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Strongly regular mappings with ANR fibers and shape

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1. Introduction.

In [7], we defined the fiber shape category FR_B which is shape theoretic category analogous to the fiber homotopy category and studied the category FR_B. In this paper, we study some properties of strongly regular mappings with ANR fibers in FR_B. We first prove the following.

(i) Let E, B be compacta and $\dim B<\infty$. If $p: E\rightarrow B$ is a strongly regular mapping with ANR fibers, then for any map $q: Y\rightarrow B$ of compacta there is a natural bijection $\Phi: [Y, E]_{q,p}\rightarrow\langle Y, E\rangle_{q,p}$, where $[Y, E]_{q,p}$ denotes the set of fiber homotopy classes of fiber maps from q to p and $\langle Y, E\rangle_{q,p}$ the set of morphisms from q to p in FR_B.

In [5], S. Ferry proved that if $f: E\rightarrow B$ is a strongly regular mapping onto a complete finite dimensional space B and $f^{-1}(b)$ is an ANR for each $b\in B$, then f is a Hurewicz fibration. If $f: E\rightarrow B$ is a Hurewicz fibration between compact ANR, then f is a shape fibration. Note that there are Hurewicz fibrations between compacta which are not shape fibrations (e.g. [**11**, p. 641]). Next, we prove the following.

(ii) Let E, B be compacta and $\dim B<\infty$. If $p: E\rightarrow B$ is a strongly regular mapping with ANR fibers, then p is a shape fibration.

As an application of (i) and (ii), we show the following.

(iii) Let E, E' and B be compacta and $\dim B<\infty$. Suppose that $p: E\rightarrow B$ and $p': E'\rightarrow B$ are strongly regular mappings with ANR fibers. If a fiber map $f: E\rightarrow E'$ from p to p' induces a strong shape equivalence, then f is a fiber homotopy equivalence.

2. Definitions.

Throughout this paper, all spaces are metric spaces and maps are continuous functions. We mean by I the unit interval $[0, 1]$ and by Q the Hilbert cube $\prod_{i=1}^{\infty}[-1, 1]$. A map $p: E\rightarrow B$ is a *strongly regular mapping* ([**1**], [**5**]) if it is a proper map and for each $b_0\in B$ and $\varepsilon>0$ there is a neighborhood U of b_0 in B such that if $b\in U$, then there exist maps $g: p^{-1}(b)\rightarrow p^{-1}(b_0)$ and $h: p^{-1}(b_0)\rightarrow p^{-1}(b)$

such that g and h move points no more than ε and gh, hg are homotopic to the identity maps on $p^{-1}(b_0)$, $p^{-1}(b)$ via homotopies which move points no more than ε, respectively.

For a subset A of a space X, A is *unstable* in X if there is a homotopy $H: X\times I\to X$ such that $H(x, 0)=x$, $H(x, t)\in X-A$, for $x\in X$, $0<t\leqq 1$. Let $p: E\to B$, $p': E'\to B$ be maps between compacta and let E and E' be subsets of compacta X and X', respectively. A map $f: X-E\to X'-E'$ is an *$F(p, p')$-map* [7] if for each $b\in B$ and each neighborhood W' of $p'^{-1}(b)$ in X' there is a neighborhood W of $p^{-1}(b)$ in X such that $f(W-E)\subset W'-E'$. $F(p, p')$-maps $f, g: X-E\to X'-E'$ are *$F(p, p')$-homotopic* ($f\underset{F(p,p')}{\simeq}g$) if there is a homotopy $H: (X-E)\times I\to X'-E'$ such that $H(x, 0)=f(x)$, $H(x, 1)=g(x)$ for $x\in X-E$ and each $b\in B$ and each neighborhood W' of $p'^{-1}(b)$ in X' there is a neighborhood W of $p^{-1}(b)$ in X such that $H((W-E)\times I)\subset W'-E'$. Such a homotopy H is called an *$F(p, p')$-homotopy*. An $F(p, p')$-map $f: X-E\to X'-E'$ is an *$F(p, p')$-homotopy equivalence* if there is an $F(p', p)$-map $g: X'-E'\to X-E$ such that $gf\underset{F(p,p)}{\simeq}1_{X-E}$ and $fg\underset{F(p',p')}{\simeq}1_{X'-E'}$, where 1_{X-E} and $1_{X'-E'}$ denote the identity maps of $X-E$ and $X'-E'$, respectively.

LEMMA ([7, Lemma 2.1]). *Let X and X' be compact* ARs *containing E as an unstable closed subset. Then there is a map $\varphi(X, X'): X\to X'$ such that*

$$(*)\qquad \varphi(X, X')|E=1_E \quad \textit{and} \quad \varphi(X, X')(X-E)\subset X'-E\,.$$

If $\varphi_1, \varphi_2: X\to X'$ are maps satisfying condition $(*)$, then there is a homotopy $H: X\times I\to X'$ such that $H(x, 0)=\varphi_1(x)$, $H(x, 1)=\varphi_2(x)$ for $x\in X$ and $H(x, t)=x$ for $x\in E$, $t\in I$ and $H((X-E)\times I)\subset X'-E$. In particular, for any map $p: E\to B$ $\varphi(X, X')|X-E: X-E\to X'-E$ is an $F(p, p)$-map and $H|(X-E)\times I: (X-E)\times I\to X'-E$ is an $F(p, p)$-homotopy.

For any compactum B, we shall define the category FR_B as follows. By $m(E)$, we mean the set of compact ARs containing E as an unstable subset. Let $X_1, X_2\in m(E)$ and $X_1', X_2'\in m(E')$. An $F(p, p')$-map $f: X_1-E\to X_1'-E'$ is *$F(p, p')$-equivalent* to an $F(p, p')$-map $g: X_2-E\to X_2'-E'$ if the following diagram is commutative up to $F(p, p')$-homotopy,

$$\begin{tikzcd}
X_1-E \arrow[r, "f"] \arrow[d, hook] & X_1'-E' \arrow[d, hook] \\
X_1 \arrow[d, "{\varphi(X_1, X_2)}"'] & X_1' \arrow[d, "{\varphi(X_1', X_2')}"] \\
X_2 & X_2' \\
X_2-E \arrow[r, "g"'] \arrow[u, hook] & X_2'-E' \arrow[u, hook]
\end{tikzcd}$$

where $\varphi(X_1, X_2)$, $\varphi(X_1', X_2')$ are maps satisfying condition (*) of the lemma. Objects of FR_B are all maps of compacta to B, and for maps $p: E\to B$ and $p': E'\to B$, morphisms from p to p' in FR_B are $F(p, p')$-equivalence classes of collections of $F(p, p')$-maps $f: X-E\to X'-E'$, $X\in m(E)$, $X'\in m(E')$. Clearly FR_B forms a category (see [7]).

3. Strongly regular mappings with ANR fibers in FR_B.

Let X_1, X_2 be disjoint subsets of a space and $f_1: X_1\to X_3$, $f_2: X_2\to X_3$ be functions. We define a function $f_1\cup f_2: X_1\cup X_2\to X_3$ by

$$f_1\cup f_2(x)=\begin{cases} f_1(x), & x\in X_1, \\ f_2(x), & x\in X_2. \end{cases}$$

Let $p: E\to B$ and $q: Y\to B$ be maps between compacta. By $[Y, E]_{q,p}$ we mean the set of fiber homotopy classes of fiber maps from q to p, and $\langle Y, E\rangle_{q,p}$ the set of morphisms from q to p in FR_B. We shall define a natural transformation $\Phi: [Y, E]_{q,p}\to\langle Y, E\rangle_{q,p}$ as follows. Let $f: Y\to E$ be a fiber map from q to p and let $M\in m(Y)$, $N\in m(E)$. Since $N\in m(E)$, there is a homotopy $H: N\times I\to N$ such that $H(x, 0)=x$, $H(x, t)\in N-E$ for $x\in N$, $0<t\leqq 1$. Choose an extension $\tilde{f}': M\to N$ of f and a map $\alpha: M\to I$ such that $\alpha^{-1}(0)=Y$. Define a map $\tilde{f}: M\to N$ by $\tilde{f}(z)=H(\tilde{f}'(z), \alpha(z))$ for $z\in M$. Then $\tilde{f}$ is an extension of f and $\tilde{f}(M-Y)\subset N-E$. Note that $\tilde{f}|M-Y: M-Y\to N-E$ is an $F(q, p)$-map. Similarly we see that if $f, g: Y\to E$ are fiber maps from q to p and f and g are fiber homotopic, then $\tilde{f}|M-Y \underset{F(q,p)}{\simeq} \tilde{g}|M-Y$, where $\tilde{f}, \tilde{g}: M\to N$ are extensions of f, g respectively such that $\tilde{f}(M-Y)\subset N-E$, $\tilde{g}(M-Y)\subset N-E$. Hence we obtain the natural transformation $\Phi: [Y, E]_{q,p}\to\langle Y, E\rangle_{q,p}$ such that for a fiber homotopy class $[f]$ of a fiber map $f: Y\to E$ from q to p, $\Phi([f])$ is the morphism from q to p induced by an $F(q, p)$-map $\tilde{f}|M-Y: M-Y\to N-E$, where $M\in m(Y)$, $N\in m(E)$ and $\tilde{f}: M\to N$ is an extension of f such that $\tilde{f}(M-Y)\subset N-E$.

Suppose that $p: E\to B$ is a strongly regular mapping with ANR fibers and $\dim B<\infty$. Embed Y into the Hilbert cube Q and consider Y as the closed subset $Y\times\{1\}$ of $Q\times I$. Then $Q\times I\in m(Y)$. Also, embed E into Q as a Z-set ($Q\in m(E)$). Then we have the following lemma.

LEMMA 3.1. *Let $f: Q\times I-Y\to Q-E$ be an $F(q, p)$-map and $f_A: A\to E$ be a map, where A is a closed subset of Y. If $f\cup f_A: (Q\times I-Y)\cup A\to Q$ is continuous, then there is a fiber map $f_Y: Y\to E$ from q to p such that $f_Y|A=f_A$ and $\tilde{f}|Q\times I-Y \underset{F(q,p)}{\simeq} f$, where $\tilde{f}: Q\times I\to Q$ is an extension of f_Y such that $\tilde{f}(Q\times I-Y)\subset Q-E$.*

PROOF. First, note that if $f, g: Q\times I-Y\to Q-E$ are $F(q, p)$-maps such that $f|Y\times[0, 1) \underset{F(q,p)}{\simeq} g|Y\times[0, 1)$ then $f \underset{F(q,p)}{\simeq} g$.

Since the fiber $p^{-1}(b_0)$, $b_0 \in B$ is an ANR and Q is a convenient AR, there is a compact ANR neighborhood M_{b_0} of $p^{-1}(b_0)$ in Q which retracts to $p^{-1}(b_0)$. Choose a neighborhood W_{b_0} of b_0 in B such that $p^{-1}(W_{b_0}) \subset M_{b_0}$. Let $R(M_{b_0}, p^{-1}(W_{b_0}))$ be the space of retractions from M_{b_0} onto some $p^{-1}(b)$, $b \in W_{b_0}$, which has the metric

$$d(r_1, r_2) = \sup\{d(r_1(x), r_2(x)) \mid x \in M_{b_0}\}, \quad r_1, r_2 \in R(M_{b_0}, p^{-1}(W_{b_0})).$$

By the proof of Ferry [**5**, Proposition 3.1], there is a closed neighborhood U_{b_0} of b_0 in W_{b_0} and a map $\varphi_{b_0}: U_{b_0} \to R(M_{b_0}, p^{-1}(W_{b_0}))$ such that $h \circ \varphi_{b_0} = 1$, where $h: R(M_{b_0}, p^{-1}(W_{b_0})) \to W_{b_0}$ is the map such that $h(r) = b$, where r retracts M_{b_0} onto $p^{-1}(b)$, $b \in W_{b_0}$. Since B is compact, there is a finite closed cover $\{U_{b_1}, U_{b_2}, \cdots, U_{b_m}\}$ of B satisfying the conditions as before. Set $Y_i = q^{-1}(U_{b_i})$ for each $i = 1, 2, \cdots, m$. Since f is an $F(q, p)$-map, there is a positive number $\varepsilon_1 < 1$ such that $f(Y_1 \times [\varepsilon_1, 1)) \subset M_{b_1} - E$. Choose a map $\alpha_1: Y \to [\varepsilon_1, 1]$ such that $\alpha^{-1}(1) = A$. Define a map $f_{A \cup Y_1}: A \cup Y_1 \to E$ by

$$(1) \qquad f_{A \cup Y_1}(y) = \begin{cases} f_A(y), & y \in A, \\ \varphi_{b_1}(q(y))(f \cup f_A(y, \alpha(y))), & y \in Y_1. \end{cases}$$

Then $f_{A \cup Y_1}$ is well-defined, because for $y \in A \cap Y_1$, $\varphi_{b_1}(q(y))(f \cup f_A(y, \alpha(y))) = \varphi_{b_1}(q(y))(f \cup f_A(y, 1)) = \varphi_{b_1}(q(y))(f_A(y)) = f_A(y)$. Also, $p \circ f_{A \cup Y_1} = q \mid A \cup Y_1$. Choose a map $\beta: Y \times I \times I \to I$ such that $\beta^{-1}(0) = Y \times \{1\} \times I$. Since E is a Z-set in Q, there is a homotopy $K: Q \times I \to Q$ such that $K(x, 0) = x$, $K(x, t) \in Q - E$ for $x \in Q$, $0 < t \leqq 1$. Define a homotopy $H_1: (Y_1 \times [0, 1) \cup (A \cap Y_1)) \times I \to Q$ by

$$(2) \qquad H_1(y, t, s) = K(\varphi_{b_1}(q(y))(f \cup f_A(y, (1-s)\cdot\alpha(y) + s\cdot t)), \beta(y, t, s)),$$
$$\text{for } (y, t, s) \in (Y_1 \times [0, 1) \cup (A \cap Y_1)) \times I.$$

Then

$$H_1(y, t, 0) = K(\varphi_{b_1}(q(y))(f \cup f_A(y, \alpha(y))), \beta(y, t, 0)),$$
$$H_1(y, t, 1) = K(\varphi_{b_1}(q(y))(f \cup f_A(y, t)), \beta(y, t, 1)),$$

for $(y, t) \in Y_1 \times [0, 1) \cup (A \cap Y_1)$ and $H_1(y, 1, s) = f_A(y)$, for $(y, 1) \in A \cap Y_1$, $s \in I$. Note that $H_1 \mid Y_1 \times [0, 1) \times \{0\} \cup (f_{A \cup Y_1} \mid Y_1)$ is continuous. Choose a map $\eta: Y \times I \times I \to I$ such that $\eta^{-1}(0) = Y \times I \times (\{0\} \cup \{1\}) \cup Y \times \{1\} \times I$. Define a homotopy $G_1: (Y_1 \times [0, 1) \cup (A \cap Y_1)) \times I \to Q$ by

$$(3) \qquad G_1(y, t, s) = K((1-s)\cdot(K(\varphi_{b_1}(q(y))(f \cup f_A(y, t)), \beta(y, t, 1))$$
$$+ s(f \cup f_A(y, t)), \eta(y, t, s)),$$
$$\text{for } (y, t, s) \in (Y_1 \times [0, 1) \cup (A \cup Y_1)) \times I.$$

Then $G_1(y, t, 0) = H_1(y, t, 1)$, $G_1(y, t, 1) = f \cup f_A(y, t)$, for $(y, t) \in Y_1 \times [0, 1) \cup (A \cap Y_1)$

and $G_1(y, 1, s)=f_A(y)$, for $(y, 1)\in A\cap Y_1$, $s\in I$. It is easy to check that $H_1|Y_1\times[0, 1)\times I: Y_1\times[0, 1)\times I\to Q-E$ and $G_1|Y_1\times[0, 1)\times I: Y_1\times[0, 1)\times I\to Q-E$ are $F(q|Y_1, p)$-homotopies, respectively. By [**7**, Lemma 3.4], we obtain an $F(q, p)$-map $f_1: Y\times[0, 1)\to Q-E$ such that $f_1 \underset{F(q, p)}{\simeq} f|Y\times[0, 1)$ and $f_1\cup f_{A\cup Y_1}: Y\times[0, 1)\cup(A\cup Y_1)\to Q$ is continuous.

If we replace A by $A\cup Y_1$, then we obtain a map $f_{A\cup Y_1\cup Y_2}: A\cup Y_1\cup Y_2\to E$ which is an extension of $f_{A\cup Y_1}$, and an $F(q, p)$-map $f_2: Y\times[0, 1)\to Q-E$ such that $f_2 \underset{F(q, p)}{\simeq} f_1$ and $f_2\cup f_{A\cup Y_1\cup Y_2}: Y\times[0, 1)\cup(A\cup Y_1\cup Y_2)\to Q$ is continuous. If we continue this process, we obtain a map $f_Y: Y\to E$, which is an extension of f_A, and an $F(q, p)$-map $f_m: Y\times[0, 1)\to Q-E$ such that $f_m \underset{F(q, p)}{\simeq} f|Y\times[0, 1)$ and $f_m\cup f_Y: Y\times I\to Q$ is continuous. Note that $f_Y: Y\to E$ is a fiber map over B. Clearly, f_Y satisfies our requirements. This completes the proof.

THEOREM 3.2. *Let E, B be compacta and* $\dim B<\infty$. *If $p: E\to B$ is a strongly regular mapping with* ANR *fibers, then for any map $q: Y\to B$ of compacta $\Phi: [Y, E]_{q,p}\to\langle Y, E\rangle_{q,p}$ is a bijection.*

PROOF. If we apply Lemma 3.1 with A replaced by the empty set, we conclude that Φ is surjective. Also, if we apply Lemma 3.1 with Y replaced by $Y\times I$, A replaced by $Y\times\{0, 1\}$ and $q: Y\to B$ replaced by the composition $q\circ\mathrm{proj}: Y\times I\to Y\to B$, we conclude that Φ is injective.

By using Theorem 3.2, we can easily prove the following.

THEOREM 3.3. *Let E, E' and B be compacta and* $\dim B<\infty$. *Suppose that $p: E\to B$ and $p': E'\to B$ are strongly regular mappings with* ANR *fibers. Then p is fiber homotopy equivalent to p' iff p is isomorphic to p' in FR_B. Moreover, if a fiber map $f: E\to E'$ from p to p' induces an isomorphism in FR_B, then it is a fiber homotopy equivalence.*

REMARK 3.4. In the statements of Theorems 3.2 and 3.3, we can not omit the condition "strongly regular mapping". Define a map $p: E=[0, 3]\to B=[0, 2]$ by $p|[0, 1]=1_{[0,1]}$, $p([1, 2])=\{1\}$ and $p(t)=t-1$ for $t\in[2, 3]$. Clearly, the map $p: E\to B$ induces an isomorphism from p to the identity map 1_B of B in FR_B, but there is no fiber map from 1_B to p. Also, it is easily seen that we cannot omit the condition "ANR fibers".

THEOREM 3.5. *Let E, B be compacta and* $\dim B<\infty$. *If $p: E\to B$ is a strongly regular mapping with* ANR *fibers, then p is a shape fibration* (see [**9**], [**10**] for the definition of shape fibration).

PROOF. Consider the composition $p\circ\mathrm{proj}: E\times Q\to E\to B$. Then, by [**2**], $p\circ\mathrm{proj}$ is a locally trivial fiber space with compact Q-manifold fibers. By [**4**], the homeomorphism group of a compact Q-manifold is an ANR. By Scharlemann [**12**, Theorem 2.1], we see that there are compact ANRs M, N and a locally trivial fiber space $\tilde{p}: M\to N$ such that $M\supset E\times Q$, $N\supset B$ and $\tilde{p}$ is an extension of $p\circ\mathrm{proj}$ with $\tilde{p}^{-1}(B)=E\times Q$. Since $\tilde{p}$ is a shape fibration, the restriction

$p \circ \mathrm{proj}$ is also. Since p is fiber homotopy equivalent to $p \circ \mathrm{proj}$, by [**6**] p is a shape fibration.

REMARK 3.6. In the statement of Theorem 3.5, we cannot omit the assumption about the fibers of p. In fact, there is a strongly regular mapping which is a locally trivial fiber space and not a shape fibration. Let E be the continuum which consists of all points in the plane having the polar coordinates (r, θ) for which $r=1$, $r=2$ or $r=(2+e^{\theta})/(1+e^{\theta})$ and B be the unit circle in the plane. Define a map $p: E \to B$ by $p(r, \theta)=(1, \theta)$. Clearly, p is a strongly regular mapping (locally trivial fiber space), but it is not a shape fibration (see [**11**, p. 641]).

THEOREM 3.7. *Let E, E' and B be compacta and* $\dim B<\infty$. *Suppose that $p: E \to B$ and $p': E' \to B$ are strongly regular mappings with* ANR *fibers. If a fiber map $f: E \to E'$ from p to p' induces a strong shape equivalence, then it is a fiber homotopy equivalence.*

PROOF. By Theorem 3.5, p and p' are shape fibrations, respectively. By [**7**, Theorem 4.1], f induces an isomorphism in FR_B. Theorem 3.3 implies that f is a fiber homotopy equivalence.

COROLLARY 3.8. *Let E, B be compacta and* $\dim B<\infty$. *If $p: E \to B$ is a strongly regular mapping with* AR *fibers, then p is shrinkable, i.e., p is a fiber homotopy equivalence from p to 1_B.*

PROOF. Since p is a cell-like map and $\dim B<\infty$, by [8], p is a hereditary shape equivalence. In particular, it is a strong shape equivalence. By Theorem 3.7, p is shrinkable.

REMARK 3.9. In the statement of Theorem 3.7, the assumption about the fibers of p cannot be omitted. In the plane R^2, put $a_0=(0, 0)$, $b_0=(1, 0)$, $a_n=(0, -1/n)$, $b_n=(1, 1/n)$, $n=1, 2, 3, \cdots$. Let $[p, q]$ be the line segment joining p and q in R^2, $p, q \in R^2$. Set $E=\bigcup_{n=0}^{\infty}[a_0, b_n] \cup \bigcup_{n=0}^{\infty}[a_n, b_0]$ and $B=[a_0, b_0]$. Define a map $p: E \to B$ by $p(x, y)=(x, 0)$, for $(x, y) \in E$. Then p is a strongly regular mapping. Also, define a map $f: E \to E$ by

$$f(x, y)=\begin{cases} (x, 0), & (x, y) \in \bigcup_{n=0}^{\infty}[a_0, b_n], \\ (x, y), & (x, y) \in \bigcup_{n=0}^{\infty}[a_n, b_0]. \end{cases}$$

Then $pf=p$ and f induces a strong shape equivalence, but f is not a fiber homotopy equivalence. In fact, f does not induce an isomorphism in FR_B.

REMARK 3.10. In the statements of Theorem 3.5, Theorem 3.7 and Corollary 3.8, we cannot omit the condition "$\dim B<\infty$". By using Taylor's example and the result of G. Kozlowski, J. V. Mill and J. Walsh [AR-maps obtained from cell-like maps, Proc. Amer. Math. Soc., **82** (1981), 299-302], we obtain a strongly regular mapping $f: X \to Q$ with AR fibers which is not shape shrinkable, where

Q is the Hilbert cube. By taking the cones of X and Q, we have the map $C(f)$: $C(X) \to C(Q) \cong Q$. Then $C(f)$ is a strong shape equivalence and a strongly regular mapping with AR-fibers, but it is not shape shrinkable. By [7, Corollary 4.4], $C(f)$ is not a shape fibration. Clearly, $C(f)$ is not shrinkable. Hence we cannot omit the condition "$\dim B < \infty$".

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