$ is an $(\in_\alpha,\in_\alpha\vee q_\beta)$-fuzzy soft BCI-subalgebra over X.*

Proof. The proof is straightforward. □

Lemma 3.18. *Let $(\mathcal{F},A)$, $(\mathcal{G},B)$ and $(\mathcal{H},C)$ be fuzzy soft sets over X. If $(\mathcal{F},A)\subset_{(\alpha,\beta)}(\mathcal{G},B)$ and $(\mathcal{G},B)\subset_{(\alpha,\beta)}(\mathcal{H},C)$, then $(\mathcal{F},A)\subset_{(\alpha,\beta)}(\mathcal{H},C)$.*

Proof. It is straightforward by Lemma 2.6. □

Let $FFF(X,E)$ denote the set of all $(\in_\alpha,\in_\alpha\vee q_\beta)$-fuzzy soft BCI-subalgebras over X. Then from Corollaries 3.12 and 3.13, we have the following results.

Theorem 3.19. *$((FFF(X,E), \widetilde{\cup}, \cap)$ is a complete distributive lattice under the order relation "$\subset_{(\alpha,\beta)}$."*

Proof. For any $(\mathcal{F},A),(\mathcal{G},B) \in FFF(X,E)$, then by Corollaries 3.12 and 3.13, $(\mathcal{F},A)\widetilde{\cup}(\mathcal{G},B) \in FFF(X,E)$ and $(\mathcal{F},A)\cap(\mathcal{G},B) \in FFF(X,E)$. It is obvious that $(\mathcal{F},A)\widetilde{\cup}(\mathcal{G},B)$ and $(\mathcal{F},A)\cap(\mathcal{G},B)$ are the least upper bound and the greatest lower bound of $(\mathcal{F},A)$ and $(\mathcal{G},B)$, respectively. It is not difficult in replacing $\{(\mathcal{F},A),(\mathcal{G},B)\}$ with an arbitrary family of $FFF(X,E)$. Therefore, $(FFF(X,E), \widetilde{\cup}, \cap)$ is a complete lattice. Now, we prove the distributive law

$$(\mathcal{F},A)\cap((\mathcal{G},B)\widetilde{\cup}(\mathcal{H},C))$$

holds for all $(\mathcal{F},A),(\mathcal{G},B),(\mathcal{H},C) \in FFF(X,E)$. Suppose that $(\mathcal{F},A)\cap((\mathcal{G},B)\widetilde{\cup}(\mathcal{H},C)) = (\tilde{I}, A\cap(\tilde{B}\cup C))$,
$((\mathcal{F},A)\cap(\mathcal{G},B))\widetilde{\cup}((\mathcal{F},A)\cap(\mathcal{H},C)) = (\tilde{J},(A\cap B)\cup(A\cap C)) = (\tilde{J}, A\cap(B\cup C))$. Now, for any $\theta \in A\cap(B\cup C)$ it follows that $\theta \in A$ and $\theta \in (B\cup C)$. We consider the following cases.

Case 1. $\theta \in A, \theta \notin B$ and $\theta \in C$. Then $\tilde{I}(\theta) = \mathcal{F}(\theta)\cap\mathcal{H}(\theta) = \tilde{J}(\theta)$.

Case 2. $\theta \in A, \theta \in B$ and $\theta \notin C$. Then $\tilde{I}(\theta) = \mathcal{F}(\theta)\cap\mathcal{G}(\alpha) = \tilde{J}(\theta)$.

Case 3. $\theta \in A, \theta \in B$ and $\theta \in C$. Then $\tilde{I}(\theta) = \mathcal{F}(\alpha)\cap(\mathcal{G}(\theta)\cup\mathcal{H}(\theta)) = (\mathcal{F}(\theta)\cap\mathcal{G}(\theta))\cup(\mathcal{F}(\theta)\cap\mathcal{H}(\theta)) = \tilde{J}(\theta)$.

Therefore, I and J are same operators, and so $(\mathcal{F},A)\cap((\mathcal{G},B)\widetilde{\cup}(\mathcal{H},C)) = ((\mathcal{F},A)\cap(\mathcal{G},B))\widetilde{\cup}\,((\mathcal{F},A)\cap(\mathcal{H},C))$. Thus, $(\mathcal{F},A)\cap((\mathcal{G},B)\widetilde{\cup}(\mathcal{H},C)) =_{(\alpha,\beta)} ((\mathcal{F},A)\cap(\mathcal{G},B))\widetilde{\cup}((\mathcal{F},A)\cap(\mathcal{H},C))$. Hence, the proof is complete. □

Theorem 3.20. *$(FFF(X,E), \cup, \widetilde{\cap})$ is a complete distributive lattice under the order relation "$\subset'_{(\alpha,\beta)}$", where for any $(\mathcal{F},A),(\mathcal{G},B) \in FFF(X,E)$, $(\mathcal{F},A) \subset'_{(\alpha,\beta)} (\mathcal{G},B)$ if and only if $B \subseteq A$ and $F(\alpha) \subseteq q_{(\alpha,\beta)}G(\alpha)$ for any $\theta \in B$.*

Proof. The proof is similar to the proof of Theorem 3.19. □

Theorem 3.21. *Let $(\mathcal{F},A)$ and $(\mathcal{G},B)$ be a $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X. Then so is $(\mathcal{F},A)\diamond(\mathcal{G},B)$.*

Proof. Let $(\mathcal{F},A)$ and $(\mathcal{G},B)$ be $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X. For any $\theta \in A\cup B$, we have the following cases.

Case 1. $\theta \in A - B$. Then $(\mathcal{F}*\mathcal{G})(\theta) = \mathcal{F}(\theta)$. It follows that that $(\mathcal{F}*\mathcal{G})$ satisfies the condition and $(T3)$ since $(\mathcal{F},A)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X.

Case 2. $\theta \in B - A$. Then $(\mathcal{F} * \mathcal{G})(\theta) = \mathcal{G}(\theta)$. It follows that $(\mathcal{F} * \mathcal{G})$ satisfies the condition $(T3)$ since $(\mathcal{G}, B)$ is an $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-subalgebra over X.

Case 3. $\theta \in A \cap B$. Then $(\mathcal{F} * \mathcal{G})(\theta) = \mathcal{F}(\theta) * \mathcal{G}(\theta)$. Now, we have to prove that $\mathcal{F}(\theta) * \mathcal{G}(\theta)$ satisfies condition (T3). For any $x, y \in X$, we have

$$\begin{aligned}
&\min\{\mu_{(F*G)[\theta]}(x), \mu_{(F*G)[\theta]}(y), \beta\} \\
&= \min\{\mu_{(F[\theta]*G[\theta])}(x), \mu_{(F[\theta]*G[\theta])}(y), \beta\} \\
&= \min\left\{\sup_{x=a*b} \min\{\mu_{\mathcal{F}[\theta]}(a), \mu_{\mathcal{G}[\theta]}(b)\},\right. \\
&\left.\sup_{y=c*d} \min\{\mu_{\mathcal{F}[\theta]}(c), \mu_{\mathcal{G}[\theta]}(d)\}, \beta\right\} \\
&\leq \sup_{x=a*b,\ y=c*b} \min\left\{\min\{\mu_{\mathcal{F}[\theta]}(a),\right. \\
&\left.\mu_{\mathcal{F}[\theta]}(c), \beta\}, \min\{\mu_{\mathcal{G}[\theta]}(b), \mu_{\mathcal{G}[\theta]}(d), \beta\}\right\} \\
&\leq \sup_{x*y=(a*b)*(c*d)} \min\left\{max\{\mu_{\mathcal{F}[\theta]}(a*c),\right. \\
&\left.\alpha\}, \max \mu_{\mathcal{G}[\theta]}(b*d), \alpha\}\right\} \\
&\leq \max\left\{\sup_{x*y=p*q} \min\{\mu_{\mathcal{F}[\theta]}(p), \mu_{\mathcal{G}[\theta]}(q)\}, \alpha\right\} \\
&= \max\{\mu_{(\mathcal{F}*\mathcal{G})}[\theta]\}(x*y), \alpha\}.
\end{aligned}$$

□

4. Conclusion

Fuzzy soft set theory has an important role in our real life development. Algebraic structures with fuzzy soft set theory have a variety of applications in different directions. In this paper $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft subalgebras and some operational properties of $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebras are developed. It is our view that these types of results can be similarly extended to other algebraic structures.

References

[1] H. Aktas and N. Cagman, *Soft sets and soft groups*, Inform. Sci., **177** (2007), 2726–2735.

[2] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, *On some new operations in soft set theory*, Comput. Math. Appl., **57** (9) (2009), 1547–1553.

[3] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, *On some new operations in soft set theory*, Comput. Math. Appl., **57** (2009), 1547–1553.
[4] A. Aygünoglu and H. Aygün, *Introduction to fuzzy soft groups*, Comput. Math. Appl., **58** (2009), 1279–1286.
[5] T. Bej and M. Pal, *Doubt Atanassovs intuitionistic fuzzy sub-implicative ideals in BCI-algebras*, Int. J. Comput. Int. Sys., **8** (2) (2015), 240–249.
[6] S. K. Bhakat and P. Das, $(\in, \in \vee q)$*-fuzzy subgroups*, Fuzzy Sets and Systems, **80** (1996), 359–368.
[7] N. Çağman and S. Enginoğlu, *Soft set theory and uni-int decision making*, European Journal of Operational Research, **207** (2) (2010), 848–855.
[8] N. Çağman and S. Enginoçglu, *Soft matrix theory and its decision making*, Comput. Math. Appl., **59** (10) (2010), 3308–3314.
[9] F. Feng, Y. B. Jun, and X. Zhao, *Soft semirings*, Comput. Math. Appl., **56** (2008), 2621–2628
[10] Y. S. Huang, BCI-Algebra, Science Press, Beijing, 2006.
[11] Y. Imai and K. Iseki, *On axiom system of propositional calculi.*, XIV. Proc. of Japan Aceademy, **42** (1966), 19–22.
[12] C. Jana, T. Senapati, M. Bhowmik, and M. Pal, *On intuitionistic fuzzy G-subalgebras of G-algebras*, Fuzzy Inf. Eng., **7.2** (2015), 195–209.
[13] C. Jana, M. Pal, T. Senapati, and M. Bhowmik, *Atanassov's intutionistic L-fuzzy G-subalgebras of G-algebras*, J. Fuzzy Math., **23.2** (2015), 195–209.
[14] C. Jana, T. Senapati, and M. Pal, *Derivation, f-derivation and generalized derivation of KUS- algebras*, Cogent Mathematics, **2** (2015), 1–12.
[15] C. Jana and M. Pal, *Applications of new soft intersection set on groups*, Ann. Fuzzy Math. Inform., **11** (6) (2016), 923–944.
[16] C. Jana, T. Senapati, and M. Pal, $(\in, \in \vee q)$*-intuitionistic fuzzy BCI-subalgebras of BCI-algebra*, J. Int. Fuzzy Syst., **31** (2016), 613–621, DOI:10.3233/IFS-162175 .
[17] C. Jana and M. Pal, *Application of (α, β)-soft intersectional sets on BCK/BCI-algebras*, Int. J. of Intelligent Systems Technologies and Applications, **16.3** (2017), 269–288.
[18] C. Jana, *Generalized* (α, Υ)*-derivation on subtraction algebras*, J. Math. Inform., **4** (2015), 71–80.
[19] C. Jana and T. Senapati, *Cubic G-subalgebras of G-algebras*, Annals of Pure and Applied Mathematics, **10.1** (2015), 105–115.
[20] C. Jana, T. Senapati, and M. Pal, *t-derivation on complicated subtraction subtraction algebras*, Journal of Discrete Mathematical Sciences and Cryptography, 2017, (to appear).
[21] C. Jana and M. Pal, *Generalized intuitionistic fuzzy ideals of BCK/BCI-algebras based on 3-valued logic and its computational study*, Fuzzy Inf. Eng., 2017, (to appear).
[22] C. Jana, M. Pal, and A. B. Saied, $(\in, \in \vee q)$*-bipolar fuzzy BCK/BCI-algebras*, Missouri J. of Math. Sci., **29.2**, (2017), 139–160.
[23] Y. B. Jun, *On (α, β)-fuzzy subalgebras of BCK/BCI-algebras*, Bull. Korean Math. Soc., **42** (4) (2005), 703–711.
[24] Y. B. Jun, *Soft BCK/BCI-algebras*, Comput. Math. Appl., **56** (2008), 1408–1413.
[25] Y. B. Jun and C. H. Park, *Applications of soft sets in ideal theory of BCK/BCI-algebras*, Inform. Sci., **178** (2008), 2466–2475.
[26] Y. B. Jun, K. J. Lee, and C. H. Park, *Fuzzy soft set theory applied to BCK/BCI-algebras*, Comput. Math. Appl., **59** (2010), 3180–319.

[27] Y. B. Jun, G. Muhiuddin, and A. Al-roqi, *Ideal theory of BCK/BCI-algebras based on double-framed soft sets*, Applied Mathematicd and Information Sciences, **7.5** (2013), 1879–1887.

[28] Y. B. Jun, M. A. Öztürk, and G. Muhiuddin, *A generalization of* $(\in, \in \vee q)$*-fuzzy subgroups*, International Journal of Algebra and Statistics, **5.1** (2016), 7–18.

[29] Y. B. Jun, S. Z. Song, and G. Muhiuddin, *Filter theory in EQ-algebras based on soft sets*, Quasigroups and Related Systems, **24** (2016), 25–32.

[30] X. Ma, J. Zhan, B. Davvaz, and Y. B. Jun, *Some kinds of* $(\in, \in \vee q)$*-interval-valued fuzzy ideals of BCI-algebras*, Inform. Sci., **178** (2008), 3738–3754.

[31] X. Ma, J. Zhan, and Y. B. Jun, *Some kinds of* $(\in_\alpha, \in_\alpha \vee q_\beta)$*-fuzzy ideals of BCI-algebras*, Comput. Math. Appl., **61** (2011), 1005–1015.

[32] P. K. Maji, R. Biswas, and A. R. Roy, *Soft set theory*, Comput. Math. Appl., **45** (2003), 555–562.

[33] P. K. Maji, A. R. Roy, and R. Biswas, *An application of soft sets in a decision making problem*, Comput. Math. Appl, **44** (2002), 1077–1083.

[34] G. Muhiuddin, A. M. Al-roqi, and S. Aldhafeeri, *Filter theory in MTL-algebras based on Uni-soft property*, Bulletin of the Iranian Mathematical Society, (2017) (to appear).

[35] G. Muhiuddin, C. H. Park, and Y. B. Jun, *Relations between regular uni-soft filters and uni-soft MV-filters in residuated lattices*, Kyungpook Mathematical Journal, **57** (1) (2017), 85–97.

[36] G. Muhiuddin, *Int-soft hyper-MV-deductive systems in hyper-MV-algebras*, Azerbaijan Journal of Mathematics, **6** (1) (2016), 39–51.

[37] G. Muhiuddin and A. M. Al-roqi, *Uni-soft Filters in R0-algebras*, Journal of Computational Analysis and Applications, **19** (1) (2015), 133–143.

[38] G. Muhiuddin, H. S. Kim, S. Z. Song, and Y. B. Jun, *Hesitant fuzzy translations and extensions of subalgebras and ideals in BCK/BCI-algebras*, J. Int. Fuzzy Syst., **32** (2017), 43–48.

[39] G. Muhiuddin and A. Al-roqi, *Subalgebras of BCK/BCI-algebras based on* (α, β)*-type fuzzy sets*, Journal of Computational Analysis and Applications, **18** (6) (2015), 1057–1064.

[40] D. Molodtsov, *Soft set theory-first results*, Comput. Math. Appl., **37** (1999), 19–31.

[41] A. Narayanan and T. Manikantan, $(\in, \in \vee q)$*-fuzzy subnear-rings and* $(\in, \in \vee q)$*-fuzzy ideals of nearrings*, J. Appl. Math. Comput., **18** (2005), 419–430.

[42] P. Pu and Y. Liu, *Fuzzy topology I. Neighbourhood structures of a fuzzy point and Moore-Smith convergence*, J. Math. Anal. Appl., **76** (1980), 571–579.

[43] A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl., **35** (1971), 512–517.

[44] Y. Yin and J. Zhan, *The characterizations of hemirings in terms of fuzzy soft h-ideals*, Neural Comput. Appl., **21** (1) (2012), 43–57.

[45] L. A. Zadeh, *Fuzzy sets*, Inform. Control, **8** (1965), 338–353.

[46] M. Zhou, D. Xiang, and J. Zhan, *The characterization of* α*-modules in terms of fuzzy soft* α*-submodules*, Appl. Math. J. Chinese Univ., **28** (2) (2013), 217–239.

MSC2010: 06D72, 06F35, 03G25

Key words and phrases: BCI-algebra; fuzzy point; fuzzy soft set; $(\in_\alpha, \in_\alpha \vee q_\beta)$-fuzzy soft BCI-algebra.

Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore, 721102, India
E-mail address: rsmo_jana.chiranjibe@mail.vidyasagar.ac.in

Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore, 721102, India
E-mail address: mmpalvu@mail.vidyasagar.ac.in