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On Oliver's p-group conjecture

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Let S be a p-group for an odd prime p. B. Oliver conjectures that a certain characteristic subgroup $\mathfrak{X}(S)$ always contains the Thompson subgroup $J(S)$. We obtain a reformulation of the conjecture as a statement about modular representations of p-groups. Using this we verify Oliver's conjecture for groups where $S/\mathfrak{X}(S)$ has nilpotence class at most two.

1. Introduction

The recently introduced concept of a p-local finite group seeks to provide a treatment of the p-local structure of a finite group G which does not refer directly to the group G itself and yet retains enough information to construct the p-localisation of the classifying space BG. Ideally one could then associate a p-local classifying space to a p-block of G, and to certain exotic fusion systems. See the survey article by Broto, Levi and Oliver [2004] for an introduction to this area.

A key open question about p-local finite groups is whether or not there is a unique centric linking system associated to each saturated fusion system. Oliver showed that this would follow from a conjecture about higher limits (see [Oliver 2004, Conjecture 2.2]) and that for odd primes this higher limits conjecture would in turn follow from a purely group-theoretic conjecture:

Conjecture [Oliver 2004, Conjecture 3.9]. *Let S be a p-group for an odd prime p. Then*

$$J(S) \leq \mathfrak{X}(S),$$

where $J(S)$ is the Thompson subgroup generated by all elementary abelian p-subgroups whose rank is the p-rank of S, and $\mathfrak{X}(S)$ is the Oliver subgroup described in Section 2.

Our main result on Oliver's conjecture is:

Theorem 1.1. *Let S be a p-group for an odd prime p. If $S/\mathfrak{X}(S)$ has nilpotency class at most two, then S satisfies Oliver's conjecture.*

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Remark. This subsumes all three cases of Oliver's Proposition 3.7 in the first case $\mathfrak{X}(S) \geq J(S)$.

The proof of Theorem 1.1 depends on a reformulation of Oliver's conjecture, for which we need to recall the terms F-module and offender. See for example [Meierfrankenfeld and Stellmacher 2006] for recent results about offenders.

Definition [Gorenstein et al. 1994, Definition 26.5]. Let G be a finite group and V a faithful $\mathbb{F}_p G$-module. If there exists a nonidentity elementary abelian p-subgroup $E \leq G$ which satisfies the inequality $|E||C_V(E)| \geq |V|$, then V is called an *F-module* for G, and E an *offending subgroup*.

Remark. F-module is short for "failure of (Thompson) factorization module". Another way to phrase the inequality is $\dim(V) - \dim(V^E) \leq \operatorname{rank}(E)$.

We will always take G to be a nontrivial p-group. Hence the $\mathbb{F}_p G$-module V is faithful if and only if it is faithful as a module for $\Omega_1(Z(G))$. We shall be interested in the stronger condition:

(PS) The restriction of V to each central order p subgroup has a nontrivial projective summand.

Remark. Projective and free are equivalent here. We are grateful to the referee for suggesting this formulation of the property. Another formulation is that every central order p element operates with minimal polynomial $(X-1)^p$: equivalence follows from the standard properties of the Jordan normal form.

Theorem 1.2. *Let $G \neq 1$ be a finite p-group. Then Oliver's conjecture holds for every finite p-group S with $S/\mathfrak{X}(S) \cong G$ if and only if G has no F-modules satisfying* **(PS)**.

Conjecture 1.3. *Let p be an odd prime and $G \neq 1$ a finite p-group. Then G has no F-modules which satisfy* **(PS)**.

Corollary 1.4. *Conjecture 1.3 is equivalent to Oliver's Conjecture* 3.9.

We prove Theorem 1.1 by verifying Conjecture 1.3 for groups of class at most two. For this we need this result:

Definition (See [Glauberman 1972]). Let V be a faithful $\mathbb{F}_p G$-module. A nonidentity element $g \in G$ is called *quadratic* if $(g-1)^2 V = 0$.

Theorem 1.5. *Suppose that p is an odd prime, G is a p-group of nilpotence class at most two, and V is a faithful $\mathbb{F}_p G$-module. If G contains a quadratic element, then so does $\Omega_1(Z(G))$.*

Structure of the paper. We prove Theorem 1.2 and Corollary 1.4 in Section 2. In Section 3 we derive a consequence of the Replacement Theorem, Theorem 3.3. Then in Section 4 we prove Theorems 1.5 and 1.1. Finally in Section 5 we discuss a class three example which cannot be handled using Theorem 3.3.

2. The reformulation of Oliver's conjecture

For the convenience of the reader we start by recapping the definition and elementary properties of $\mathfrak{X}(S)$, as given in [Oliver 2004, §3].

Definition [Oliver 2004, Definition 3.1]. Let S be a p-group and $K \lhd S$ a normal subgroup. A Q-series leading up to K consists of a series of subgroups

$$1 = Q_0 \leq Q_1 \leq \cdots \leq Q_n = K$$

such that each Q_i is normal in S, and such that

$$[\Omega_1(C_S(Q_{i-1})), Q_i; p-1] = 1$$

holds for each $1 \leq i \leq n$. The unique largest normal subgroup of S which admits such a Q-series is called $\mathfrak{X}(S)$, the Oliver subgroup of S.

Lemma 2.1 (Oliver). *If $1 = Q_0 \leq Q_1 \leq \cdots \leq Q_n = K$ is such a Q-series and $H \lhd G$ also admits a Q-series, then there is a Q-series leading up to HK which starts with $Q_0, \ldots, Q_n$.*

Hence there is indeed a unique largest subgroup admitting a Q-series, and this subgroup $\mathfrak{X}(S)$ is characteristic in S. In addition, $\mathfrak{X}(S)$ is centric in S: recall that $P \leq S$ is centric if $C_S(P) = Z(P)$.

Proof. See [Oliver 2004, pp. 334–5]. $\square$

Now we can start to derive the reformulation of Oliver's conjecture.

Lemma 2.2. *Let S be a finite p-group with $\mathfrak{X}(S) < S$. Then the induced action of $G := S/\mathfrak{X}(S)$ on $V := \Omega_1\big(Z(\mathfrak{X}(S))\big)$ satisfies* **(PS)**.

Proof. Pick $g \in S$ such that $1 \neq g\mathfrak{X}(S) \in \Omega_1(Z(G))$. Then $\langle \mathfrak{X}(S), g\rangle \lhd S$ and so $[V, g; p-1] \neq 1$, by maximality of $\mathfrak{X}(S)$. So the minimal polynomial of the action of g does not divide $(X-1)^{p-1}$. But it has to divide $(X-1)^p = X^p - 1$. So $(X-1)^p$ is the minimal polynomial. This is the reformulation of **(PS)**. $\square$

Proof of Theorem 1.2. Suppose first that no F-module for G satisfies **(PS)**, and that $S/\mathfrak{X}(S) \cong G$. Let us prove Oliver's Conjecture for G. By Lemma 2.2 the induced action of G on $V := \Omega_1\big(Z(\mathfrak{X}(S))\big)$ satisfies **(PS)**, so by assumption there are no offending subgroups.

Let $E \leq S$ be an elementary abelian subgroup not contained in $\mathfrak{X}(S)$. It suffices for us to show that $\mathfrak{X}(S)$ contains an elementary abelian subgroup of greater rank than E. We can split E up as $E = E_1 \times E_2 \times E_3$, with $E_1 = E \cap V \leq V^E$ and $E_1 \times E_2 = E \cap \mathfrak{X}(S)$. By assumption, $1 \neq E_3$ embeds in $S/\mathfrak{X}(S) \cong G$. As there are no offenders, we have $\dim(V) - \dim(V^{E_3}) > \operatorname{rank}(E_3)$. But $V^{E_3} = V^E$. So $V \times E_2$ lies in $\mathfrak{X}(S)$ and has greater rank than E.

Conversely suppose that the $\mathbb{F}_p G$-module V is an F-module and satisfies **(PS)**. Set S to be the semidirect product $S = V \rtimes G$ defined by this action. From Lemma 2.3 below we see that $V = \mathfrak{X}(S)$. As V is an F-module, there is an offender: an elementary abelian subgroup $1 \neq E \leq G$ with $\dim(V) - \dim(V^E) \leq \operatorname{rank}(E)$. This means that $W := V^E \times E$ is an elementary abelian subgroup which does not lie in $V = \mathfrak{X}(S)$ but does have rank at least as great as that of $\mathfrak{X}(S)$. So $W \leq J(S)$ and therefore $J(S) \not\leq \mathfrak{X}(S)$. □

Lemma 2.3. *Suppose that V is an $\mathbb{F}_p G$-module which satisfies* **(PS)**. *Let S be the semidirect product $S = V \rtimes G$ defined by this action. Then $V = \mathfrak{X}(S)$.*

Proof. First we prove that V is a maximal normal abelian subgroup of S: clearly it is abelian and normal. If A is a normal abelian subgroup strictly containing V, then $A = V \rtimes H$ for some nontrivial abelian $H \lhd G$. As H is nontrivial and normal it contains an order p element g of $Z(G)$. Since V satisfies **(PS)**, it follows that g acts on V with minimal polynomial $(X-1)^p$. But that is a contradiction, as A is abelian. So V is indeed maximal normal abelian.

We now argue as in the proof of Oliver's Lemma 3.2. Since V is maximal normal abelian, it is centric in S: for if not then $V < C_S(V) \lhd S$, and so $C_S(V)/V$ has nontrivial intersection with the centre of S/V. Picking an $x \in C_S(V)$ whose image in $C_S(V)/V$ is a nontrivial element of this intersection, we obtain a strictly larger normal abelian subgroup $\langle V, x\rangle$, a contradiction. Hence $\Omega_1(C_S(V)) = V$.

Moreover, since V is normal abelian and $p > 2$, there is a Q-series $1 < V$. So by Lemma 2.1 there is a Q-series leading up to $\mathfrak{X}(S)$ with $Q_1 = V$. If $V < \mathfrak{X}(S)$ then there is $Q_1 < Q_2 \lhd S$ with $[V, Q_2; p-1] = 1$. But this cannot happen, because by the argument of the first paragraph of this proof there is a $g \in Q_2$ whose action on V has minimal polynomial $(X-1)^p$. So $V = \mathfrak{X}(S)$. □

Proof of Corollary 1.4. Immediate from Theorem 1.2. If $\mathfrak{X}(S) = S$ then Oliver's Conjecture holds automatically. □

3. The Replacement Theorem

We shall need the following lemma, which is a special case of the Replacement Theorem and its proof in [Huppert and Blackburn 1982, X, 3.3].

Lemma 3.1. *Suppose that $G \neq 1$ is elementary abelian, that V is a faithful $\mathbb{F}_p G$-module, and that G contains no quadratic elements. Let us write*

$$T = \{(H, W) \mid H \leq G \text{ and } W \text{ is a subspace of } V^H\}.$$

Suppose that $(H, W) \in T$ with $H \neq 1$. Then there is $(K, U) \in T$ with $K < H$, $W \subsetneq U \subsetneq V$ and $|H \times W| = |K \times U|$.

Proof. Let us set

$$I = \{v \in V \mid (h-1)v \in W \text{ for every } h \in H\},$$
$$J = \{v \in V \mid (h-1)v \in I \text{ for every } h \in H\}.$$

If $1 \neq h \in H$ then $(h-1)^2 v \neq 0$ for some $v \in V$. Then $v \notin I$, for otherwise $(h-1)v \in W$ and so $(h-1)^2 v = 0$. So $I \subsetneq V$, and therefore $W \subsetneq I \subsetneq J$ by the usual orbit length argument. Pick $v_0 \in J \setminus I$ and set U to be the subspace spanned by W and $\{(h-1)v_0 \mid h \in H\}$. Set $K = \{h \in H \mid (h-1)v_0 \in W\}$. So $U \supsetneq W$ by choice of v_0. Also $U \subseteq I \subsetneq V$. If $h, h' \in H$ then

$$(hh'-1)v_0 = (h-1)v_0 + (h'-1)v_0 + (h-1)(h'-1)v_0,$$

and so

$$(hh'-1)v_0 \equiv (h-1)v_0 + (h'-1)v_0 \pmod{W}. \tag{3-1}$$

So $K \leq H$, and in fact $K < H$ by choice of v_0. By (3-1) it also follows that $|H : K| = p^r$ for $r = \dim U - \dim W$. Finally $U \subseteq V^K$, for if $k \in K$ and $u \in U$, then

$$u = \sum_{h \in H} \lambda_h (h-1)v_0 + w$$

for suitable $\lambda_h \in \mathbb{F}_p$, $w \in W$. So

$$(k-1)u = \sum_{h \in H} \lambda_h (h-1)(k-1)v_0 = 0,$$

since $(k-1)v_0 \in W \subseteq V^H$. □

Corollary 3.2. *Suppose as in Lemma 3.1 that $(H, W) \in T$ and $H \neq 1$. Then $|H \times W| < |V|$.*

Proof. By induction on $|H|$. By the lemma we may reduce $|H|$ whilst keeping $|H \times W|$ constant. This process only stops when we arrive at (K, U) with $K = 1$. But $U \subsetneq V$ by the lemma. □

The following result is presumably well known to those familiar with Thompson factorization.

Theorem 3.3. *Suppose that p is an odd prime, G is a finite group, V is a faithful $\mathbb{F}_p G$-module, and $E \leq G$ is a nonidentity elementary abelian p-subgroup. If E is an offender, then it must contain a quadratic element.*

Proof. Without loss of generality $E = G$. Apply Corollary 3.2 to the pair

$$(G, V^G) \in T. \qquad \square$$

Remark. Pursuing this direction further, it might be worthwhile to investigate potential applications of the $P(G, V)$-theorem in the theory of p-local finite groups. The properties of the Thompson subgroup $J(S)$ which Chermak describes in his comments on the motivation for the $P(G, V)$-theorem [Chermak 1999, Remark 2] are the same properties which led to $J(S)$ featuring in Oliver's conjecture. And Timmesfeld's Replacement Theorem plays an important part in the proof of the $P(G, V)$-theorem.

4. Nilpotence class at most two

We can now start work on the proof of Theorem 1.1.

Lemma 4.1. *Suppose that p is an odd prime, that $G \neq 1$ is a finite p-group, and that V is a faithful $\mathbb{F}_p G$-module. Suppose that $A, B \in G$ are such that $C := [A, B]$ is a nontrivial element of $C_G(A, B)$. If C is nonquadratic, then so are A and B.*

Proof. By symmetry it suffices to prove that B is nonquadratic. So suppose that B is quadratic. Denote by α, β, γ the action matrices on V of $A - 1$, $B - 1$ and $C - 1$ respectively.

By assumption we have $\gamma^2 \neq 0$ and $\beta^2 = 0$. As C commutes with A and B, we have $\alpha\gamma = \gamma\alpha$ and $\beta\gamma = \gamma\beta$. Since $[A, B] = C$, we have $AB = BAC$ and therefore

$$\alpha\beta - \beta\alpha = \gamma(1 + \beta + \alpha + \beta\alpha). \tag{4-1}$$

Evaluating $\beta \cdot (4\text{-}1) \cdot \beta$, we deduce that $\gamma\beta\alpha\beta = 0$. So when we evaluate $\beta \cdot (4\text{-}1) + (4\text{-}1) \cdot \beta$, we find that $\gamma(2\beta + \beta\alpha + \alpha\beta) = 0$. Let us write $\lambda = -\frac{1}{2}$ and $\delta = \gamma\beta$. Then we have

$$\delta = \lambda(\delta\alpha + \alpha\delta).$$

From this one sees by induction upon $r \geq 1$ that

$$\delta = \lambda^r \sum_{s=0}^{r} \binom{r}{s} \alpha^s \delta \alpha^{r-s}.$$

Since the order of A is a power of p, it follows that $(A - 1)$ and its action matrix α are nilpotent. From this we deduce that $\delta = 0$, that is $\gamma\beta = 0$. Applying this to $\gamma \cdot (4\text{-}1)$ we see that $\gamma^2(1 + \alpha) = 0$. As α is nilpotent it follows that $\gamma^2 = 0$, a contradiction. So $\beta^2 \neq 0$ after all. □

Proof of Theorem 1.5. We suppose that $\Omega_1(Z(G))$ has no quadratic elements, and show that G has none either. Suppose $1 \neq B \in Z(G)$. Then there is an $r \geq 0$ with $1 \neq B^{p^r} \in \Omega_1(Z(G))$. So B^{p^r} is not quadratic. Hence $(B - 1)^{2p^r} = (B^{p^r} - 1)^2$ has nonzero action. So $(B - 1)^2$ has nonzero action, and $Z(G)$ contains no quadratic elements.

If $B \notin Z(G)$ then the nilpotency class is two and there is an element $A \in G$ with $1 \neq [A, B] \in Z(G)$. So $(B-1)^2$ has nonzero action by Lemma 4.1. □

Corollary 4.2. *Suppose that p is an odd prime, $G \neq 1$ a finite p-group and V an $\mathbb{F}_p G$-module which satisfies* **(PS)**. *If the nilpotence class of G is at most two then V cannot be an F-module.*

Proof. As p is odd, condition **(PS)** means that there are no quadratic elements in $\Omega_1(Z(G))$. Then Theorem 1.5 says that there are no quadratic elements in G. So by Theorem 3.3 there are no offenders. □

Proof of Theorem 1.1. Follows from Corollary 4.2 and Theorem 1.2 if $\mathfrak{X}(S) < S$. If $\mathfrak{X}(S) = S$ then there is nothing to prove. □

5. A class 3 example

Theorem 1.5 was a key step in the proof of Theorem 1.1. We now give an example which shows that Theorem 1.5 does not apply to groups of nilpotence class three.

Let G be the semidirect product $G = K \rtimes L$, where the $K = \mathbb{F}_3^3$ is elementary abelian of order 3^3, $L = \langle A \rangle$ is cyclic of order 3, and the action of L on $v \in K$ is given by

$$AvA^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot v.$$

Observe that G is isomorphic to the wreath product $C_3 \wr C_3$, as the action of A permutes the following basis of K cyclically: $(0, 0, 1)$, $(0, 1, 1)$, $(1, 2, 1)$.

Setting $B = (0, 0, 1)$, $C = (0, 1, 0)$ and $D = (1, 0, 0)$ we obtain the following presentation of G, where we take $[A, B]$ to mean $ABA^{-1}B^{-1}$.

$$G = \left\langle A, B, C, D \;\middle|\; \begin{array}{lll} A^3 = B^3 = C^3 = D^3 = 1, & & D \text{ central}, \\ [B, C] = 1, & [A, B] = C, & [A, C] = D \end{array} \right\rangle,$$

From this we deduce that matrices $\alpha, \beta, \gamma, \delta \in M_n(\mathbb{F}_3)$ induce a representation $\rho\colon G \to GL_n(\mathbb{F}_3)$ with

$$\rho(A) = 1 + \alpha, \quad \rho(B) = 1 + \beta, \quad \rho(C) = 1 + \gamma, \quad \rho(D) = 1 + \delta,$$

if and only if the following relations are satisfied, where $[\alpha, \beta]$ now of course means $\alpha\beta - \beta\alpha$:

$$\begin{aligned} &\alpha^3 = \beta^3 = \gamma^3 = \delta^3 = 0, \\ &[\alpha, \delta] = [\beta, \delta] = [\gamma, \delta] = [\beta, \gamma] = 0, \\ &[\alpha, \beta] = \gamma(1 + \beta)(1 + \alpha), [\alpha, \gamma] = \delta(1 + \gamma)(1 + \alpha). \end{aligned} \tag{5-1}$$

Now we consider what it means for such a representation to satisfy **(PS)**. Here,

$$Z(G) = \langle D \rangle$$

is cyclic of order 3. So we need both $(\rho(D)-1)^2$ and $(\rho(D^2)-1)^2$ to be nonzero. That is, δ^2 and $(\delta^2+2\delta)^2 = \delta^2(1+\delta+\delta^2)$ should both be nonzero. But $1+\delta+\delta^2$ is invertible, since δ is nilpotent.

We deduce therefore that matrices $\alpha, \beta, \gamma, \delta \in GL_n(\mathbb{F}_3)$ induce a representation of G satisfying **(PS)** if and only if they satisfy the inequality

$$\delta^2 \neq 0 \tag{5-2}$$

in addition to (5-1).

Using GAP [2007] we obtained the following matrices in $GL_8(\mathbb{F}_3)$. The reader is invited to check[1] that they satisfy the relations (5-1) and (5-2).

$$\delta = \begin{pmatrix} 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&2\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}, \qquad \gamma = \begin{pmatrix} 0&0&1&1&2&2&2&1\\ 0&0&1&0&1&1&0&0\\ 0&0&0&0&0&0&2&2\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix},$$

$$\beta = \begin{pmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}, \qquad \alpha = \begin{pmatrix} 2&2&0&2&0&1&0&1\\ 1&1&2&2&0&0&0&0\\ 0&0&2&2&0&0&0&0\\ 0&0&1&1&0&0&0&0\\ 0&0&1&2&0&0&1&1\\ 0&0&1&1&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0 \end{pmatrix}.$$

Observe that $\beta^2 = 0$. So although this module satisfies **(PS)**, the elementary abelian subgroups $\langle B \rangle$ and $\langle B, C, D \rangle$ both contain B, a quadratic element. So we must find another way to show that they are not offenders: Theorem 3.3 does not apply.

Remark. More generally, we are not currently able to decide Conjecture 1.3 either way for the wreath product group $H \wr C_3$, where the group H on the bottom is an elementary abelian 3-group.

[1] See http://users.minet.uni-jena.de/~green/Documents/matTest.g for a GAP script that performs these checks.

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