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Links with homotopically trivial complements are trivial

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1. Introduction

A smooth (resp. PL locally flat or locally flat) m-component link L stands for m embedded disjoint n-spheres $L_1 \cup \cdots \cup L_m$ in S^{n+2}. A knot is nothing but a 1-component link. A smooth (resp. PL locally flat or locally flat) m-component link is called trivial if it bounds m smoothly (resp. PL locally flatly or locally flatly) embedded disjoint $(n+1)$-disks. The complement of a trivial knot has the homotopy type of a circle S^1. The converse is known to be true for a locally flat knot. The converse is also known to be true for a smooth (or PL locally flat) knot when $n \neq 2$ ([9] for $n \geq 4$, [14] for $n = 3$ and [13] for $n = 1$). The complement of a trivial m-component link has the homotopy type of a one point union $(\vee_{i=1}^{m} S_i^1) \vee (\vee_{j=1}^{m-1} S_j^{n+1})$ of circles and $(n+1)$-spheres. So, there arises a natural question whether a link is trivial if the complement of the link has the homotopy type of a one point union of spheres. One of the purposes of this paper is to settle this question affirmatively provided that $n \neq 2$:

THEOREM 1. *Let $L \subset S^{n+2}$ be a smooth (resp. PL locally flat or locally flat) m-component link such that $S^{n+2} - L$ has the homotopy type of $(\vee_m S^1) \vee (\vee_{m-1} S^{n+1})$. Suppose that $n \neq 2$. Then L is trivial.*

A one point union of spheres has a special property that it is covered by two subsets which are contractible. This property itself is not a homotopy type invariant and a better notion is that it has Lusternik-Schnirelmann category one. The category $\operatorname{cat} X$ of a space X is the least integer n such that X can be covered by $n+1$ number of open subsets each of which is contractible to a point in X. In particular, cat X is a homotopy type invariant and $\operatorname{cat}((\vee_m S^1) \vee (\vee_{m-1} S^{n+1})) = 1$. We know that $\pi_1(X)$ is a free group if X is a manifold and $\operatorname{cat} X = 1$ (cf. [5]).

A locally flat knot (S^{n+2}, S^n) is topologically unknotted if and only if the category of its complement is one [11]. In fact, $\operatorname{cat}(S^{n+2} - S^n) = 1$ if and only if $S^{n+2} - S^n$ has the homotopy type of S^1. So, a smooth (or PL locally flat) knot (S^{n+2}, S^n) is unknotted if and only if $\operatorname{cat}(S^{n+2} - S^n) = 1$ when $n \neq 2$.

By Theorem 1 of [8] the link complement $S^{n+2} - L$ has the homotopy type of $(\vee_m S^1) \vee (\vee_{m-1} S^{n+1})$ if $\mathrm{cat}(S^{n+2} - L) = 1$. So, Theorem 1 implies the following theorem.

THEOREM 2. *Let L be a smooth (resp. PL locally flat or locally flat) m-component link in S^{n+2}. Suppose that $n \neq 2$. Then L is trivial if and only if* $\mathrm{cat}(S^{n+2} - L) = 1$.

A classical link L is trivial if $\pi_1(S^3 - L)$ is free by the loop theorem [13]. So, Theorem 1 is already known for $n = 1$. If $S^{n+2} - L$ has the homotopy type of $(\vee_m S^1) \vee (\vee_{m-1} S^{n+1})$, we will show that L is a boundary link in §2. Then, we see that L is trivial by the unlinking criterion of boundary links due to Gutiérrez ([6] for $n \geq 4$ and use the splitting theorem [1] for $n = 3$).

The dimensional restriction can be removed for the case of knot by considering homeomorphism rather than diffeomorphism but remains unknown for the case of 2-dimensional link [4].

Theorem 2 has been conjectured by Professor T. Matumoto to whom the author would like to express his sincere gratitude for suggesting the problem.

2. Proof of Theorem 1

In the proof the link exterior $E = S^{n+2} - \mathrm{Int}\, N(L)$ is more useful than the link complement $S^{n+2} - L$ where $N(L)$ denotes a tubular neighborhood of L; E is a compact manifold with boundary $\partial E = \partial N(L)$ and has the homotopy type of the link complement. By the unlinking criterion of boundary links due to Gutiérrez [1], [6] it suffices to show that L is a boundary link if $S^{n+2} - L$ has the homotopy type of $(\vee_m S^1) \vee (\vee_{m-1} S^{n+1})$. We recall the definition of a boundary link; a smooth (resp. PL locally flat or locally flat) m-component link is boundary if it bounds a Seifert manifold which consists of m disjoint compact smooth (resp. PL locally flat or locally flat) $(n+1)$-submanifolds with connected boundary. We remark that an element of $\pi_1(E)$ is called meridian if it is conjugate to a generator of the fundamental group of some component $S^1 \times S^n$ of ∂E. We will find m number of meridians $m_1, \dots, m_m$ which generate $\pi_1(E)$; this is a necessary and sufficient condition for the link to be boundary in our case that $\pi_1(E)$ is a free group by [6, p. 493, Prop. 3] and [10, p. 109, Cor. 2.12].

Let $i(k)\colon \partial N(L_k) \to E$ be the inclusion map for k. Then, $i(k)_*\colon \pi_n(\partial N(L_k)) \to \pi_n(E)$ is a 0-map for any k, since $\pi_n(E) = 0$. So, it suffices to show the following proposition in order to prove Theorem 1.

PROPOSITION 2.1. *Let L be an m-component link and $i(k)\colon \partial N(L_k) \to E$ be* *the inclusion map for a component* L_k of the link L. *Suppose that* $\pi_1(E)$ *is a free group and* $i(k)_*\colon \pi_n(\partial N(L_k)) \to \pi_n(E)$ *is a* 0*-map for any* k. *Then,* L *is boundary.*

PROOF. By induction on m we will prove Proposition 2.1. Note that $\pi_1(E)$ is a free group F_m of rank m. The case when $m = 1$ is proven because $\pi_1(E)$ is isomorphic to an infinite cyclic group $\mathbf{Z}$ and it is generated by any meridian. So, we may assume that $m \geq 2$. Suppose that Proposition 2.1 is true for any j-component sublink L' of $L (j < m)$ which satisfies the assumption in Proposition 2.1 that $i'(k)_*\colon \pi_n(\partial N(L_k)) \to \pi_n(E')$ is a 0-map for any component L_k of L' and $\pi_1(E')$ is a free group. Here L' is a sublink of L with the exterior E' and $i'(k)\colon \partial N(L_k) \to E'$ is the inclusion for a component L_k of L'. We fix m number of meridians $m'_1, m'_2, \ldots, m'_m$ of L corresponding to the components $\partial N(L_1), \partial N(L_2), \ldots, \partial N(L_m)$ of ∂E. Let $H_i (\subset \pi_1(E))$ be an infinite cyclic subgroup generated by m'_i.

Even when $(E, \partial E)$ admits no triangulation, $(E, \partial E)$ has the simple homotopy type of a finite Poincaré complex by [7, III, §4] and we will denote by $(E, \partial E)$ this finite Poincaré complex instead of the original link exterior in this case. Let $p\colon \tilde{E} \to E$ be the universal covering of E and put $\partial\tilde{E} = p^{-1}(\partial E)$. Let $H_c^*(X; \mathbf{Z})$ denote the cohomology with compact support of X. Since the CW complex pair $(\tilde{E}, \partial\tilde{E})$ has the proper homotopy type of the universal covering of the original link exterior, we can apply the Poincaré duality theorem for the non-compact manifold and see that the left $\mathbf{Z}[F_m]$-module $H_{n+1}(\tilde{E}, \partial\tilde{E}; \mathbf{Z})$ is anti-$\mathbf{Z}[F_m]$ isomorphic to the right $\mathbf{Z}[F_m]$-module $H_c^1(\tilde{E}; \mathbf{Z})$ and the left $\mathbf{Z}[F_m]$-module $H_n(p^{-1}(\partial N(L_i)); \mathbf{Z})$ is anti-$\mathbf{Z}[F_m]$ isomorphic to the right $\mathbf{Z}[F_m]$-module $H_c^1(p^{-1}(\partial N(L_i)); \mathbf{Z})$.

We recall the relationship between homology of free coverings and cohomology of groups. Let $C_\#(X)$ denote the cellular chain complex of a cellular complex X and $C_c^\#(X)$ denote the cellular cochain complex with compact support of a cellular complex X. We see that $C_c^\#(\tilde{E})$ is cochain equivalent to the cochain complex $\mathrm{Hom}_{\mathbf{Z}[F_m]}(C_\#(\tilde{E}), \mathbf{Z}[F_m])$, and that $C_c^\#(p^{-1}(\partial N(L_i)))$ is cochain equivalent to the cochain complex $\mathrm{Hom}_{\mathbf{Z}[F_m]}(C_\#(p^{-1}(\partial N(L_i))), \mathbf{Z}[F_m])$ as in the proof of Lemma 2.1 (2) of [8]. Let $H^*(H\colon \mathbf{Z}G)$ denote the cohomology of a group H with coefficient $\mathbf{Z}G$. Note that the kernel of $\mathbf{Z}[F_m]$-homomorphism between finitely generated projective $\mathbf{Z}[F_m]$-modules is a finitely generated projective $\mathbf{Z}[F_m]$-module. In fact, it is projective because $\mathbf{Z}[F_m]$ has the global dimension two due to [12, p. 326, Cor. 2.7], and finitely generated because $\mathbf{Z}[F_m]$ is coherent [2, p. 137, Th. (2.1)], [16, p. 158, Prop.]. Note that the cellular chain complexes $\{C_\#(\tilde{E}), \partial_\#\}$ and $\{C_\#(p^{-1}(\partial N(L_i))), \partial'_\#\}$ are chain complexes of finitely generated free $\mathbf{Z}[F_m]$-modules and $\mathbf{Z}[F_m]$-homomorphisms.

Since $\tilde{E}$ is the universal covering, $H_1(\tilde{E}; \mathbf{Z}) = 0$. Then, we have the following projective resolution of $\mathbf{Z}$ over $\mathbf{Z}[\pi_1(E)] = \mathbf{Z}[F_m]$: $0 \to \operatorname{Ker} \partial_2 \subsetneq C_2(\tilde{E}) \xrightarrow{\partial_2} C_1(\tilde{E}) \xrightarrow{\partial_1} C_0(\tilde{E}) \to C_0(\tilde{E})/\operatorname{Im} \partial_1 \cong \mathbf{Z} \to 0$. Hence, we get $H_c^1(\tilde{E}; \mathbf{Z}) \cong H^1(\operatorname{Hom}_{\mathbf{Z}[F_m]}(C_\#(\tilde{E}), \mathbf{Z}[F_m])) \cong H^1(\pi_1(E); \mathbf{Z}[\pi_1(E)])$.

Let M_i denote one of the connected components of $p^{-1}(\partial N(L_i))$. Note that the cellular chain complex $\{C_\#(M_i), \partial''_\#\}$ of M_i is a chain complex of finitely genereted free $\mathbf{Z}[H_i]$-modules. Since M_i is the universal covering of $p(M_i)$, $H_1(M_i; \mathbf{Z}) = 0$. Then, we have the following projective resolution of $\mathbf{Z}$ over $\mathbf{Z}[H_i]$: $0 \to \operatorname{Ker} \partial''_2 \subsetneq C_2(M_i) \xrightarrow{\partial''_2} C_1(M_i) \xrightarrow{\partial''_1} C_0(M_i) \to C_0(M_i)/\operatorname{Im} \partial''_1 \cong \mathbf{Z} \to 0$. We have the natural $\mathbf{Z}[\pi_1(E)]$-isomorphisms q_j: $C_j(M_i) \otimes_{\mathbf{Z}[H_i]} \mathbf{Z}[\pi_1(E)] \xrightarrow{\cong} C_j(p^{-1}(\partial N(L_i)))$ such that $q_{j-1} \circ (\partial''_j \otimes_{\mathbf{Z}[H_i]} id_{\mathbf{Z}[\pi_1(E)]}) = \partial'_j \circ q_j$ for any j. Hence, we get $H_c^1(p^{-1}(\partial N(L_i)); \mathbf{Z}) = H^1(\operatorname{Hom}_{\mathbf{Z}[F_m]}(C_\#(p^{-1}(\partial N(L_i))), \mathbf{Z}[F_m])) \cong H^1(\operatorname{Hom}_{\mathbf{Z}[F_m]}(C_\#(M_i) \otimes_{\mathbf{Z}[H_i]} \mathbf{Z}[\pi_1(E)], \mathbf{Z}[F_m])) \cong H^1(H_i; \mathbf{Z}[\pi_1(E)])$.

We consider the following commutative diagram:

$$\begin{tikzcd}
H_{n+1}(\tilde{E}, \partial\tilde{E}; \mathbf{Z}) \arrow[r, "\partial_*"] & H_n(\partial\tilde{E}; \mathbf{Z}) = \bigoplus_{i=1}^m H_n(p^{-1}(\partial N(L_i)); \mathbf{Z}) \\
H_c^1(\tilde{E}; \mathbf{Z}) \arrow[u, "\cong"] \arrow[r] \arrow[d, equal] & \bigoplus_{i=1}^m H_c^1(p^{-1}(\partial N(L_i)); \mathbf{Z}) \arrow[u, "\cong"] \arrow[d, equal] \\
H^1(\pi_1(E); \mathbf{Z}[\pi_1(E)]) \arrow[r, "r=(r_1, r_2, \dots, r_m)"] & \bigoplus_{i=1}^m H^1(H_i; \mathbf{Z}[\pi_1(E)]),
\end{tikzcd}$$

where r_i: $H^1(\pi_1(E); \mathbf{Z}[\pi_1(E)]) \to H^1(H_i; \mathbf{Z}[\pi_1(E)])$ is the restriction map induced by the inclusion $H_i \subsetneq \pi_1(E)$ and the vertical maps are Poincaré duality isomorphisms for non-compact manifolds.

Because $\partial\tilde{E}$ is an $(n+1)$-dimensional non-compact manifold, we have $H_{n+1}(\partial\tilde{E}; \mathbf{Z}) = 0$. So, the kernel of ∂_* is isomorphic to $H_{n+1}(\tilde{E}; \mathbf{Z})$ by the homology long exact sequence of $(\tilde{E}, \partial\tilde{E})$. Then, we see that the kernel of r is isomorphic to $H_{n+1}(\tilde{E}; \mathbf{Z}) \cong \mathbf{Z}^{m-1}$ by the above commutative diagram. Hence, $\bigcap_{i=1}^m \operatorname{Ker} r_i = \operatorname{Ker} r \neq \{0\}$ when $m \geq 2$. We quote the following theorem:

THEOREM 2.2 ([15, p. 75, 1.1. Theorem]). *Let G be a finitely generated group and let H_i, $1 \leq i \leq m$, be subgroups of G. Let r_i, $1 \leq i \leq m$, denote the restriction maps $H^1(G; \mathbf{Z}G) \to H^1(H_i; \mathbf{Z}G)$. If the intersection of the kernels of r_i is non-zero, then, either*
a) *G has a non-trivial decomposition $G = G_1 *_F G_2$ with F finite and each H_i is conjugate to a subgroup of G_1 or G_2; or*
b) *G has a non-trivial decomposition $G = G_1 *_F$ with F finite and each H_i is conjugate to a subgroup of G_1.*

We can apply Theorem 2.2 to the case that $G = \pi_1(E)$ and H_i is the infinite cyclic subgroup generated by m_i' $(1 \le i \le m)$, since we have shown that $\bigcap_{i=1}^m \operatorname{Ker} r_i \neq \{0\}$. Since $\pi_1(E)$ is a free group, we see that $\pi_1(E)$ doesn't have a non-trivial decomposition of the case b), and we see that $\pi_1(E)$ has a non-trivial decomposition $\pi_1(E) = G_1 * G_2$ such that each H_i is conjugate to a subgroup of G_1 or G_2.

To complete the induction step we need the following two lemmas.

LEMMA 2.3. *Let H be a normal subgroup of A and K be a normal subgroup of B. If N is the normal closure of the subgroup of $A * B$ generated by H and K, then $(A * B)/N \cong (A/H) * (B/K)$.*

This algebraic lemma can be proven easily, since $(A * B)/N$ is obtained by adding the relators H and K to the relators of $A * B$ by [10, p. 71, Th. 2.1].

LEMMA 2.4. *Let $L = L_1 \cup \cdots \cup L_m$ be an m-component link and $L(i) = L_1 \cup \cdots \cup L_{i-1} \cup L_{i+1} \cup \cdots \cup L_m$ $(1 \le i \le m)$ be the $(m-1)$-component sublink of L with the exterior $E(i)$. Suppose that $i(k)_*\colon \pi_n(\partial N(L_k)) \to \pi_n(E)$ is a 0-map for any k and $\pi_1(E)$ is a free group. Then $i'(k)_*\colon \pi_n(\partial N(L_k)) \to \pi_n(E(i))$ is a 0-map for the inclusion $i'(k)\colon \partial N(L_k) \to E(i)$ $(k \neq i)$ and $\pi_1(E(i))$ is a free group.*

PROOF OF LEMMA 2.4. The first statement is easy: Since $i(k)_*\colon \pi_n(\partial N(L_k)) \to \pi_n(E)$ is a 0-map for any k and $i'(k)$ is the composition of $i(k)$ and the inclusion $E \subsetneq E(i)$, we see that $i'(k)_*\colon \pi_n(\partial N(L_k)) \to \pi_n(E(i))$ is a 0-map $(k \neq i)$.

Now we will prove the other statement. Since each component $\partial N(L_k)$ of ∂E is homeomorphic to $S^1 \times S^n$ and $i(k)_*\colon \pi_n(\partial N(L_k)) \to \pi_n(E)$ is a 0-map for any k, we see that $i(k)_*\colon H_n(p^{-1}(\partial N(L_k)); \mathbf{Z}) \to H_n(\tilde{E}; \mathbf{Z})$ is a 0-map for any k and hence $H_n(\partial \tilde{E}; \mathbf{Z}) \to H_n(\tilde{E}; \mathbf{Z})$ is a 0-map for the inclusion $\partial \tilde{E} \subsetneq \tilde{E}$. Then $\partial_*\colon H_{n+1}(\tilde{E}, \partial \tilde{E}; \mathbf{Z}) \to H_n(\partial \tilde{E}; \mathbf{Z})$ is surjective by the homology long exact sequence of $(\tilde{E}, \partial \tilde{E})$. So, r is surjective by the above commutative diagram, and hence each r_i is surjective.

So, we can use the following result given in [3].

PROPOSITION 2.5 ([3, p. 246]). *Let H be an infinite cyclic subgroup of the finitely generated group G. If the restriction map $res\colon H^1(G, \mathbf{Z}G) \to H^1(H, \mathbf{Z}G)$ is surjective, then H is a free factor of G.*

By Proposition 2.5 there is a free group K_i such that $\pi_1(E) = H_i * K_i$ for any i. Hence, by Lemma 2.3 the fundamental group $\pi_1(E(i)) = \pi_1(E)/NH_i$ of the exterior of the $(m-1)$-component sublink $L(i)$ of L is isomorphic to the free group K_i, where NH_i is the normal closure of H_i. The proof of Lemma 2.4 is complete. q.e.d.

Recall that $\pi_1(E)$ has a non-trivial decomposition $\pi_1(E) = G_1 * G_2$ such that each H_i is conjugate to a subgroup of G_1 or G_2. Since $\pi_1(E) = N\langle m_i'; 1 \leq i \leq m\rangle$, by Lemma 2.3 and reordering indices, there are an integer ℓ with $2 \leq \ell \leq m-1$ and $g_i \in \pi_1(E)$ with $1 \leq i \leq m$ such that $g_i m_i' g_i^{-1} \in G_1$ for $1 \leq i \leq \ell$ and $g_i m_i' g_i^{-1} \in G_2$ for $\ell + 1 \leq i \leq m$. Here $N\langle m_i'; 1 \leq i \leq m\rangle$ denotes the normal closure of the subgroup generated by $m_1', m_2', \dots, m_m'$.

Now we will show that G_1, G_2 are isomorphic to the fundamental groups of the exteriors of the sublinks of L which satisfy the assumption in Proposition 2.1 and $g_i m_i' g_i^{-1} \in G_1$ or G_2 correspond to those meridians. Let NK denote the normal closure in $\pi_1(E)$ of a subgroup K and $N_i K_i$ denote the normal closure in G_i of a subgroup K_i of $G_i (i = 1, 2)$ and $\langle x_i; 1 \leq i \leq k\rangle$ denote the subgroup of G_1, G_2 or $\pi_1(E)$ generated by k number of elements $x_1, x_2, \dots, x_k$. Since $N\langle m_i'; \ell + 1 \leq i \leq m\rangle = N(N_2\langle g_i m_i' g_i^{-1}; \ell + 1 \leq i \leq m\rangle))$, the fundamental group $\pi_1(E)/N\langle m_i'; \ell + 1 \leq i \leq m\rangle$ of the exterior of the sublink $L_1 \cup \cdots \cup L_\ell$ of L is isomorphic to $G_1 * (G_2/N_2\langle g_i m_i' g_i^{-1}; \ell + 1 \leq i \leq m\rangle)$ by Lemma 2.3. Because $\pi_1(E) = N\langle m_i'; 1 \leq i \leq m\rangle$, by Lemma 2.3 we get $(G_1/N_1\langle g_i m_i' g_i^{-1};\ 1 \leq i \leq \ell\rangle) * (G_2/N_2\langle g_i m_i' g_i^{-1};\ \ell + 1 \leq i \leq m\rangle) = \pi_1(E)/N\langle m_i'; 1 \leq i \leq m\rangle = \{1\}$. Then, $G_2/N_2\langle g_i m_i' g_i^{-1}; \ell + 1 \leq i \leq m\rangle = \{1\}$. Hence, G_1 is isomorphic to the fundamental group of the exterior of the sublink $L_1 \cup \cdots \cup L_\ell$ of L, and $g_i m_i' g_i^{-1} \in G_1$ $(1 \leq i \leq \ell)$ correspond to its meridians. Similarly G_2 is isomorphic to the fundamental group of the exterior of the sublink $L_{\ell+1} \cup \cdots \cup L_m$ of L, and $g_i m_i' g_i^{-1} \in G_2 (\ell + 1 \leq i \leq m)$ correspond to its meridians.

By an inductive argument on the number of compoments of sublinks, Lemma 2.4 implies that two sublinks $L_1 \cup \cdots \cup L_\ell$ and $L_{\ell+1} \cup \cdots \cup L_m$ of L satisfy the assumption in Proposition 2.1. Then, by the inductive hypothesis in the proof of Proposition 2.1, we have that $L_1 \cup \cdots \cup L_\ell$ and $L_{\ell+1} \cup \cdots \cup L_m$ are boundary links. Hence, as mentioned above, we see that each G_i is generated by the meridians in it, that is, there exist $h_i \in \pi_1(E)$ $(1 \leq i \leq m)$ such that $h_i \in G_1$ $(1 \leq i \leq \ell)$ and G_1 is generated by $m_i = h_i g_i m_i' g_i^{-1} h_i^{-1}$ $(1 \leq i \leq \ell)$, and $h_i \in G_2$ $(\ell + 1 \leq i \leq m)$ and G_2 is generated by $m_i = h_i g_i m_i' g_i^{-1} h_i^{-1}$ $(\ell + 1 \leq i \leq m)$. Hence $\pi_1(E)$ is generated by m number of meridians $\{m_i\}$. This implies that L is boundary as mentioned above. q.e.d.

Now L is a boundary link by Proposition 2.1. So, L is trivial by the unlinking criterion [1], [6]. This completes the proof of Theorem 1.

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