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An optional unrelated question RRT model

Jeong S. Sihm, Anu Chhabra and Sat N. Gupta

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We propose a modified unrelated question randomized response technique (RRT) model which allows respondents the option of answering a sensitive question directly without using the randomization device if they find the question non-sensitive. This situation has been handled before by Gupta, Tuck, Spears Gill, and Crowe using the split sample approach. In this work we avoid the split sample approach, which requires larger total sample size. Instead, we estimate the prevalence of the sensitive characteristic by using an optional unrelated question RRT model, but the corresponding sensitivity level is estimated from the same sample by using the traditional binary unrelated question RRT model of Greenberg, Abul-Ela, Simmons, and Horvitz. We compare the simulation results of this new model with those of the split-sample based optional unrelated question RRT model of Gupta et al. and the simple unrelated question RRT model of Greenberg et al. Computer simulations show that the new binary response and quantitative response models have the smallest variance among the three models when they have the same sample size.

1. Introduction

Social Desirability Bias (SDB) is the tendency in survey respondents to reply to a sensitive question in a socially desirable manner as opposed to replying truthfully. In order to encourage truthful answers, several techniques have been developed for sensitive survey questions. These include the indirect response technique of [Warner 1965] and the unrelated question technique of [Greenberg et al. 1969], both belonging to the family of randomized response techniques (RRT) models.

Several variations of the original RRT models, both binary response and quantitative response models, have been discussed by researchers, including Mangat and Singh [1990], Gupta [2001], Gupta et al. [2002; 2010; 2013a; 2013b], Christofides [2003], Mehta et al. [2012], and Sihm et al. [2015]. Among the many RRT procedures, we will focus here on the binary response unrelated question RRT model

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of [Greenberg et al. 1969], as well as the quantitative response unrelated question RRT model of [Greenberg et al. 1971].

Gupta et al. [2013b] demonstrated that an optional unrelated question RRT model can lead to improvement in estimating the prevalence of the sensitive characteristic. They used the split sample approach because they estimated both the prevalence of the sensitive characteristic and sensitivity level of the question from the same set of responses. We will introduce in this paper new binary and quantitative optional unrelated question RRT models without using the split sample approach and estimate the prevalence of the sensitive characteristic and the sensitivity level of the question from two different sets of responses from the same sample. We will demonstrate by an extensive simulation study that the proposed models work better than the optional unrelated question RRT models of [Gupta et al. 2013b] and the traditional unrelated question RRT models of [Greenberg et al. 1969; 1971] for a fixed sample size.

2. Unrelated question RRT models

2.1. *Unrelated question models.*

2.1.1. *Binary response model.* This model was first introduced in [Greenberg et al. 1969]. In this model, you use a randomization device to ask a respondent the sensitive binary question with preassigned probability p_a and an innocuous question (whose prevalence is already known) with probability of $1 - p_a$.

Let π_a be the known prevalence of an unrelated characteristic and π be the unknown prevalence of the sensitive characteristic. Let P_y be the probability of a "yes" response from a respondent. Then P_y can be expressed as

$$P_y = p_a\pi + (1 - p_a)\pi_a. \tag{1}$$

Solving for π, we have

$$\pi = \frac{P_y - (1 - p_a)\pi_a}{p_a}.$$

This leads to the estimator of [Greenberg et al. 1969],

$$\hat{\pi}_g = \frac{\widehat{P}_y - (1 - p_a)\pi_a}{p_a}, \tag{2}$$

where $\widehat{P}_y$ is the proportion of "yes" responses in the survey. It is known that $\hat{\pi}_g$ is an unbiased estimator with its variance given by

$$\mathrm{Var}(\hat{\pi}_g) = \frac{P_y(1 - P_y)}{np_a^2}. \tag{3}$$

2.1.2. *Quantitative response model.* Very much like the binary response model, in this model the researcher will also ask a sensitive question with preassigned probability p_a and an innocuous question with probability $1 - p_a$.

Let μ_y and σ_y^2 respectively be the known mean and variance of an unrelated question and μ_x and σ_x^2 respectively be the unknown mean and variance of the sensitive question in the population. Let Z be the reported response from a respondent. Then Z can be expressed as

$$Z = \begin{cases} X & \text{with probability } p_a \text{ (sensitive question)}, \\ Y & \text{with probability } 1 - p_a \text{ (nonsensitive question)}, \end{cases}$$

with

$$E(Z) = \mu_z = p_a \mu_x + (1 - p_a)\mu_y, \tag{4}$$

$$\begin{aligned} \operatorname{Var}(Z) &= p_a E(X^2) + (1 - p_a)E(Y^2) - \mu_z^2 \\ &= p_a(\sigma_x^2 + \mu_x^2) + (1 - p_a)(\sigma_y^2 + \mu_y^2) - \mu_z^2. \end{aligned} \tag{5}$$

Solving (4) for μ_x, we have

$$\mu_x = \frac{\mu_z - (1 - p_a)\mu_y}{p_a}.$$

This leads to the estimator of [Greenberg et al. 1971],

$$\hat{\mu}_g = \frac{\bar{Z} - (1 - p_a)\mu_y}{p_a}, \tag{6}$$

where $\bar{Z}$ is the sample mean of the quantitative responses in the survey.

It is known that $\hat{\mu}_g$ is an unbiased estimator with its variance given by

$$\operatorname{Var}(\hat{\mu}_g) = \frac{1}{np_a^2} \operatorname{Var}(Z) = \frac{1}{np_a^2}\big(\sigma_y^2 + p_a(\sigma_x^2 - \sigma_y^2) + p_a(1 - p_a)(\mu_x - \mu_y)^2\big). \tag{7}$$

2.2. ***Optional unrelated question models.*** These models were proposed in [Gupta et al. 2013b] as a generalization of the original unrelated question models of [Greenberg et al. 1969; 1971] by giving respondents the option of responding to the sensitive question directly if they consider the question nonsensitive, while they can still give a scrambled response by using the model of [Greenberg et al. 1969] for a binary response and by using the model of [Greenberg et al. 1971] for a quantitative response if they feel the question is sensitive.

2.2.1. *Binary response model.* Let π_a be the known prevalence of an unrelated characteristic, π be the unknown prevalence of the sensitive characteristic, p be the preassigned probability of the respondent selecting the sensitive question, and ω be the unknown sensitivity level of the survey question in the population. Sensitivity level means the proportion of respondents in the population who would consider the question sensitive and subsequently opt to use a randomization device.

The probability of a "yes" response (P_y) in this model can be expressed as

$$P_y = (1-\omega)\pi + \omega(\pi p + (1-p)\pi_a). \tag{8}$$

Using two independent samples with sample sizes n_1 and n_2 respectively, and assuming that p_1 and p_2 are two different preassigned probabilities of the respondents selecting the sensitive question in the two samples, (8) can be written as

$$P_{y_1} - \pi = \omega(1-p_1)(\pi_a - \pi) \quad \text{and} \quad P_{y_2} - \pi = \omega(1-p_2)(\pi_a - \pi). \tag{9}$$

Solving for π, we have

$$\pi = \frac{\lambda P_{y_2} - P_{y_1}}{\lambda - 1} \quad (p_1, p_2 \neq 1,\ \ p_1 \neq p_2,\ \ \pi_a \neq \pi), \quad \text{where } \lambda = \frac{p_1 - 1}{p_2 - 1}. \tag{10}$$

Equation (10) leads to the unbiased estimator for π of [Gupta et al. 2013b],

$$\hat{\pi}_{gu} = \frac{\lambda \widehat{P}_{y_2} - \widehat{P}_{y_1}}{\lambda - 1}, \tag{11}$$

with its variance given by

$$\operatorname{Var}(\hat{\pi}_{gu}) = \frac{1}{(\lambda-1)^2}\left(\lambda^2 \frac{P_{y_2}(1-P_{y_2})}{n_2} + \frac{P_{y_1}(1-P_{y_1})}{n_1}\right). \tag{12}$$

Similarly from (9), we have

$$\omega = \frac{P_{y_1} - P_{y_2}}{(p_2 - p_1)\pi_a + (1-p_2)P_{y_1} - (1-p_1)P_{y_2}} \quad (p_1 \neq p_2,\ \pi_a \neq \pi), \tag{13}$$

which leads to an estimator of ω given by

$$\hat{\omega}_{gu^*} = \frac{\widehat{P}_{y_1} - \widehat{P}_{y_2}}{(p_2 - p_1)\pi_a + (1-p_2)\widehat{P}_{y_1} - (1-p_1)\widehat{P}_{y_2}}. \tag{14}$$

Gupta et al. [2013b] show that, up to first-order Taylor approximation, $\hat{\omega}_{gu^*}$ is an unbiased estimator for ω with its variance given by

$$\operatorname{Var}(\hat{\omega}_{gu^*}) = \frac{(p_2-p_1)^2\left((\pi_a-P_{y_2})^2 \frac{P_{y_1}(1-P_{y_1})}{n_1} + (\pi_a-P_{y_1})^2 \frac{P_{y_2}(1-P_{y_2})}{n_2}\right)}{\left((p_2-p_1)\pi_a + (1-p_2)P_{y_1} - (1-p_1)P_{y_2}\right)^4}. \tag{15}$$

2.2.2. *Quantitative response model.* Let μ_y and σ_y^2 respectively be the known mean and variance of an innocuous question, μ_x and σ_x^2 respectively be the unknown mean and variance of the sensitive question in the population, p_1 and p_2 be the preassigned probabilities of answering the sensitive survey question in two subsamples, and ω be the unknown sensitivity level of the survey question in the population. Let Z_i be the reported response from a respondent in the i-th subsample ($i = 1, 2$). Then Z_i can be expressed as

$$Z_i = \begin{cases} X_i & \text{with probability } (1-\omega)+\omega p_i \text{ (sensitive question)}, \\ Y_i & \text{with probability } \omega(1-p_i) \text{ (nonsensitive question)}, \end{cases}$$

with

$$E(Z_i) = \mu_{z_i} = ((1-\omega)+\omega p_i)\mu_x + \omega(1-p_i)\mu_y \quad (i = 1, 2), \tag{16}$$

$$\begin{aligned} \operatorname{Var}(Z_i) &= ((1-\omega)+\omega p_i)E(X_i^2) + \omega(1-p_i)E(Y_i^2) - \mu_{z_i}^2 \\ &= ((1-\omega)+\omega p_i)(\sigma_x^2+\mu_x^2) + \omega(1-p_i)(\sigma_y^2+\mu_y^2) - \mu_{z_i}^2. \end{aligned} \tag{17}$$

Equation (16) can be rearranged as

$$\mu_{z_i} - \mu_x = \omega(p_i - 1)(\mu_x - \mu_y) \quad (i = 1, 2). \tag{18}$$

Solving (18) for μ_x, we have

$$\mu_x = \frac{\mu_{z_1} - \lambda\mu_{z_2}}{1-\lambda} \quad (p_1, p_2 \neq 1,\ \ p_1 \neq p_2,\ \ \mu_x \neq \mu_y), \quad \text{where } \lambda = \frac{p_1 - 1}{p_2 - 1}. \tag{19}$$

This leads to the unbiased estimator for μ_x of [Gupta et al. 2013b],

$$\hat{\mu}_{gu} = \frac{\hat{\mu}_{z_1} - \lambda\hat{\mu}_{z_2}}{1-\lambda} = \frac{\bar{Z}_1 - \lambda\bar{Z}_2}{1-\lambda}, \tag{20}$$

with its variance given by

$$\operatorname{Var}(\hat{\mu}_{gu}) = \frac{1}{(1-\lambda)^2}\left(\frac{\operatorname{Var}(Z_1)}{n_1} + \lambda^2\frac{\operatorname{Var}(Z_2)}{n_2}\right). \tag{21}$$

Solving (18) for ω, we have

$$\omega = \frac{\mu_{z_1} - \mu_{z_2}}{(p_2 - p_1)\mu_y + (1-p_2)\mu_{z_1} - (1-p_1)\mu_{z_2}} \quad (p_1 \neq p_2,\ \mu_x \neq \mu_y), \tag{22}$$

which leads to

$$\hat{\omega}_{gu^{**}} = \frac{\bar{Z}_1 - \bar{Z}_2}{(p_2 - p_1)\pi_a + (1-p_2)\bar{Z}_1 - (1-p_1)\bar{Z}_2}. \tag{23}$$

Gupta et al. [2013b] show that, up to first-order Taylor approximation, $\hat{\omega}_{gu^{**}}$ is an unbiased estimator for ω with its variance given by

$$\operatorname{Var}(\hat{\omega}_{gu^{**}}) = \frac{(p_2-p_1)^2\left((\mu_y-\mu_{z_2})^2\frac{\operatorname{Var}(Z_1)}{n_1}+(\mu_{z_1}-\mu_y)^2\frac{\operatorname{Var}(Z_2)}{n_2}\right)}{\left((p_2-p_1)\pi_a+(1-p_2)\mu_{z_1}-(1-p_1)\mu_{z_2}\right)^4}. \tag{24}$$

3. The proposed model

The main motivation for this model is to avoid the split sample approach which requires unnecessarily larger total sample size. We do this by asking respondents two questions. Question 1 is the main research question, which the respondent answers using the optional model of [Gupta et al. 2013b]. Question 2 is the auxiliary question about whether or not the underlying research question is sensitive enough for the respondent to opt for a scrambled response. Respondents will answer Question 2 by using the original model of [Greenberg et al. 1969].

3.1. ***Binary response model.*** Let π_a be the known prevalence of an unrelated innocuous characteristic, π_b be the known prevalence of another unrelated innocuous characteristic, π be the unknown prevalence of the sensitive characteristic, p_a be the preassigned probability of the respondent selecting the sensitive question in answering Question 1, p_b be the preassigned probability of the respondent selecting the question about sensitivity in answering Question 2, and ω be the unknown sensitivity level of the survey question in the population.

Let P_{y_i} be the probability of a "yes" response from a respondent to Question i $(i=1,2)$. We have

$$P_{y_1} = (1-\omega)\pi+\omega(\pi p_a+(1-p_a)\pi_a), \tag{25}$$

$$P_{y_2} = p_b\omega+(1-p_b)\pi_b. \tag{26}$$

Solving (25) and (26) for π and ω respectively, we have

$$\pi = \frac{P_{y_1}-(1-p_a)\omega\pi_a}{1-(1-p_a)\omega} \quad \text{and} \quad \omega = \frac{P_{y_2}-(1-p_b)\pi_b}{p_b}, \tag{27}$$

which lead to the estimators

$$\hat{\pi}_p = \frac{\widehat{P}_{y_1}-(1-p_a)\hat{\omega}_{p^*}\pi_a}{1-(1-p_a)\hat{\omega}_{p^*}} \quad \text{and} \quad \hat{\omega}_{p^*} = \frac{\widehat{P}_{y_2}-(1-p_b)\pi_b}{p_b}, \tag{28}$$

where $\widehat{P}_{y_i}$ is the proportion of "yes" responses in the sample to Question i $(i=1,2)$.

Notice that $\hat{\omega}_{p^*}$ is an unbiased estimator with its variance given by

$$\operatorname{Var}(\hat{\omega}_{p^*}) = \frac{P_{y_2}(1-P_{y_2})}{np_b^2}. \tag{29}$$

After applying first-order Taylor expansion to $\hat{\pi}_p$, we have

$$\hat{\pi}_p \approx \hat{\pi}(P_{y_1}, \omega) + \left.\frac{\partial \hat{\pi}(\widehat{P}_{y_1}, \hat{\omega}_{p^*})}{\partial \widehat{P}_{y_1}}\right|_{P_{y_1},\omega} (\widehat{P}_{y_1} - P_{y_1}) + \left.\frac{\partial \hat{\pi}(\widehat{P}_{y_1}, \hat{\omega}_{p^*})}{\partial \hat{\omega}_{p^*}}\right|_{P_{y_1},\omega} (\hat{\omega}_{p^*} - \omega) \tag{30}$$

$$= \frac{P_{y_1} - \omega(1-p_a)\pi_a}{1-(1-p_a)\omega} + \frac{\widehat{P}_{y_1} - P_{y_1}}{1-(1-p_a)\omega} + \frac{(1-p_a)(P_{y_1}-\pi_a)(\hat{\omega}_{p^*}-\omega)}{(1-(1-p_a)\omega)^2}. \tag{31}$$

Up to first-order Taylor approximation, $\hat{\pi}_p$ is an unbiased estimator for π with its variance given by

$$\mathrm{Var}(\hat{\pi}_p) = \frac{1}{(1-(1-p_a)\omega)^2} \cdot \frac{P_{y_1}(1-P_{y_1})}{n} + \frac{(1-p_a)^2(P_{y_1}-\pi_a)^2}{(1-(1-p_a)\omega)^4} \cdot \frac{P_{y_2}(1-P_{y_2})}{np_b^2}. \tag{32}$$

3.2. ***Quantitative response model.*** Let μ_y and σ_y^2 respectively be the known mean and variance of an innocuous question, μ_x and σ_x^2 respectively be the unknown mean and variance of the sensitive question in the population, p_a be the preassigned probability of the respondent selecting the sensitive question in answering Question 1, and ω be the unknown sensitivity level of the survey question in the population. Let Z be the reported response to Question 1 from a respondent. Then Z can be expressed as

$$Z = \begin{cases} X & \text{with probability } (1-\omega)+\omega p_a \text{ (sensitive question)}, \\ Y & \text{with probability } \omega(1-p_a) \text{ (nonsensitive question)}, \end{cases}$$

with

$$E(Z) = \mu_z = ((1-\omega)+\omega p_a)\mu_x + \omega(1-p_a)\mu_y, \tag{33}$$

$$\begin{aligned} \mathrm{Var}(Z) &= ((1-\omega)+\omega p_a)E(X^2) + \omega(1-p_a)E(Y^2) - \mu_z^2 \\ &= ((1-\omega)+\omega p_a)(\sigma_x^2+\mu_x^2) + \omega(1-p_a)(\sigma_y^2+\mu_y^2) - \mu_z^2. \end{aligned} \tag{34}$$

Let π_b be the known prevalence of a binary innocuous characteristic for Question 2 and p_b be the preassigned probability of the respondent selecting the question about sensitivity in answering Question 2. We have the probability of a "yes" response to Question 2 given by

$$P_y = p_b\omega + (1-p_b)\pi_b. \tag{35}$$

Solving (33) and (35) for μ_x and ω respectively, we have

$$\mu_x = \frac{\mu_z - \mu_y(1-p_a)\omega}{1-(1-p_a)\omega} \quad \text{and} \quad \omega = \frac{P_y - (1-p_b)\pi_b}{p_b}, \tag{36}$$

which lead to the estimators

$$\hat{\mu}_p = \frac{\bar{Z} - \mu_y(1-p_a)\hat{\omega}_{p^{**}}}{1-(1-p_a)\hat{\omega}_{p^{**}}} \quad \text{and} \quad \hat{\omega}_{p^{**}} = \frac{\widehat{P}_y - (1-p_b)\pi_b}{p_b}, \tag{37}$$

where $\widehat{P}_y$ is the proportion of "yes" responses to Question 2 in the sample.

Notice that $\hat{\omega}_{p^{**}}$ is an unbiased estimator with the variance given by

$$\operatorname{Var}(\hat{\omega}_{p^{**}}) = \frac{P_y(1-P_y)}{np_b^2}. \tag{38}$$

After applying first-order Taylor expansion to $\hat{\mu}_p$, we have

$$\hat{\mu}_p \approx \hat{\mu}(\mu_z,\omega) + \frac{\partial \hat{\mu}(\hat{\mu}_z,\hat{\omega}_{p^{**}})}{\partial \hat{\mu}_z}\bigg|_{\mu_z,\omega}(\hat{\mu}_z-\mu_z) + \frac{\partial \hat{\mu}(\hat{\mu}_z,\hat{\omega}_{p^{**}})}{\partial \hat{\omega}_{p^{**}}}\bigg|_{\mu_z,\omega}(\hat{\omega}_{p^{**}}-\omega) \tag{39}$$

$$= \frac{\mu_z-\mu_y(1-p_a)\omega}{1-(1-p_a)\omega} + \frac{\hat{\mu}_z-\mu_z}{1-(1-p_a)\omega} + \frac{(1-p_a)(\mu_z-\mu_y)(\hat{\omega}_{p^{**}}-\omega)}{(1-(1-p_a)\omega)^2}$$

$$= \frac{\mu_z-\mu_y(1-p_a)\omega}{1-(1-p_a)\omega} + \frac{\bar{Z}-\mu_z}{1-(1-p_a)\omega} + \frac{(1-p_a)(\mu_z-\mu_y)(\hat{\omega}_{p^{**}}-\omega)}{(1-(1-p_a)\omega)^2}. \tag{40}$$

Up to first-order Taylor approximation, $\hat{\mu}_p$ is an unbiased estimator for μ_x with the variance given by

$$\operatorname{Var}(\hat{\mu}_p) = \frac{1}{(1-(1-p_a)\omega)^2}\cdot\frac{\operatorname{Var}(Z)}{n} + \frac{(1-p_a)^2(\mu_z-\mu_y)^2}{(1-(1-p_a)\omega)^4}\cdot\frac{P_y(1-P_y)}{np_b^2}. \tag{41}$$

4. Simulation results

In this section, simulation results are presented for our estimators, $\hat{\pi}_p$, $\hat{\omega}_{p^*}$, $\hat{\mu}_p$, and $\hat{\omega}_{p^{**}}$. We compare simulation results of the proposed models with the results from other models. Specifically, we compare $\hat{\pi}_p$ with $\hat{\pi}_g$ and $\hat{\pi}_{gu}$, and $\hat{\omega}_{p^*}$ with $\hat{\omega}_{gu^*}$ for binary response. Likewise, for the quantitative response models, we compared $\hat{\mu}_p$ with $\hat{\mu}_g$ and $\hat{\mu}_{gu}$, and $\hat{\omega}_{p^{**}}$ with $\hat{\omega}_{gu^{**}}$.

All the simulations were conducted by using the R programming language (http://www.R-project.org). For the binary response models, two parameters, π and ω, were allowed to vary while all the other variables were fixed. We used 10000 iterations with $p_a = 0.85$, $\pi_a = 0.7$, $p_b = 0.5$, $\pi_b = 0.1$, $p_1 = 0.85$, and $p_2 = 0.15$. For the quantitative response models, two parameters, μ_x and ω, were allowed to vary while all the other variables were fixed. Again we used 10000 iterations with $p_a = 0.85$, $\mu_y = 7.0$, $p_b = 0.6$, $\pi_b = 0.1$, $p_1 = 0.85$, and $p_2 = 0.15$. For the distributions of X and Y, we used the Poisson distributions with the parameters μ_x and μ_y respectively, as done in [Gupta et al. 2013b]. Also notice that all models have the same total sample size ($n = 1000$). For the optional unrelated question RRT models with split samples, the optimal sample ratios were used according to the formulas in [Gupta et al. 2013b].

4.1. ***Simulation of $\hat{\pi}$ and $\hat{\omega}$ for binary case.*** Table 1 shows that the theoretical $\operatorname{Var}(\hat{\pi}_p)$ values are always the smallest in comparison with the theoretical $\operatorname{Var}(\hat{\pi}_g)$

		$\pi = 1.0$	$\pi = 2.0$	$\pi = 3.0$	$\pi = 4.0$	$\pi = 5.0$
New model	EMean($\hat{\pi}_p$)	0.099923	0.199995	0.300015	0.400170	0.500034
	EVar($\hat{\pi}_p$)	0.000101	0.000169	0.000220	0.000243	0.000256
	TVar($\hat{\pi}_p$)	0.000103	0.000172	0.000220	0.000249	0.000258
	EMean($\hat{\omega}_{p*}$)	0.100110	0.099778	0.099747	0.100091	0.099960
	EVar($\hat{\omega}_{p*}$)	0.000360	0.000355	0.000366	0.000365	0.000355
	TVar($\hat{\omega}_{p*}$)	0.000360	0.000360	0.000360	0.000360	0.000360
Simple unrelated	EMean($\hat{\pi}_g$)	0.099875	0.200037	0.300155	0.400311	0.500007
	EVar($\hat{\pi}_g$)	0.000209	0.000273	0.000316	0.000336	0.000342
	TVar($\hat{\pi}_g$)	0.000213	0.000276	0.000319	0.000342	0.000345
Optional unrelated with optimal split	EMean($\hat{\pi}_{gu}$)	0.100290	0.199921	0.300257	0.399756	0.499761
	EVar($\hat{\pi}_{gu}$)	0.000203	0.000345	0.000445	0.000492	0.000495
	TVar($\hat{\pi}_{gu}$)	0.000207	0.000341	0.000436	0.000493	0.000510
	EMean($\hat{\omega}_{gu*}$)	0.097581	0.098096	0.093820	0.090546	0.073563
	EVar($\hat{\omega}_{gu*}$)	0.004716	0.010569	0.021178	0.041900	0.102535
	TVar($\hat{\omega}_{gu*}$)	0.004727	0.010600	0.020670	0.041002	0.094970
	Optimal n_1	831	843	847	849	850
	Optimal n_2	169	157	153	151	150
New model	EMean($\hat{\pi}_p$)	0.099817	0.200152	0.299968	0.399772	0.500247
	EVar($\hat{\pi}_p$)	0.000122	0.000188	0.000240	0.000264	0.000276
	TVar($\hat{\pi}_p$)	0.000127	0.000194	0.000240	0.000267	0.000275
	EMean($\hat{\omega}_{p*}$)	0.300150	0.299862	0.300151	0.300172	0.300387
	EVar($\hat{\omega}_{p*}$)	0.000659	0.000628	0.000623	0.000641	0.000641
	TVar($\hat{\omega}_{p*}$)	0.000640	0.000640	0.000640	0.000640	0.000640
Simple unrelated	EMean($\hat{\pi}_g$)	0.099657	0.200152	0.299961	0.399758	0.500268
	EVar($\hat{\pi}_g$)	0.000210	0.000274	0.000320	0.000338	0.000344
	TVar($\hat{\pi}_g$)	0.000213	0.000276	0.000319	0.000342	0.000345
Optional unrelated with optimal split	EMean($\hat{\pi}_{gu}$)	0.100291	0.199864	0.299693	0.400126	0.500136
	EVar($\hat{\pi}_{gu}$)	0.000247	0.000366	0.000437	0.000496	0.000508
	TVar($\hat{\pi}_{gu}$)	0.000247	0.000367	0.000450	0.000497	0.000509
	EMean($\hat{\omega}_{gu*}$)	0.298015	0.299102	0.295495	0.288993	0.278986
	EVar($\hat{\omega}_{gu*}$)	0.005779	0.010766	0.019549	0.037961	0.090777
	TVar($\hat{\omega}_{gu*}$)	0.005654	0.010818	0.019634	0.037539	0.085578
	Optimal n_1	813	834	843	848	851
	Optimal n_2	187	166	157	152	149

Table 1. Simulation results of binary models: trials $= 10000$, $p_a = 0.85$, $\pi_a = 0.7$, $p_b = 0.5$, $\pi_b = 0.1$, $p_1 = 0.85$, $p_2 = 0.15$, $n = 1000$. Continued on next two pages.

		$\pi = 1.0$	$\pi = 2.0$	$\pi = 3.0$	$\pi = 4.0$	$\pi = 5.0$
New model	EMean($\hat{\pi}_p$)	0.100065	0.199972	0.299964	0.400173	0.500032
	EVar($\hat{\pi}_p$)	0.000150	0.000219	0.000258	0.000289	0.000294
	TVar($\hat{\pi}_p$)	0.000153	0.000217	0.000262	0.000287	0.000293
	EMean($\hat{\omega}_{p*}$)	0.500294	0.499501	0.500029	0.499672	0.499882
	EVar($\hat{\omega}_{p*}$)	0.000846	0.000840	0.000819	0.000840	0.000824
	TVar($\hat{\omega}_{p*}$)	0.000840	0.000840	0.000840	0.000840	0.000840
Simple unrelated	EMean($\hat{\pi}_g$)	0.100058	0.199923	0.299879	0.400215	0.499955
	EVar($\hat{\pi}_g$)	0.000214	0.000281	0.000316	0.000341	0.000348
	TVar($\hat{\pi}_g$)	0.000213	0.000276	0.000319	0.000342	0.000345
Optional unrelated with optimal split	EMean($\hat{\pi}_{gu}$)	0.099788	0.200111	0.300070	0.400095	0.500009
	EVar($\hat{\pi}_{gu}$)	0.000282	0.000377	0.000458	0.000506	0.000522
	TVar($\hat{\pi}_{gu}$)	0.000281	0.000387	0.000460	0.000500	0.000508
	EMean($\hat{\omega}_{gu*}$)	0.499378	0.495986	0.496737	0.491159	0.483003
	EVar($\hat{\omega}_{gu*}$)	0.006075	0.010543	0.018355	0.035501	0.081250
	TVar($\hat{\omega}_{gu*}$)	0.006043	0.010546	0.018314	0.034180	0.076555
	Optimal n_1	807	830	842	849	852
	Optimal n_2	193	170	158	151	148
New model	EMean($\hat{\pi}_p$)	0.099826	0.200188	0.300180	0.399989	0.500289
	EVar($\hat{\pi}_p$)	0.000176	0.000238	0.000284	0.000307	0.000314
	TVar($\hat{\pi}_p$)	0.000180	0.000242	0.000285	0.000309	0.000313
	EMean($\hat{\omega}_{p*}$)	0.700494	0.700062	0.699990	0.700182	0.700184
	EVar($\hat{\omega}_{p*}$)	0.000951	0.000958	0.000996	0.000954	0.000958
	TVar($\hat{\omega}_{p*}$)	0.000960	0.000960	0.000960	0.000960	0.000960
Simple unrelated	EMean($\hat{\pi}_g$)	0.099812	0.200224	0.300152	0.399974	0.500196
	EVar($\hat{\pi}_g$)	0.000216	0.000276	0.000318	0.000342	0.000348
	TVar($\hat{\pi}_g$)	0.000213	0.000276	0.000319	0.000342	0.000345
Optional unrelated with optimal split	EMean($\hat{\pi}_{gu}$)	0.099964	0.199881	0.299798	0.399785	0.499802
	EVar($\hat{\pi}_{gu}$)	0.000309	0.000399	0.000476	0.000501	0.000508
	TVar($\hat{\pi}_{gu}$)	0.000308	0.000403	0.000466	0.000500	0.000505
	EMean($\hat{\omega}_{gu*}$)	0.698348	0.697556	0.698043	0.695540	0.686860
	EVar($\hat{\omega}_{gu*}$)	0.006058	0.009892	0.017114	0.031608	0.072975
	TVar($\hat{\omega}_{gu*}$)	0.006026	0.009977	0.016837	0.030600	0.068462
	Optimal n_1	808	831	844	850	854
	Optimal n_2	192	169	156	150	146

Table 1 (continued).

		$\pi = 1.0$	$\pi = 2.0$	$\pi = 3.0$	$\pi = 4.0$	$\pi = 5.0$
New model	EMean($\hat{\pi}_p$)	0.099938	0.199859	0.300231	0.399827	0.499915
	EVar($\hat{\pi}_p$)	0.000210	0.000268	0.000303	0.000332	0.000325
	TVar($\hat{\pi}_p$)	0.000209	0.000269	0.000310	0.000332	0.000334
	EMean($\hat{\omega}_{p^*}$)	0.899699	0.900364	0.900596	0.900153	0.899261
	EVar($\hat{\omega}_{p^*}$)	0.000993	0.001002	0.000974	0.000995	0.000982
	TVar($\hat{\omega}_{p^*}$)	0.001000	0.001000	0.001000	0.001000	0.001000
Simple unrelated	EMean($\hat{\pi}_g$)	0.099951	0.199827	0.300278	0.399858	0.499884
	EVar($\hat{\pi}_g$)	0.000217	0.000278	0.000310	0.000343	0.000336
	TVar($\hat{\pi}_g$)	0.000213	0.000276	0.000319	0.000342	0.000345
Optional unrelated with optimal split	EMean($\hat{\pi}_{gu}$)	0.099964	0.200086	0.299842	0.400039	0.499913
	EVar($\hat{\pi}_{gu}$)	0.000333	0.000414	0.000474	0.000506	0.000496
	TVar($\hat{\pi}_{gu}$)	0.000329	0.000414	0.000470	0.000499	0.000502
	EMean($\hat{\omega}_{gu^*}$)	0.898232	0.898642	0.897893	0.888967	0.886717
	EVar($\hat{\omega}_{gu^*}$)	0.005918	0.009252	0.015552	0.027088	0.065652
	TVar($\hat{\omega}_{gu^*}$)	0.005709	0.009163	0.015084	0.027281	0.060890
	Optimal n_1	815	836	847	853	856
	Optimal n_2	185	164	153	147	144

Table 1 (end).

values of the traditional unrelated question RRT model and the theoretical Var($\hat{\pi}_{gu}$) values of the split sample optional unrelated question RRT model. Similarly, Var($\hat{\omega}_{p^*}$) is always smaller than Var($\hat{\omega}_{gu^*}$).

In Table 1, the variances of the proposed models consistently have the smallest value. For each model, the theoretical and empirical variances match very well. First-order Taylor approximation was used to calculate Var($\hat{\pi}_p$) and Var($\hat{\omega}_{gu^*}$).

Notice that Var($\hat{\pi}_{gu}$) > Var($\hat{\pi}_g$), except for one case in Table 1. In this case, different models are used, the former a 2-parameter model and the latter a 1-parameter model.

4.2. ***Simulation of $\hat{\mu}_x$ and $\hat{\omega}$ for quantitative case.*** Table 2 shows that the Var($\hat{\mu}_p$) values are always the smallest in comparison with the Var($\hat{\mu}_g$) values of the traditional unrelated question RRT model and the Var($\hat{\mu}_{gu}$) values of the split sample optional unrelated question RRT model. Similarly, Var($\hat{\omega}_{p^{**}}$) is always smaller than Var($\hat{\omega}_{gu^{**}}$).

In Table 2, the variances of the proposed models are consistently the smallest. For each model, the theoretical and empirical variances match very well. First-order Taylor approximation was used to calculate the theoretical values of Var($\hat{\mu}_p$) and Var($\hat{\omega}_{gu^{**}}$).

		$\pi = 1.0$	$\pi = 2.0$	$\pi = 3.0$	$\pi = 4.0$	$\pi = 5.0$
New model	EMean($\hat{\mu}_p$)	1.000132	1.999822	2.999639	3.999132	4.999228
	EVar($\hat{\mu}_p$)	0.001693	0.002556	0.003328	0.004309	0.005253
	TVar($\hat{\mu}_p$)	0.001880	0.002664	0.003490	0.004358	0.005268
	EMean($\hat{\omega}_{p^{**}}$)	0.099985	0.100201	0.100164	0.099987	0.099953
	EVar($\hat{\omega}_{p^{**}}$)	0.000249	0.000245	0.000252	0.000252	0.000252
	TVar($\hat{\omega}_{p^{**}}$)	0.000250	0.000250	0.000250	0.000250	0.000250
Simple unrelated	EMean($\hat{\mu}_g$)	0.998261	2.000599	3.000724	3.999652	4.998115
	EVar($\hat{\mu}_g$)	0.009062	0.008097	0.007823	0.007660	0.008019
	TVar($\hat{\mu}_g$)	0.008983	0.008218	0.007806	0.007747	0.008042
Optional unrelated with optimal split	EMean($\hat{\mu}_{gu}$)	1.000980	2.001495	2.998708	3.999821	4.999394
	EVar($\hat{\mu}_{gu}$)	0.004008	0.005554	0.007014	0.008719	0.010369
	TVar($\hat{\mu}_{gu}$)	0.003965	0.005506	0.007094	0.008756	0.010504
	EMean($\hat{\omega}_{gu^{**}}$)	0.099302	0.098822	0.100371	0.098259	0.096624
	EVar($\hat{\omega}_{gu^{**}}$)	0.001197	0.002033	0.003726	0.007750	0.019916
	TVar($\hat{\omega}_{gu^{**}}$)	0.001162	0.002018	0.003724	0.007716	0.020077
	Optimal n_1	777	809	828	839	845
	Optimal n_2	223	191	172	161	155
New model	EMean($\hat{\mu}_p$)	0.999257	2.000800	2.999809	3.997820	5.000697
	EVar($\hat{\mu}_p$)	0.003161	0.003674	0.004222	0.005034	0.005707
	TVar($\hat{\mu}_p$)	0.003512	0.003912	0.004429	0.005064	0.005817
	EMean($\hat{\omega}_{p^{**}}$)	0.300102	0.300129	0.300195	0.300102	0.299864
	EVar($\hat{\omega}_{p^{**}}$)	0.000475	0.000476	0.000487	0.000491	0.000469
	TVar($\hat{\omega}_{p^{**}}$)	0.000477	0.000477	0.000477	0.000477	0.000477
Simple unrelated	EMean($\hat{\mu}_g$)	0.999620	2.001652	2.999560	3.998422	5.000657
	EVar($\hat{\mu}_g$)	0.008981	0.008030	0.007813	0.007787	0.007886
	TVar($\hat{\mu}_g$)	0.008983	0.008218	0.007806	0.007747	0.008042
Optional unrelated with optimal split	EMean($\hat{\mu}_{gu}$)	1.000295	2.000283	2.999919	3.999949	5.000366
	EVar($\hat{\mu}_{gu}$)	0.007276	0.008010	0.008841	0.009987	0.010842
	TVar($\hat{\mu}_{gu}$)	0.007258	0.007911	0.008746	0.009780	0.011036
	EMean($\hat{\omega}_{gu^{**}}$)	0.299975	0.298695	0.299377	0.299276	0.294327
	EVar($\hat{\omega}_{gu^{**}}$)	0.002125	0.002947	0.004738	0.008544	0.020036
	TVar($\hat{\omega}_{gu^{**}}$)	0.002111	0.002957	0.004594	0.008385	0.019787
	Optimal n_1	757	784	807	826	838
	Optimal n_2	243	216	193	174	162

Table 2. Simulation results of quantitative models: trials = 10000, $p_a = 0.85$, $\mu_y = 7.0$, $p_b = 0.6$, $\pi_b = 0.1$, $p_1 = 0.85$, $p_2 = 0.15$, $n = 1000$. Continued on the next two pages.

		$\pi = 1.0$	$\pi = 2.0$	$\pi = 3.0$	$\pi = 4.0$	$\pi = 5.0$
New model	EMean($\hat{\mu}_p$)	1.000104	2.000087	3.000718	4.001133	5.000078
	EVar($\hat{\mu}_p$)	0.004778	0.004885	0.005055	0.005810	0.006419
	TVar($\hat{\mu}_p$)	0.005204	0.005213	0.005416	0.005815	0.006409
	EMean($\hat{\omega}_{p^{**}}$)	0.499925	0.500087	0.500174	0.500064	0.499728
	EVar($\hat{\omega}_{p^{**}}$)	0.000626	0.000628	0.000623	0.000612	0.000613
	TVar($\hat{\omega}_{p^{**}}$)	0.000623	0.000623	0.000623	0.000623	0.000623
Simple unrelated	EMean($\hat{\mu}_g$)	1.001130	1.999331	3.000900	4.001092	4.999751
	EVar($\hat{\mu}_g$)	0.008974	0.008272	0.007543	0.007785	0.008200
	TVar($\hat{\mu}_g$)	0.008983	0.008218	0.007806	0.007747	0.008042
Optional unrelated with optimal split	EMean($\hat{\mu}_{gu}$)	0.999785	2.000865	2.998736	3.999293	4.999645
	EVar($\hat{\mu}_{gu}$)	0.010030	0.009858	0.010053	0.010752	0.011474
	TVar($\hat{\mu}_{gu}$)	0.010021	0.009904	0.010105	0.010627	0.011484
	EMean($\hat{\omega}_{gu^{**}}$)	0.499831	0.498203	0.499094	0.497134	0.496861
	EVar($\hat{\omega}_{gu^{**}}$)	0.002595	0.003521	0.004917	0.008461	0.019331
	TVar($\hat{\omega}_{gu^{**}}$)	0.002614	0.003421	0.004978	0.008500	0.019160
	Optimal n_1	762	782	802	820	835
	Optimal n_2	238	218	198	180	165
New model	EMean($\hat{\mu}_p$)	1.000434	1.999794	3.000376	3.999070	4.998875
	EVar($\hat{\mu}_p$)	0.006498	0.006200	0.006230	0.006572	0.007062
	TVar($\hat{\mu}_p$)	0.006956	0.006570	0.006457	0.006617	0.007051
	EMean($\hat{\omega}_{p^{**}}$)	0.700136	0.700524	0.699319	0.700041	0.700131
	EVar($\hat{\omega}_{p^{**}}$)	0.000695	0.000686	0.000703	0.000678	0.000668
	TVar($\hat{\omega}_{p^{**}}$)	0.000690	0.000690	0.000690	0.000690	0.000690
Simple unrelated	EMean($\hat{\mu}_g$)	0.999969	2.000120	3.000306	3.999057	4.998973
	EVar($\hat{\mu}_g$)	0.009033	0.008287	0.007767	0.007932	0.008049
	TVar($\hat{\mu}_g$)	0.008983	0.008218	0.007806	0.007747	0.008042
Optional unrelated with optimal split	EMean($\hat{\mu}_{gu}$)	0.999089	2.000346	2.999364	4.000602	4.998395
	EVar($\hat{\mu}_{gu}$)	0.012147	0.011724	0.011007	0.011255	0.011901
	TVar($\hat{\mu}_{gu}$)	0.012241	0.011503	0.011193	0.011309	0.011855
	EMean($\hat{\omega}_{gu^{**}}$)	0.700095	0.699526	0.700043	0.698377	0.697909
	EVar($\hat{\omega}_{gu^{**}}$)	0.002767	0.003456	0.004917	0.008352	0.018523
	TVar($\hat{\omega}_{gu^{**}}$)	0.002757	0.003508	0.004967	0.008260	0.018173
	Optimal n_1	777	790	805	820	834
	Optimal n_2	223	210	195	180	166

Table 2 (continued).

		$\pi = 1.0$	$\pi = 2.0$	$\pi = 3.0$	$\pi = 4.0$	$\pi = 5.0$
New model	EMean($\hat{\mu}_p$)	0.997806	1.999640	2.999296	4.000093	5.000968
	EVar($\hat{\mu}_p$)	0.008691	0.007787	0.007430	0.007428	0.007794
	TVar($\hat{\mu}_p$)	0.008770	0.007986	0.007554	0.007475	0.007749
	EMean($\hat{\omega}_{p^{**}}$)	0.900024	0.900222	0.899738	0.899885	0.900270
	EVar($\hat{\omega}_{p^{**}}$)	0.000684	0.000673	0.000662	0.000664	0.000681
	TVar($\hat{\omega}_{p^{**}}$)	0.000677	0.000677	0.000677	0.000677	0.000677
Simple unrelated	EMean($\hat{\mu}_g$)	0.998399	2.000097	2.999036	4.000156	5.001079
	EVar($\hat{\mu}_g$)	0.009059	0.008152	0.007844	0.007811	0.008153
	TVar($\hat{\mu}_g$)	0.008983	0.008218	0.007806	0.007747	0.008042
Optional unrelated with optimal split	EMean($\hat{\mu}_{gu}$)	1.000834	1.997784	2.999923	3.999134	4.999262
	EVar($\hat{\mu}_{gu}$)	0.013713	0.012646	0.011814	0.011912	0.011936
	TVar($\hat{\mu}_{gu}$)	0.013854	0.012677	0.012003	0.011828	0.012148
	EMean($\hat{\omega}_{gu^{**}}$)	0.898653	0.900103	0.898165	0.898256	0.897476
	EVar($\hat{\omega}_{gu^{**}}$)	0.002611	0.003239	0.004693	0.007889	0.016584
	TVar($\hat{\omega}_{gu^{**}}$)	0.002582	0.003297	0.004657	0.007755	0.016864
	Optimal n_1	800	807	815	825	834
	Optimal n_2	200	193	185	175	166

Table 2 (end).

5. Conclusion

We propose a modification of the optional unrelated question models of [Gupta et al. 2013b] for binary and quantitative responses by avoiding the split sample approach, which requires a larger total sample size. Rather than trying to estimate prevalence and sensitivity simultaneously, we estimate them independently by asking two questions. The simulation study shows that the proposed models always achieve smaller variances of the estimators for both binary and quantitative response cases. As shown in the tables, $\operatorname{Var}(\hat{\pi}_p)$, $\operatorname{Var}(\hat{\omega}_{p^*})$, $\operatorname{Var}(\hat{\mu}_p)$, and $\operatorname{Var}(\hat{\omega}_{p^{**}})$ are respectively smaller than $\operatorname{Var}(\hat{\pi}_g)$ and $\operatorname{Var}(\hat{\pi}_{gu})$, $\operatorname{Var}(\hat{\omega}_{gu^*})$, $\operatorname{Var}(\hat{\mu}_g)$ and $\operatorname{Var}(\hat{\mu}_{gu})$, and $\operatorname{Var}(\hat{\omega}_{gu^{**}})$.

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