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Asset liability management for Tanzania : pension funds by stochastic programming

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Abstract. We present a long-term model of asset liability management for Tanzania pension funds. The pension system is pay-as-you-go where contributions are used to pay current benefits. The pension plan is a final salary defined benefit. Two kinds of pension benefits, a commuted (at retirement) and a monthly (old age) pension are considered. A decisive factor for a long-term asset liability management is that, Tanzania pension funds face an increase of their members' life expectancy, which will cause the retirees to contributors dependence ratio to increase. We present a stochastic programming approach which allocates assets with the best return to raise the asset value closer to the level of liabilities. The model is based on work by Kouwenberg in 2001, with features from Tanzania pension system. In contrast to most asset liability management models for pension funds by stochastic programming, liabilities are modeled by using number of years of life expectancy for monthly benefit. Scenario trees are generated by using Monte Carlo simulation. Numerical results suggest that, in order to improve the long-term sustainability of the Tanzania pension fund system, it is necessary to make reforms concerning the contribution rate, investment guidelines and formulate target funding ratios to characterize the pension funds' solvency situation.

Key words: Pay-as-you-go pension fund, asset liability management, stochastic programming, scenario trees.
AMS 2010 Mathematics Subject Classification : 62P05, 90C15.

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Résumé. Nous présentons un modèle à long terme de gestion actif-passif pour les fonds de pension tanzaniens. Le système de retraite est un système de répartition qui utilise les cotisations pour payer les prestations courantes. Le régime de retraite est basé sur les points du dernier salaire. Deux types de prestations de retraite, une rente viagère (à la retraite) et une rente mensuelle (vieillesse) sont pris en compte. Un facteur décisif pour une gestion actif-passif à long terme est que les fonds de pension tanzaniens sont confrontés à une augmentation de l'espérance de vie de leurs membres, ce qui entraînera une augmentation du taux de dépendance des retraités à contributeurs. Nous présentons une approche de programmation stochastique consistant à allouer des actifs offrant le meilleur rendement afin d'élever la valeur de l'actif plus près du niveau des passifs. Le modèle est basé sur les travaux de Kouwenberg (2001), avec des caractéristiques du système de retraite de la Tanzanie. Contrairement à la plupart des modèles de gestion actif-passif pour les fonds de pension par programmation stochastique, les passifs sont modélisés en utilisant le nombre d'années d'espérance de vie pour les prestations mensuelles. Les arbres de scénario sont générés à l'aide de la simulation de Monte Carlo. Des résultats numériques suggèrent que, pour améliorer la viabilité à long terme du système de fonds de pension tanzanien, il est nécessaire de procéder à des réformes concernant le taux de cotisation, les directives d'investissement et la formulation de ratios de financement objectifs pour caractériser la situation de solvabilité des fonds de pension.

1. Introduction

A pension is a generic term for single or periodic payments to a beneficiary, which replaces a former income of an employee in case of reaching a certain age, or in the case of disability or death. A pension fund is considered to be an organization, obliged with paying pensions and it has a task of making benefit payments to members who have ended their active earnings carrier. The payments to be made to the retirees must be in accordance with a benefit condition that prescribes the flow of payments to which each member in the fund is entitled.

There are two major kinds of pension fund schemes, known as defined contribution (DC) and defined benefit (DB). A defined contribution scheme specifies how much a member will contribute, often as a fixed percentage of salary. The fixed percentage of the salary is called contribution rate. The benefit is determined by the size of accumulated contributions in an individual account that participates in a profit sharing. At an agreed age or state, pension benefits are paid as a lump sum or as regular payments, depending on contributions as well as development of invested funds. A defined benefit scheme specifies a level of benefit, usually based on salary in relation to near retirement (final salary), or on salary throughout employment (career average salary plans). This level is usually defined according to a benefit formula as a function of the salary and years of the service. The contributions from employer and employee are accumulated to meet the level of benefit. Contributions may be increased or decreased depending on investment performance or demographic experience. The main difference between DC and DB pension schemes is the way in which the financial risk is treated. In DC plans, the financial risk is borne by the contributors. In DB scheme, the financial risk is borne by the sponsors of the scheme. Recently, due to the demographic evolution and the development of the equity market, DC schemes have become popular in the global pension market (Gao, 2008).

A pay-as-you-go pension is a system where current benefit payouts for retirees are paid by using contributions from current members. To be sustainable, it requires a balance between the benefits paid to the retirees and the contributions made by the current members. The world demography is however changing rapidly with increased life expectancy and decreased fertility rate (Johnson, 2004; Batini et al., 2006), and the retirees population is growing, compared to the working population, in most countries of the world (Bos et al., 1992). The increasing ratio of retirees to working population is bringing various policy responses. Parametric reforms tinker with pay-as-you-go defined benefit pension schemes by reducing benefits, and also raising taxes and eliminating the incentives for early retirement will be necessary.

There are recent applications of Asset Liability Management (ALM) for pension funds by stochastic programming for several countries. Dert (1995) studied asset liability management by chance constraints for Dutch pension funds and Kouwenberg (2001) studied multistage stochastic programming for a Dutch pension fund. Dupačová and Polívka (2009) studied ALM for Czech pension funds, Hilli et al. (2007) for a Finnish pension company, Mulvey et al. (2000) for a Towers Perrin-Tillinghast pension fund in America, and Geyer and Ziemba (2008) for an Innovest Austrian pension fund. Klein Haneveld et al. (2010) studied ALM with integrated chance constraints for Dutch pension funds while Hussin et al. (2014) studied two-stage stochastic programming using integrated chance constraints for a Malaysia pension fund. Bogentoft et al. (2001) and Bai and Ma (2009) study ALM with CVaR constraints for Dutch and China pension funds, respectively.

Several studies of the challenges facing pay-as-you-go pension systems due to changes in demography have been conducted. Among these are Humberto et al. (2016) who studied a sustainability framework for pay-as-you-go pension system. This study forecasts that the net present value of expenditure on pensions in the US will exceed the net present value of contributions through the period $2015-2089$. Ai et al. (2015) develop a benchmark risk measure for pension sponsors by obtaining a total asset requirement for sustaining the pension plan with respect to the risk of increased longevity.

We develop an asset liability management for Tanzania pension funds by stochastic programming. As an application, the largest pension fund in Tanzania, the National Social Security Fund (NSSF) is considered. According to the Social Security Regulatory Authority (SSRA), this fund has about 44% of the total pension funds population in 2015 and the retirees to contributors dependency ratio is the lowest among Tanzania pension funds.

The remainder of this paper is organized as follows. The Tanzania pension fund system is presented in Section 2. Section 3 outlines an asset liability management for pension funds by stochastic programming. The model is developed in Section 4 while Section 5 explains the scenario tree generation. Numerical results and simulations are presented in Section 6 while Section 7 gives the summary and conclusion.

2. Tanzania pension system

The pension fund system in mainland Tanzania is characterized by five pension funds serving a small subset of the population. These are Parastatal Public Pension (PPF), Public Service Pension Fund (PSPF), National Social Security Fund (NSSF), Local Authority Pension Fund (LAPF) and Government Employees Provident Fund (GEPF). In year 2015, the system covered 2.14 millions members which was about 4% of the total population and 10% of the working population. All funds were converted to pay-as-you-go defined benefit between 1999 and 2013. These funds are regulated and supervised by the Social Security Regulatory Authority (SSRA).

The SSRA was established under the social security regulatory authority Act No. 8 of 2008 and amended by Act No. 5 of 2012, with the main objective of supervising and regulating the social security sector. This authority started its operations at the end of the year 2010. It may set contribution rates payable to a fund by members and minimum benefit payable to its beneficiaries. But before adjusting contribution rate and minimum benefit payable, the authority should undertake or cause a fund to undertake actuarial valuation. Currently, the contribution rate is 20% of the monthly salary. An employee commonly pays 5% and the employer pays 15%, while in some funds including NSSF, an employee and employer each pays 10%.

The five aforementioned pension funds offer different kinds of benefits, on short and long term. Two of the benefits paid by the funds are commuted and monthly benefit. Commuted benefit is a part of pension payable as a lump sum at retirement. The monthly benefit is the part of pension converted as regular monthly payments after retirement on the condition of retiree survival. A member who attains an age of fifty-five years may at any time thereafter opt to retire, but if he does not, he may continue be working until the compulsory retirement age of sixty. To receive pension a member should have contributed for 15 years, that is, 180 months provided a member has met other conditions set in the enabling legislation of the respective fund and attained the retirement age. The current benefit formula was issued in 2014 by SSRA and aims to offer a commuted benefit of 25% of highest average final salary (average of the highest three salaries in the last 10 years preceding retirement) for all contributing months of a beneficiary paid at retirement. The monthly benefit aims to a benefit of 75% of average final salary

Table 1: Asset investment limits

Asset	Lower limit	Upper limit
Government Security	20%	70%
Real Estate	0%	30%
Loans	0%	20%
Fixed Deposit	0%	35%

for all contributing months, paid every month after retirement. This benefit aims at a minimum pension payable to members not less than 40% of the prescribed lowest minimum wage. The authority may where necessary set rates of indexation of members benefit to the current level of earnings of contributors.

The investment of pension funds had grown up to 7.8 trillion Tanzania shillings in $2014/15$. The Bank of Tanzania in consultation with SSRA issue guidelines regarding pension funds investment activities. These guidelines prescribe limits for investments in various asset categories to foster risk diversification and limit excessive concentration of risk. Pension funds may invest in the following asset categories: fixed deposits, government security, corporate bonds, loans to government, loans to corporate and cooperative societies, equities, property and licensed collective schemes. Since our ALM model is limited to a long-term strategic decision, a small set of asset categories should be sufficient, as recommended by Kouwenberg (2001). Therefore, we consider four asset categories only, which are government security, real estate, loans and fixed deposit. Government security is a low risk asset with high returns, real estate is a low risk asset with low return. Further, loans have high risk and high return while fixed deposit has low risk and high returns. The recent investment guidelines were issued by Bank of Tanzania in 2015, as shown in Table 1.

The pension fund may exceed the upper limits in the event of an increase in the market price of assets, reevaluation, bonus issues or transfer of investment from one category to another provided that, no new investment shall be done for those categories until such times, when the investments are restored to the limits prescribed in the guidelines. Such excess should be reported immediately to the Bank of Tanzania.

We consider these guidelines to be regulatory and not practical. We therefore modify some of the limits to make them more practical and suitable for modeling. Since loans have high risk, we decrease its upper bound to 10%. We increase the lower bound for real estate to 20% since it is a low risk asset. Also, pension funds need to participate into real estate activities that directly support their stakeholders (employees and employers). Table 2 displays the modified limits.
Table 3 gives the anticipated remaining life expectancy of members of Tanzania pension funds at different periods of time, by age and sex as given in Isaka (2016).

Table 2: Modified asset investment limits

Asset	Lower limit	Upper limit
Government Security	20%	70%
Real Estate	20%	30%
Loans	0%	10%
Fixed Deposit	0%	35%

Table 3: Remaining life expectancy of members at different time, by age and sex

	Men			Women		
Year	At 20	at 40	At 60	At 20	At 40	At 60
2013	54.6	37.5	20.8	55.7	39.1	22.2
2038	57.1	39.2	21.8	58.1	40.4	22.9
2063	59.7	41.0	22.9	61.2	42.5	24.2
2088	61.8	42.6	23.9	63.6	44.2	25.4

3. Asset liability management for pension funds by stochastic programming

Stochastic programming is an approach for modeling decision problems that involve parameters that are not known at the time of making decisions. In an application of stochastic programming, these uncertainties are modeled as random parameters in a discrete time model with a finite planning horizon (Dupačová and Polívka, 2009). Stochastic programming has been proven to be an efficient approach in designing effective strategies in wealth and asset liability management in practice (Hilli et al., 2007).

3.1. Multistage stochastic programming

In multistage stochastic programming, decisions x_t are taken in time stages $t = 1, \ldots, T$. The initial decision x_1 is followed by a random realization ξ_1, the next decision, x_2, is followed by the realization ξ_2, and so on. A multistage decision problem may allow a decision x_T at the terminal time such that

$$x_1 \rightarrow \xi_1 \rightarrow x_2 \rightarrow ... \rightarrow x_{T-1} \rightarrow \xi_{T-1} \rightarrow x_T \tag{1}$$

or terminate with the last observation ξ_T, such that

$$x_1 \rightarrow \xi_1 \rightarrow x_2 \rightarrow ... \rightarrow x_T \rightarrow \xi_T. \tag{2}$$

Basing on Shapiro et al. (2009), the sequence ξ_t for $t = 1, \ldots, T$ is a stochastic process, and we let $\xi_{[t]} = (\xi_1, \ldots, \xi_t)$ denote the information process up to time t. The decision x_t taken at stage t depends on the information data $\xi_{[t]}$, but not on the future realization, which is the nonanticipativity property, the basic requirement of stochastic programming .

The generic form of a T-stage stochastic programming problem can be written in a nested formulation as

$$\min_{x_1 \in \mathcal{X}_1} f_1(x_1) + \mathbb{E}\left[\min_{x_2 \in \mathcal{X}_2(x_1,\xi_2)} f_2(x_2,\xi_2) + \mathbb{E}\left[\cdots + \mathbb{E}\left[\min_{x_T \in \mathcal{X}_T(x_{T-1},\xi_T)} f_T(x_T,\xi_T)\right]\right]\right], \quad (3)$$

where $\mathbb{E}$ is the expectation operator, the function $f_1 : \mathbb{R}^{n_1} \to \mathbb{R}$ is continuous and deterministic and the set $\mathcal{X}_1 \subset \mathbb{R}^{n_1}$ is deterministic. Further, $x_t \in \mathbb{R}^{n_t}, t = 1, \ldots, T$, are decision variables and $f_t : \mathbb{R}^{n_t} \times \mathbb{R}^{m_t} \to \mathbb{R}$ are continuous functions at stages $t = 2, \ldots, T$. The multistage problem is linear if the objective functions and the constraint functions are linear.

The formulation which is often used in stochastic optimization models, is

$$\begin{aligned} &\min_{x_1,x_2,..,x_T} \mathbb{E}\left[f_1(x_1) + f_2(x_2(\xi_{[2]}),\xi_2) + \cdots + f_T(x_T(\xi_{[T]}),\xi_T)\right] \\ &\text{subject to} \quad x_1 \in \mathcal{X}_1, \\ &x_t(\xi_{[t]}) \in \mathcal{X}_t(x_{t-1}(\xi_{[t-1]}),\xi_t), \quad t = 2,\ldots,T. \end{aligned} \quad (4)$$

In this formulation the decision variable $x_t = x_t(\xi_{[t]}),\ t = 1, \ldots, T$, is considered as a function of the data process $\xi_{[t]}$ up to time t.

3.2. Scenario tree

In stochastic programming, the uncertainty of parameter values are described by a scenario tree. The scenario tree branches off every random parameter in each time stage. This approach requires a finite discrete distribution, that is, a limited number of possible values of the random parameters. According to Kouwenberg (2001), the performance of stochastic programming can be improved by choosing an appropriate scenario generation method.

Following Pflug and Pichler (2014), scenario trees are circle free directed graphs, with a unique root, for which the distance of all leaves nodes from the root is equal to $T - 1$. Scenario trees carry probability valuations on nodes and on arcs.

The tree consists of N nodes and for each node n, except the root, a predecessor pred(n) is defined. The nodes of the tree are dissected into a node set at each stage, called N_t such that

N_1 is the root node,
N_T are terminal nodes (leaves),
$N_t,\ t = 2, \ldots, T-1,$ are intermediate nodes (inner nodes).

For each $t = 2, \ldots, T$ and all $n \in N_t$, pred$(n) \in N_{t-1}$. A scenario s corresponds to path of nodes from the root node to a terminal node in N_T.

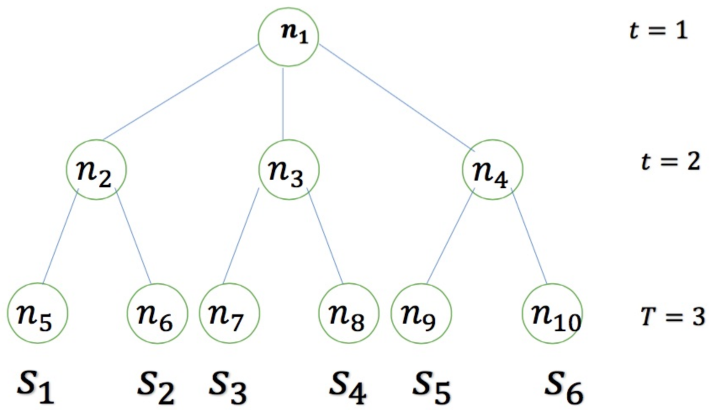


Fig. 1: Scenario tree for $T = 3$ stages, with 10 nodes, and 6 scenarios

Figure 1 shows an example of a scenario tree for a process explained above with $T = 3$ stages, 10 nodes, and 6 scenarios. Stages are presented by $t = 1, 2, 3$. The root node n_1 is the predecessor of nodes n_2, n_3 and n_4 at stage $t = 2$. Each of the nodes at this stage has two successors at stage $T = 3$. Nodes at stage $t = 2$ are predecessors of nodes at stage $T = 3$.

3.3. Application to asset liability management for pension funds

Asset liability management for a pension fund is a risk management approach, which takes into account the assets and liabilities, in reference to different restrictions from policies that can be applied. The main characteristic of financial institutions is the solvency, which is the ability to meet long-term obligations. The management of a pension fund should make a decision on acceptable policies that guarantee with a high likelihood that the solvency is sufficient during the planning horizon, which is up to several decades. In order to take sound decisions, the management needs to identify financial risks that may affect the solvency.

The solvency characterization differs from country to country. Some use the ratio of asset value to liability (funding ratio) to define the solvency (Dert, 1995). When the funding ratio equals 100%, it means that the value of the assets will exactly match the value of the liabilities. The ratio greater than 100% means over-funding, that is, the fund will have a surplus. But if it is less than 100%, the fund's assets value will not cover all liabilities. Other countries use defined financial levels to characterize the solvency (Hilli et al., 2007). These levels are used as an early warning system so that the institution and regulators can take actions before a possible bankruptcy.

Management of assets involve decisions on investments while the liability consists of future pension payments. The future assets returns, liabilities, streams of contributions and benefit payments are unknown at the time of making a decision. Stochastic programming models these uncertainties as random parameters in a discrete time model with a finite planning horizon consisting of time stages $t = 1, \ldots, T$.

At stage t a decision x_t, which is selling and buying of assets, is made. The decision is on the diversification of the current wealth of the fund among I asset categories. The major uncertainty for pension funds is the return of each asset investment at every time stage. Considering random asset return rates $r_{1t}, \ldots, r_{It}$ at stages $t = 1, \ldots, T-1$, a random process with a known distribution is formed.
The first stage is deterministic and a decision is made by specifying the transaction amounts $x_1 = (x_{11}, \ldots, x_{I1})$ in each asset. At stage t, the decision $x_t = (x_{1t}, \ldots, x_{It})$ is a function $x_t = x_t(\xi_{[t]})$ of the available information $\xi_{[t]} = (\xi_1, \ldots, \xi_t)$ of the data process up to stage t. The sequence of functions $x_t = x_t(\xi_{[t]}), \quad t = 2, \ldots, T-1$, defines the implementable policy of the decision process.

The objective may represent the expected utility while the scenario-wise objectives represent discounted sums of utility over time. Sometimes, a part of the objective may be interpreted as a penalty function. This is appropriate for the class of portfolio selection problems in which asset liability management falls. Such objectives tend to penalize the violation of goal constraints, such as goals for the solvency of the fund.

4. Problem formulation

We here present a multistage stochastic programming model for Tanzania pension funds. The sequence of decisions are made for a planning horizon of 50 years. The stages are indexed by $t = 0, \ldots, T$, where $t = 0$ denotes the present time and $t = T$ is the end of the planning horizon. Random realizations at stages are represented by nodes of a scenario tree. We denote by N_t the number of nodes of the scenario tree in stage t. The predecessor of the node n is denoted by $\hat{n}$.

There are I asset categories indexed by i. The definitions of parameters and variables are given below.

Deterministic parameters

X_i^{ini}	Initial amount held in asset i
m^{ini}	Initial cash position
$cr^{lo/up}$	Lower/upper bound for contribution rate
$dcr^{lo/up}$	Lower/upper bound for the decrease/increase of contribution rate
F^{min}	Minimum funding ratio
F^{end}	Required funding ratio at the end of the planning horizon
lb/ub	Lower/upper bound for portion of asset mix
t_i^p/t_i^s	Transaction cost for purchasing/selling of asset category i
λ	Positive parameter for specifying risk aversion

Random parameters

B_{tn}	Benefit payment at node n of stage t
L_{tn}	Liabilities at node n of stage t
S_{tn}	Total salaries of members at node n of stage t
r_{itn}	Rate of return of asset category i at node n of stage t

Decision variables

X_{itn}^h	Amount held in asset category i at node n of stage t
X_{itn}^p	Amount purchased of asset category i at node n of stage t
X_{itn}^s	Amount sold of asset category i at node n of stage t
A_{tn}	Asset value at node n of stage t
cr_{tn}	Contribution rate at node n of stage t
cr_n^{end}	Contribution rate at node n of the end of the horizon, needed to lift the funding ratio to F^{end}
Z_{tn}	Deficit relative to the minimum funding ratio F^{min} at node n of stage t

4.1. Asset inventory constraints

These constraints describe the dynamic change of amount in assets investment in each stage. No rebalance is made (no decision is taken) at the end of the horizon.

$$X_{i01}^h = X_i^{ini} + X_{i01}^p - X_{i01}^s, \quad i = 1, \dots, I \tag{5}$$

$$X_{itn}^h = (1 + r_{itn})X_{i,t-1,\hat{n}}^h + X_{itn}^p - X_{itn}^s, \\ n = 1, \dots, N_t, \; t = 1, \dots, T-1, \; i = 1, \dots, I \tag{6}$$

Here, equation (5) describes the initial amount invested at stage $t = 0$.

4.2. Cash balance constraints

These constraints specify that the cash inflow is equal to the cash outflow. There are two sources of cash inflow, which are contributions from members and selling of assets. Cash outflow is the benefit paid to retirees and purchasing of assets. Transaction costs for selling and purchasing of assets are incorporated. Borrowing and lending variables are not included since there is a loan asset. There is no cash balance at the end of the horizon.

$$cr_{01}S_{01} + m^{ini} + \sum_{i=1}^{I}(1 - t_i^s)X_{i01}^s - B_{01} - \sum_{i=1}^{I}(1 + t_i^p)X_{i01}^p = 0 \tag{7}$$

$$cr_{tn}S_{tn} + \sum_{i=1}^{I}(1 - t_i^s)X_{itn}^s - B_{tn} - \sum_{i=1}^{I}(1 + t_i^p)X_{itn}^p = 0,$$
$$n = 1, \ldots, N_t, \; t = 1, \ldots, T-1 \tag{8}$$

4.3. Total asset value

At the end of each stage, the fund should measure its asset value, which is used to measure deficit and the solvency. Asset value is the value of previous period asset holdings and the returns of each asset in the current stage.

$$A_{tn} = \sum_{i=1}^{I}(1 + r_{itn})X_{i,t-1,\hat{n}}^h, \; n = 1, \ldots, N_t, \; t = 1, \ldots, T \tag{9}$$

4.4. Goal constraints

The pension fund sets a minimum funding ratio F^{min} and defines it as a goal. At the end of each stage, the deficit is measured. When the funding ratio is less than the minimum funding ratio then the deficit is penalized in the objective function. To assure a final wealth of the pension fund, the funding ratio is restored to the target level F^{end} at the end of the planning horizon by setting the contribution rate to cr_n^{end}.

$$A_{tn} \geq F^{min}L_t - Z_{tn}, \; n = 1, \ldots, N_t, \; t = 1, \ldots, T-1 \tag{10}$$
$$A_{Tn} \geq F^{end}L_T - cr_n^{end}S_{Tn}, \; n = 1, \ldots, N_T \tag{11}$$
$$Z_{tn} \geq 0, \; n = 1, \ldots, N_t, \; t = 1, \ldots, T-1$$
$$cr_n^{end} \geq 0, \; n = 1, \ldots, N_T \tag{12}$$

4.5. Contribution rates constraints

The level and change of contribution rate are bounded.

$$cr^{lo} \leq cr_{tn} \leq cr^{up}, \; n = 1, \ldots, N_t, \; t = 0, \ldots, T-1 \tag{13}$$
$$dcr^{lo} \leq cr_{tn} - cr_{t-1,\hat{n}} \leq dcr^{up}, \; n = 1, \ldots, N_t, \; t = 1, \ldots, T-1 \tag{14}$$

4.6. Asset weight mix boundaries

The asset weight mix is bounded. Bank of Tanzania gives asset mix limits through specified investment guidelines as described in the Table 1.

$$lb\sum_{i=1}^{I} X_{itn}^{h} \leq X_{itn}^{h} \leq ub\sum_{i=1}^{I} X_{itn}^{h}, \quad n=1,\dots,N_t, \ t=0,\dots,T-1, \ i=1,\dots,I \tag{15}$$

4.7. Objective

We use the objective function of Kouwenberg (2001), which is to minimize the sum of the average contributions rates, while taking into account the risk aversion of the pension fund and the state of the fund at the end of the planning horizon. Risk aversion is modeled with a quadratic penalty on deficits Z_{tn}.

$$\min \sum_{t=0}^{T-1}\left(\sum_{n=1}^{N_t}\frac{cr_{tn}}{N_t}\right) + \lambda\sum_{t=1}^{T}\sum_{n=1}^{N_t}\frac{1}{N_t}\left(\frac{Z_{tn}}{L_{tn}}\right)^2 + \sum_{n=1}^{N_T}\frac{cr_n^{end}}{N_T} \tag{16}$$

Here, λ is the positive risk aversion penalty parameter.

5. Scenario tree generation

5.1. Asset returns scenarios

Asset returns scenarios provide the information about possible future returns of assets. Each asset scenario should also include a salary increase in order to transform the real expected values of the benefits and liabilities into nominal values. Salaries are the rates of change of GDP per capita of working population as described by the World Bank. To model asset returns , a vector autoregressive model (VAR) is used as discussed in Kouwenberg (2001).

$$y_t = v + Ay_{t-1} + u_t, \ t=1,\dots,T \tag{17}$$

$$y_{it} = \ln(1+r_{it}), \ t=1,\dots,T, \ i=1,\dots,I \tag{18}$$

Here r_{it} is the rate of return of asset i in stage t. The returns of each asset are transformed to $\ln(1+r_{it})$ to avoid heteroscedasticity problems. Further, $y_t = (y_{1t},\dots,y_{It})^T$ is an $I\times 1$ random vector of continuously compounded rates, A is a fixed $I\times I$ matrix of coefficients, $v=(v_1,\dots,v_I)^T$ is a fixed $I\times 1$ vector of intercept terms allowing of nonzero mean $E(y_t)$. Finally $u_t=(u_{1t},\dots,u_{It})^T$ is an I-dimensional white noise with $E(u_t)=0$, $E(u_tu_t^T)=\Sigma_u$ and $E(u_tu_s^T)=0$ for $s\neq t$. The covariance matrix Σ_u is assumed to be nonsingular.

To include the number of years between stages, the following relation is incorporated.

$$r_{it} = (1+\beta_i)^{\tau} - 1 + \overline{r}_{it}\sigma_i\sqrt{\tau} \tag{19}$$

Here, $\overline{r}_{it}$ is the rate of return produced by the vector autoregressive model, β_i is the mean return of asset i, σ_i is the standard deviation of the rate of return of asset i, and τ is the number of years between stages.

5.2. Liability scenarios

Liabilities depend on expected future benefits. When a member makes a contribution, an expected benefit to be paid in future is created. Tanzania pension funds pay different kinds of pension benefits. These include maternity, gratuity, education, death gratuity, commuted pension and monthly pensions. In our work, only commuted and monthly benefits are considered and other benefits are ignored. This simplification is due to the fact that other benefits are significantly smaller, and it is optimistic with respect to the fund's sustainability. Commuted pension is a part of benefit paid as a lump sum at retirement while monthly pension is a part of benefit paid in terms of regular payments every month from retirement until the death of the retiree. These two benefits depend on the number of months that a member has been contributing to the fund and a retiree's average annual earnings in the best three of the last 10 years preceding retirement.

To receive pension benefit a member should contribute for at least 180 months and reach an age of retirement. In 2014 SSRA issued the harmonization rule formula for commuted and monthly benefits (SSRA, 2014). The commuted benefit, denoted CB, is given by

$$CB = \frac{1}{580} \times m \times S_{fin} \times 12.5 \times 25\%, \tag{20}$$

where m is the number of months a participant has been contributing to the fund, S_{fin} is the average final salary, $\frac{1}{580}$ is the annual accrual factor, 12.5 is the commutation factor at retirement, and 25% is the commutation rate of the annual full amount of the pension.

A monthly pension, denoted MB, is given by

$$MB = \frac{1}{580} \times m \times S_{fin} \times 75\% \times \frac{1}{12}, \tag{21}$$

where 75% is the commutation rate of the annual full amount of the pension. In our calculations, the monthly pension is regularly revised to follow the growth in average salary for the working population. Also, MB is converted to annual benefit instead of monthly.

In our calculations, a final annual average salary of a member aged k years in year t is obtained from

$$S_{fin_{tk}} = S_t \times (1 + d)^{60-k}. \tag{22}$$

Here, d is the annual salary growth rate and S_t is the annual average salary in year t.

The total expected commuted benefit in year t for members of age k is

$$CB_{tk} = P_t^{60-k} \times n_k \times \frac{1}{580} \times m_k \times S_{fin_{tk}} \times 12.5 \times 25\%, \tag{23}$$

where P_t^{60-k} is the probability of a member aged k years in year t to live $60-k$ years more, that is, until retirement, n_k is the number of members aged k years in year t, and m_k is the average number of months that members of age k have contributed.

Total expected yearly benefit in year t for members aged k is

$$MB_{tk} = P_t^{60-k} \times n_k \times \frac{1}{580} \times m_k \times S_{fin_{tk}} \times 75\% \times Ep_{60_{tk}}, \tag{24}$$

where $Ep_{60_{tk}}$ is the remaining life expectancy in years in year t for a member aged k years, when he reaches the age of sixty, as shown in Table 3.

The total expected benefit B_{tk} in year t for members of age k is

$$B_{tk} = CB_{tk} + MB_{tk}.$$

Liability is the discounted present value of expected total benefit. The total liability in year t is therefore given by

$$L_t = \sum_{k=20}^{59} \frac{B_{tk}}{(1+r)^{60-k}}, \tag{25}$$

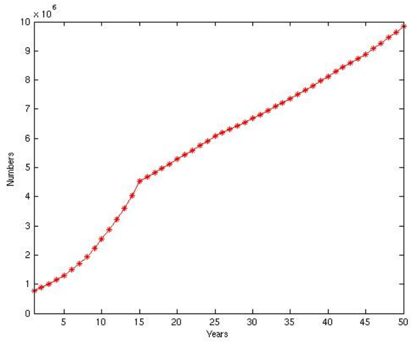
where $r = 5\%$ is a discount factor. This factor is in line with other assumptions and modeling used by World Bank economists (PolicyNote, 2014). In our calculations, we assume that initial members had been creating liabilities before the start of the horizon.

6. Numerical results

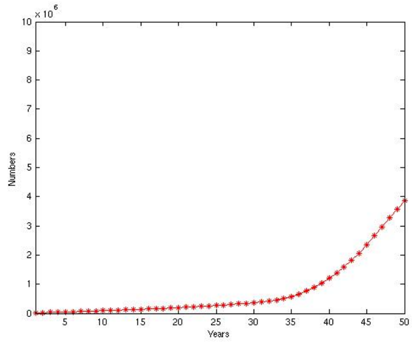
Basing on John et al. (2017) and assumptions therein, we project the future status of the members, that is, how many will survive, retire and being paid commuted and monthly pensions. We update the status of each member year by year according to predicted mortality rates before retirement age and expected lifetime thereafter. This projection shows that the fund is facing a mass increase of retirees in the long future. The amount of contributions will not cover benefit payouts.

The projected number of members in future 50 years is shown in the Figure 2a. The projection shows a fast growth in number of members for the first 15 years, to reach around 20% of the working population, and then the number of members grows slower.

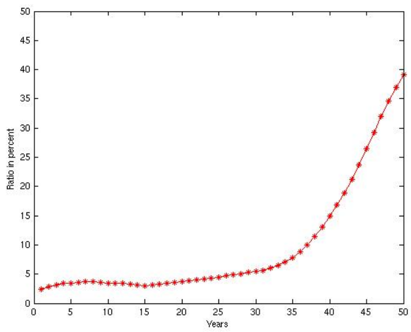
Figure 2b shows that the number of retirees grows slowly in the beginning of the time horizon and then grows fast after 35 years. This will cause an increase of the retirees to contributors dependency ratio, as shown in the Figure 2c. The dependency ratio starts in the beginning year at around 2% and increases to around 39%, which is very high, at the end of the horizon. This is an adverse situation since the



(a) Members growth



(b) Retirees growth



(c) Retirees to contributors ratio

Fig. 2: Projected population

contributions from a 100 members is by far not sufficient to pay benefit payouts for 39 retirees.

We use the vector autoregressive model as described in Section 5.1 to model stochastic factors and then this model is in turn used for generating data for the scenario tree. The asset data from NSSF annual reports of 2001 to 2011 are used. Assets considered in this study are government security (Gs), real estate (Re), loans (Lo) and fixed deposit (Fd). Further, each scenario includes salary (Sa) growth, which is based on the yearly rate of changes of GDP per capita of working population for 2001 to 2011. Table 4 displays the means and standard deviations of the asset returns and the salary, while Table 5 displays their correlation matrix.

To avoid any problem with spurious and unstable asset returns, lagged variables are not used for the real estate, loans and fixed deposit assets. Estimation of vector autoregressive coefficients is done by using a least square method as discussed by Drijver (2005) and Dert (1995). The result is displayed in Table 6 while Table 7 displays the estimated correlation matrix of the residuals.

Monte Carlo simulation together with Cholesky decomposition are used to generate a scenario tree for the stochastic programming model. Cholesky decomposition

Table 4: Means and standard deviations

	Mean	Std. Dev
Sa	0.0377	0.0112
Gs	0.0919	0.0164
Re	0.0346	0.0080
Lo	0.0647	0.0151
Fd	0.0665	0.0233

Table 5: Correlations

	Sa	Gs	Re	Lo	Fd
Sa	1				
Gs	-0.0159	1			
Re	0.3700	0.2220	1		
Lo	-0.5090	-0.0139	-0.5330	1	
Fd	0.2990	0.2780	0.5400	-0.5040	1

Table 6: Coefficients of the vector autoregressive model

Sa	$\ln(1+\text{Sa}_t) = 0.0381 + 0.0651\times \ln(1+\text{Sa}_{t-1}) + e_{1t}$
Gs	$\ln(1+\text{Gs}_t) = 0.0885 + 0.0189\times\ln(1+\text{Gs}_{t-1}) + e_{2t}$
Re	$\ln(1+\text{Re}_t) = 0.0212 + e_{3t}$
Lo	$\ln(1+\text{Lo}_t) = 0.0769 + e_{4t}$
Fd	$\ln(1+\text{Fd}_t) = 0.0493 + e_{5t}$

Table 7: Residual correlations for vector autoregressive model.

	Sa	Gs	Re	Lo	Fd
Sa	1				
Gs	-0.1799	1			
Re	-0.2261	0.1369	1		
Lo	-0.0899	-0.3379	-0.1589	1	
Fd	0.1971	0.2781	0.5028	-0.3596	1

is applied to preserve the covariance structure of rate of asset returns. Future returns are estimated using equations (17), (18) and (19). The number of years between stages is $2, 3, 5, 10, 10, 10$, and 10 years respectively. The stages contain a tree structure with $1, 20, 5, 5, 2, 2, 2$, and 2 nodes, which gives 8000 scenarios. For each node a different random vector of distribution of residual is used to generate

Table 8: Deterministic parameters

cr^{lo}	cr^{up}	dcr^{lo}	dcr^{up}	F^{min}	F^{end}	B_{01}	t_i^p/t_i^s	λ	m^{ini}	S_{01}
0.10	0.20	-0.02	0.02	80%	100%	358	0.02	4	3500	4994

the scenario for the asset returns.

To generate the scenario tree for benefit payouts and liabilities, we use the projected fund population displayed in the Figures 2a and 2b. To get the tree shaped structure, needed for stochastic programming, we use the salary tree, to generate benefit payouts and liability trees. The stochastic programming model is solved by the AMPL/Cplex package. It is assumed that the fund has an initial cash position and holds assets in all categories. The deterministic parameters are given in the Table 8. Cash are in billions Tanzanian shillings.

6.1. First case: using current investment limits

We start by using the investment limit values as specified in Table 1. We set the funding ratio at the end of the time horizon F^{end} to 100%, which means that the asset value will match liabilities exactly.

First stage decision asset mix:

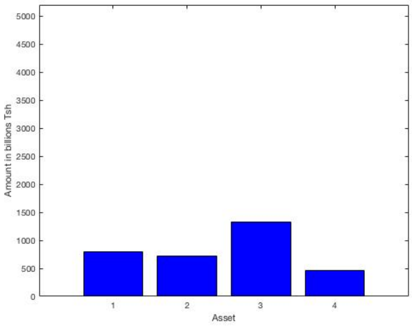
In order to attain its goal, the fund has to make first stage decisions. We use the initial asset holding shown in the Figure 3a, in which loans holding is beyond the limit (see Table 1). This is allowed by Tanzania pension system with some conditions as stated in Section 2. As shown in the Figure 3b, the model makes a decision of selling all real estate assets, which have low risk but also have low return compared to other assets. Also, the model decides to buy more government security and fixed deposit, which have low risk but high returns, as displayed on Figure 3c, while holding the initial value of loans. This means that, the first stage decision on asset allocation is to hold government security, loans, fixed deposit as shown in the Figure 3d.

Average contribution rates:

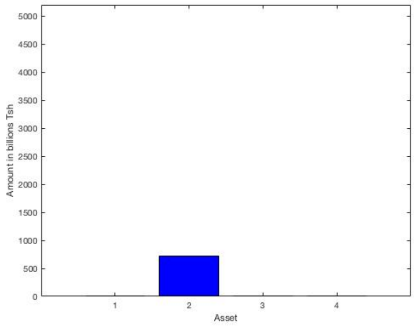
The average contribution rate is 10%, which is the minimal possible, for all stages before the end of the time horizon. At the horizon the fund needs no contribution to raise the funding ratio to F^{end}, as shown in the Figure 4.

Average asset values:

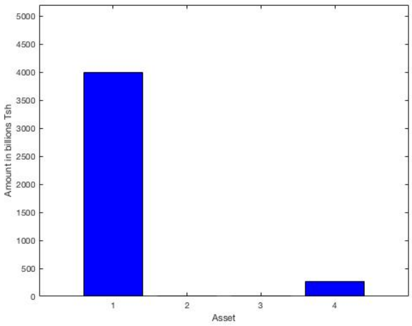
The Figure 5 shows that, the average asset values grow across the time horizon with an average annual return rate of 8.5%.



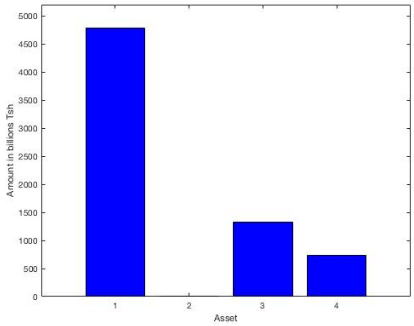
(a) Initial holdings



(b) Asset sold



(c) Asset bought



(d) Asset hold

Fig. 3: First stage decision on asset allocation

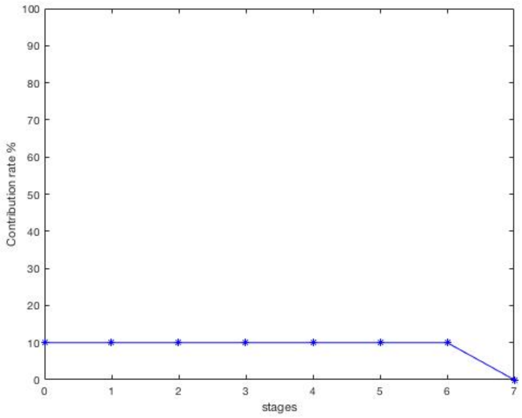


Fig. 4: Average contribution rates

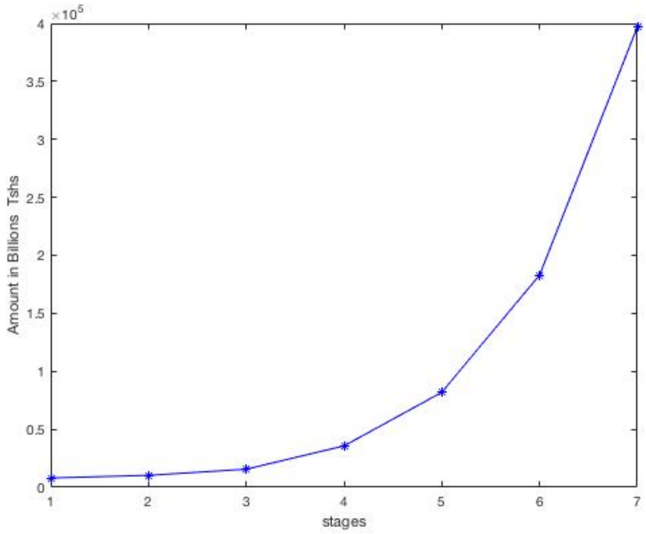


Fig. 5: Average asset values

6.2. Second case: using the modified investment limits

We here use modified investment limits as specified in the Table 2. But we still consider the assumptions for contribution rate, asset holdings and funding ratio as in the first case.

First stage decision asset mix:

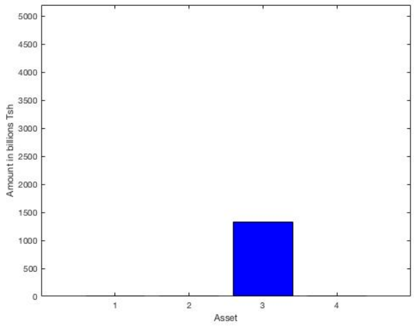
The model makes the first stage decision of selling all of the high risk asset loans, as shown in the Figure 6a. Also, the model decides to buy more government security, real estate and fixed deposit, which are low risk assets, as displayed on Figure 6b. This means that, the first stage decision on asset allocation is to hold government security, real estate and fixed deposit, as shown in the Figure 6c.

Average contribution rates:

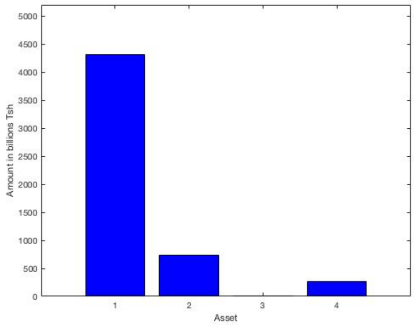
The average contribution starts at a rate of 20%. At the horizon the average final contribution rate to raise the funding ratio to F^{end} is 28%, as shown by the Figure 7.

Average asset values:

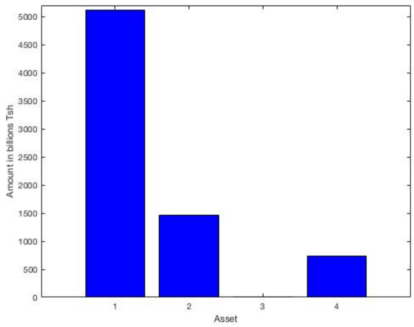
The Figure 8 shows that, average asset values grow across the horizon with an average annual return rate of 8.2%.



(a) Asset sold



(b) Asset bought



(c) Asset hold

Fig. 6: First stage decision on asset allocation

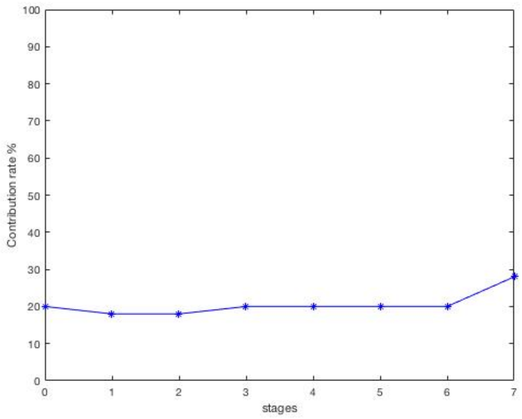


Fig. 7: Average contribution rates

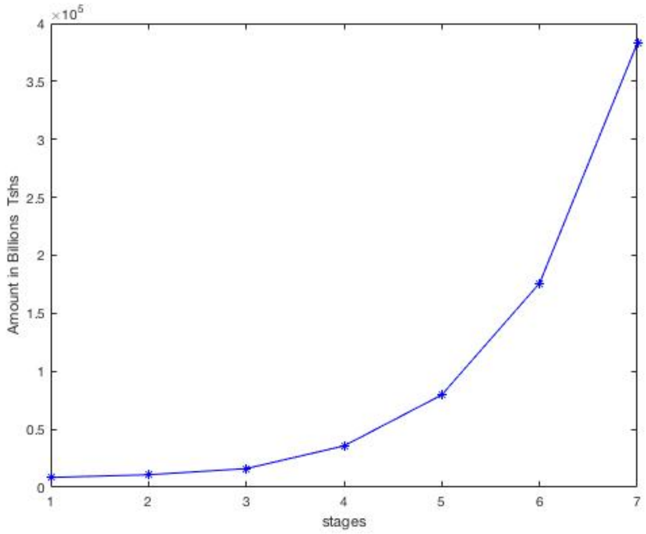


Fig. 8: Average asset values

6.3. Funding ratio

Using the modified investment limits specified in Table 2, we study how the funding ratio Fr_t at stage t of the fund changes. We use the relation

$$Fr_t = \overline{A}_t / \overline{L}_t, \tag{26}$$

where $\overline{A}_t$ and $\overline{L}_t$ are average values for asset values and liabilities, respectively, at stage t. The Figure 9 shows that the funding ratio starts by decreasing fast while towards the end of the horizon, the funding ratio raises towards the target funding ratio F^{end} of 100%.

6.4. The difference between contributions and benefit payouts

Since this fund is pay-as-you-go, it is of interest to study the difference between contributions and benefit payouts throughout the stages. Basing on the case of modified investment limits, according to Table 2, we use the equation (8) to study the changes in this difference.

$$cr_{tn}S_{tn} + \sum_{i=1}^{I}(1-t_i^s)X_{itn}^s - B_{tn} - \sum_{i=1}^{I}(1+t_i^p)X_{itn}^p = 0,$$
$$n = 1,\dots,N_t,\ t = 1,\dots,T-1.$$

If

$$cr_{tn}S_{tn} - B_{tn} < 0 \tag{27}$$

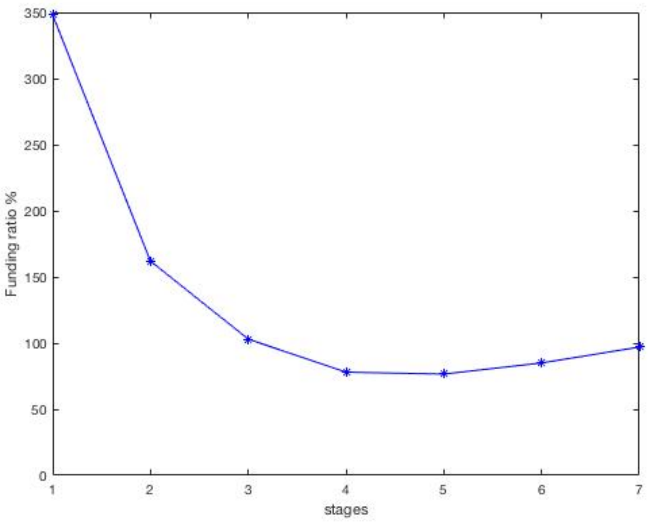


Fig. 9: Funding ratio

holds, then the fund should sell assets according to the terms $(1-t_i^s)X_{itn}^s$, to pay benefit payouts rather than buying assets, according to the terms $(1+t_i^s)X_{itn}^p$, for investment.

Let the average difference between contributions and benefit payouts at stage t be dc_t, then

$$dc_t = \overline{cr_t S_t} - \overline{B}_t, \quad (28)$$

where $\overline{cr_t S_t}$ is the average contribution at stage t and $\overline{B}_t$ is the average benefit at stage t. Figure 10 shows that, this difference is first increasing, but in the last stage, it decreases to a large negative value. This means that, at the horizon the fund uses asset values to pay benefit payouts.

6.5. Increasing contribution rate

We here study the effect of increasing contribution rates using the case of modified investment limits, according to Table 2. We allow the upper bound of the contribution rate cr^{up} to increase to 25%. As shown by Figure 11, the initial contribution rates are then higher while the rates at stages 5 and 6 are lower, than when the maximal contribution rate is 20%.
Considering the Figure 12, the difference between contributions and benefit payouts is slightly higher compared to when the maximal contribution rate is 20%. But still at the horizon this value becomes very negative. Hence, before the end of the horizon, the fund makes higher surplus which is invested to improve the asset values, but towards the end of the horizon assets are instead consumed.

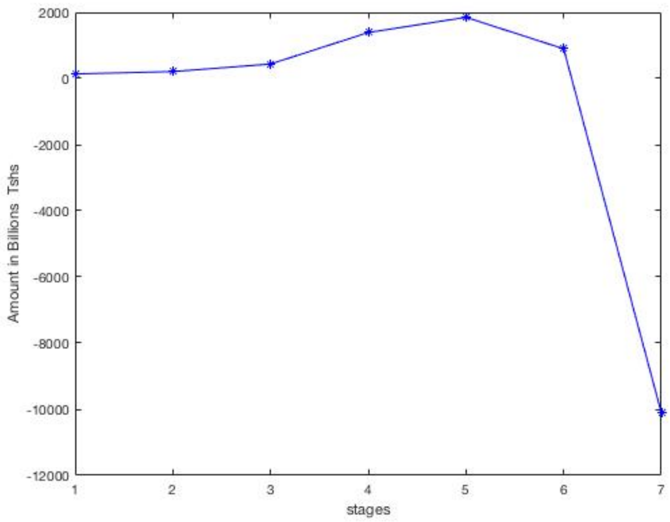


Fig. 10: Difference between contributions and benefit payouts

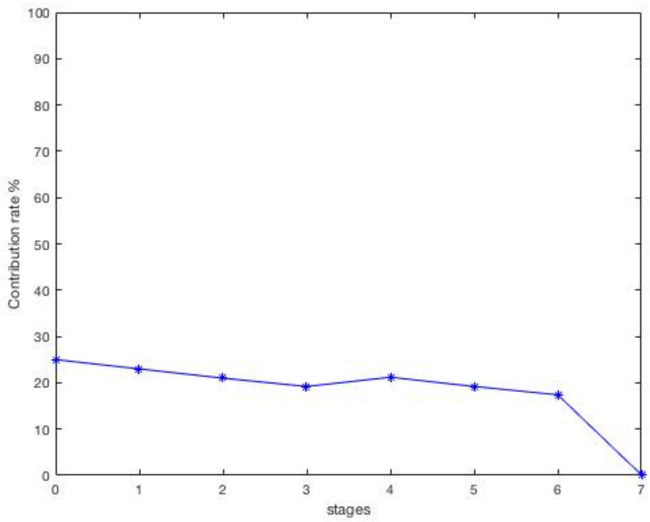


Fig. 11: Average contribution rates

6.6. The effect of funding ratio on fund's sustainability

We study the effect of changes in funding ratios for the modified investment limits case. When the ratios F^{min} and F^{end} are both 80%, average contribution rates behave as in the Figure 4. The difference between contributions and benefit payouts

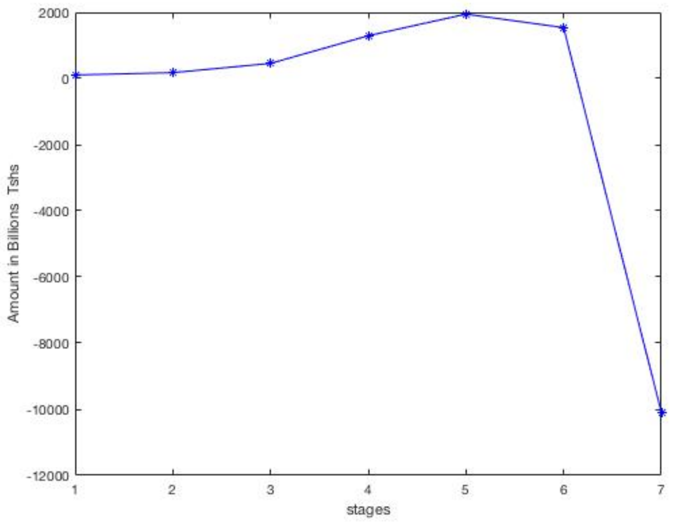


Fig. 12: Difference between contributions and benefit payouts

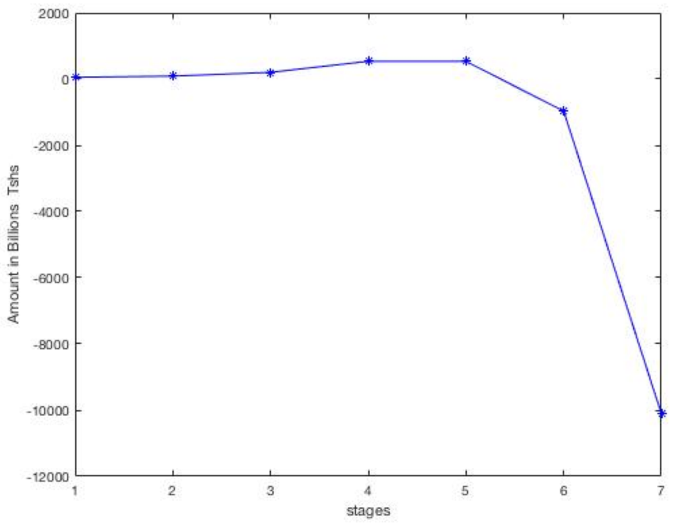


Fig. 13: Difference between contributions and benefit payouts

grows very slowly and then decreases to very negative values after stage 5, which is earlier than in Figures 10 and 12 . This means that by setting a low funding ratio, contributions will be low, and the fund must use its asset value for the last 20 years to pay benefit payouts, which will cause assets to deplete.

7. Conclusion

We present an asset liability management model for Tanzania pension fund by stochastic programming. The pension system is a pay-as-you-go defined benefit where the current benefits are paid by using contributions from current members of the fund. This system is an inter-generation contract which is largely affected by changes in demography. It is shown that, as the old age population increases much in relation to the number of contributing members, the pension payments exceed the contributions, and the asset value is used to pay benefits instead of being investing for covering future benefit payments and liabilities.

Our study suggests that in order to improve the long-term sustainability of the Tanzania pension fund system, it is necessary to make reforms concerning contribution rates and investment guidelines. Also the authority should formulate conditions for the solvency of funds.

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