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Correction of a proof in the paper "Approximate roots, toric resolutions and deformations of a plane branch"

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Abstract. We correct the proof of Theorem 3.8 in [**GP2**].

The proof of Theorem 3.8 in [**GP2**] is not correct from the sentence starting at line 5 to the end of line 14 of page 995. The correction is done by the replacing this part by the following:

If $\alpha(x)F_1^{\eta_1}\cdots F_{j-1}^{\eta_{j-1}}$ with $\mathrm{ord}_x\alpha(x) = \eta_0 \geq 0$, is a term appearing in the $(F_1,\dots,F_j)$-expansions of F_{j+1}, different from $F_j^{N_j}$ and $\alpha_0^{(j)}(x)F_1^{\eta_1^{(j)}}\cdots F_{j-1}^{\eta_{j-1}^{(j)}}$, then the straight line condition implies that

$$(1/E_j)N_j\bar{B}_j < (1/E_j)\big(\eta_0\bar{B}_0 + \cdots + \eta_{j-1}\bar{B}_{j-1}\big). \tag{1}$$

The inequality (1) is strict since $\sum_{i=0}^{j-1}\eta_i\bar{B}_i \neq \sum_{i=0}^{j-1}\eta_i^{(j)}\bar{B}_i$ by the numerical properties of the semigroup Γ generated by $\bar{B}_0,\dots,\bar{B}_G$, see Lemma 1.15.

We re-embed the germ $(C,0)$ defined by $F=0$, in $(\mathbf{C}^{G+1},0)$ by setting

$$u_0 = x,\ u_1 = F_1,\ \dots,\ u_G = F_G. \tag{2}$$

We also set the weight of u_i equal to $\bar{B}_i$, for $i=0,\dots,G$. The equations defining the embedding of $(C,0)$ are obtained by making the replacement (2) in the $(F_1,\dots,F_j)$-expansion of F_{j+1} for $j=1,\dots,G$. The inequalities of the form (1) together with $N_j\bar{B}_j = \sum_{i=0}^{j-1}\eta_i^{(j)}\bar{B}_i$ for $j=1,\dots,G$ and $N_j\bar{B}_j < \bar{B}_{j+1}$ for $j=1,\dots,G-1$, are precisely the weight conditions on the equations of defining the embedding $(C,0)\subset(\mathbf{C}^{G+1},0)$ indicated in Proposition 39 of [**GP**]. By the proof of Theorem 6.1 in [**G-T**] or by Theorem 2, page 1867 in [**GP**] one toric modication of $\mathbf{C}^{G+1}$ provides a simultaneous embedded resolution of both the

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monomial curve $(C^\Gamma, 0)$ parametrized by $u_i = t^{\bar{B}_i}$, $i = 0, \dots, G$ and the germ $(C, 0) \subset (\mathbf{C}^{G+1}, 0)$. In addition, the strict transforms of both curves intersect the exceptional divisor at exactly one point. In particular, this implies that the normalization of $(C, 0)$ is smooth, and the germ $(C, 0)$ is irreducible hence F is irreducible. $\square$

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References

[G-T] R. Goldin and B. Teissier, Resolving singularities of plane analytic branches with one toric morphism, Resolution of Singularities, A research textbook in tribute to Oscar Zariski, (eds. H. Hauser, J. Lipman, F. Oort and A. Quiros), Progr. Math., **181**, Birkhäuser, Basel, 2000, pp. 315–340.

[GP] P. D. González Pérez, Toric embedded resolutions of quasi-ordinary hypersurface singularities, Ann. Inst. Fourier (Grenoble), **53** (2003), 1819–1881.

[GP2] P. D. González Pérez, Approximate roots, toric resolutions and deformations of a plane branch, J. Math. Soc. Japan, **62** (2010), 975–1004.

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