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THE UNIVALENCE CONDITIONS FOR A FAMILY OF INTEGRAL OPERATORS

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ABSTRACT. The main object of the present paper is to discuss some univalence conditions for a family of integral operators. Several other closely-related results are also considered.

1. INTRODUCTION AND PRELIMINARIES

Let $\mathcal{U}=\{z \in \mathbb{C} : |z| < 1\}$ be the open unit disk and $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

which are analytic in $\mathcal{U}$ and satisfy the following usual normalization condition

$$f(0) = f'(0) - 1 = 0.$$

Also, let $\mathcal{S}$ denote the subclass of $\mathcal{A}$ consisting of functions $f(z)$ which are univalent in $\mathcal{U}$.
We begin by recalling a theorem dealing with a univalence criterion, which will be required in our present work.
In [1], Pascu gave the following univalence criterion for the functions $f \in \mathcal{A}$.

Theorem 1.1. (Pascu [1]). *Let $f \in \mathcal{A}$ and $\beta \in \mathbb{C}$. If $\mathrm{Re}(\beta) > 0$ and*

$$\frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)}\left|\frac{zf''(z)}{f'(z)}\right| \leq 1 \, (z \in \mathcal{U})$$

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then the function $F_\beta(z)$ *given by*

$$F_\beta(z) = \left(\beta \int_0^z t^{\beta-1} f'(t) dt\right)^{\frac{1}{\beta}}$$

is in the univalent function class $\mathcal{S}$ *in* $\mathcal{U}$.

In this paper, we consider three general families of integral operators. The first family of integral operators, studied by Breaz and Breaz [2], is defined as follows:

$$F_{\alpha_1,\alpha_2,\cdots,\alpha_n,\beta}(z) = \left(\beta \int_0^z t^{\beta-1} \prod_{i=1}^{n} \left(\frac{f_i(t)}{t}\right)^{\frac{1}{\alpha_i}} dt\right)^{\frac{1}{\beta}} \tag{1.1}$$

The second family of integral operators was introduced by Breaz and Breaz [3] and it has the following form:

$$G_{\alpha_1,\alpha_2,\cdots,\alpha_n}(z) = \left(\left(\sum_{i=1}^{n} (\alpha_i - 1) + 1\right) \int_0^z \prod_{i=1}^{n} (f_i(t))^{\alpha_i - 1} dt\right)^{\frac{1}{\left(\sum_{i=1}^{n}(\alpha_i-1)+1\right)}} \tag{1.2}$$

Finally, Breaz and Breaz [4] considered the following family of integral operators

$$H_{\alpha_1,\alpha_2,\cdots,\alpha_n,\beta}(z) = \left(\beta \int_0^z t^{\beta-1} \prod_{i=1}^{n} \left(\frac{f_i(t)}{t}\right)^{\alpha_i} dt\right)^{\frac{1}{\beta}} \tag{1.3}$$

In the present paper, we propose to investigate further univalence conditions involving the general families of integral operators defined by (1.1), (1.2) and (1.3).

2. Main results

Theorem 2.1. *Let the functions* $f_i \in \mathcal{A}, (i \in \{1, \cdots, n\})$ *and* α_i, β *be complex numbers with* $\mathrm{Re}(\beta) \geq 0$ *for all* $i \in \{1, 2, \cdots, n\}$. *If*

$$(a). \quad 4\sum_{i=1}^{n} \frac{1}{|\alpha_i|} \leq \mathrm{Re}(\beta), \quad \mathrm{Re}(\beta) \in (0, 1)$$

or

$$(b). \quad \sum_{i=1}^{n} \frac{1}{|\alpha_i|} \leq \frac{1}{4}, \quad \mathrm{Re}(\beta) \in [1, \infty)$$

then the function $F_{\alpha_1,\alpha_2,\cdots,\alpha_n,\beta}(z)$ *defined by (1.1) is in the class* $\mathcal{S}$.

Proof. We define the function

$$h(z) = \int_0^z \prod_{i=1}^{n} \left(\frac{f_i(t)}{t}\right)^{\frac{1}{\alpha_i}} dt. \tag{2.1}$$

Because $f_i \in \mathcal{S}$, for $i \in \{1, 2, \cdots, n\}$, we have

$$\left|\frac{z f_i'(z)}{f_i(z)}\right| \leq \frac{1 + |z|}{1 - |z|} \tag{2.2}$$

for all $z \in \mathcal{U}$.
Now, we calculate for $h(z)$ the derivates of the first and second order.
From (2.1) we obtain

$$h'(z) = \prod_{i=1}^{n} \left(\frac{f_i(z)}{z} \right)^{\frac{1}{\alpha_i}}$$

and

$$h''(z) = \frac{1}{\alpha_i} \sum_{i=1}^{n} \left(\frac{f_i(z)}{z} \right)^{\frac{1-\alpha_i}{\alpha_i}} \left(\frac{z f_i'(z) - f_i(z)}{z^2} \right) \prod_{\substack{k=1 \\ k \neq i}}^{n} \left(\frac{f_k(z)}{z} \right)^{\frac{1}{\alpha_i}}.$$

After the calculus we obtain that

$$\frac{zh''(z)}{h'(z)} = \sum_{i=1}^{n} \frac{1}{\alpha_i} \left(\frac{z f_i'(z)}{f_i(z)} - 1 \right)$$

which readily shows that

$$\begin{aligned} \frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| &= \frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left| \sum_{i=1}^{n} \frac{1}{\alpha_i} \left(\frac{z f_i'(z)}{f_i(z)} - 1 \right) \right| \\ &\leq \frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \sum_{i=1}^{n} \frac{1}{|\alpha_i|} \left| \frac{z f_i'(z)}{f_i(z)} - 1 \right| \\ &\leq \frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \sum_{i=1}^{n} \frac{1}{|\alpha_i|} \left(\left| \frac{z f_i'(z)}{f_i(z)} \right| + 1 \right). \end{aligned} \tag{2.3}$$

From (2.2) and (2.3) we obtain

$$\begin{aligned} \frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| &\leq \frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \sum_{i=1}^{n} \frac{1}{|\alpha_i|} \left(\frac{1+|z|}{1-|z|} + 1 \right) \\ &\leq \frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \sum_{i=1}^{n} \frac{1}{|\alpha_i|} \left(\frac{2}{1-|z|} \right) \\ &\leq \frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left(\frac{2}{1-|z|} \right) \sum_{i=1}^{n} \frac{1}{|\alpha_i|}. \end{aligned} \tag{2.4}$$

Now, we consider the cases

$$i1). \quad 0 < \mathrm{Re}(\beta) < 1.$$

We have

$$1 - |z|^{2\mathrm{Re}(\beta)} \leq 1 - |z|^2 \tag{2.5}$$

for all $z \in \mathcal{U}$.
From (2.4) and (2.5), we have

$$\frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| \leq \frac{1-|z|^2}{\mathrm{Re}(\beta)} \left(\frac{2}{1-|z|} \right) \sum_{i=1}^{n} \frac{1}{|\alpha_i|}$$

$$\leq \frac{4}{\mathrm{Re}(\beta)} \sum_{i=1}^{n} \frac{1}{|\alpha_i|}. \tag{2.6}$$

Using the condition (a). and (2.6) we have

$$\frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| \leq 1 \tag{2.7}$$

for all $z \in \mathcal{U}$.

$$i2). \quad \mathrm{Re}(\beta) \geq 1.$$

We have

$$\frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \leq 1-|z|^2 \tag{2.8}$$

for all $z \in \mathcal{U}$.
From (2.8) and (2.4) we have

$$\frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| \leq \left(1-|z|^2\right) \left(\frac{2}{(1-|z|)} \right) \sum_{i=1}^{n} \frac{1}{|\alpha_i|}$$

$$\leq 4 \sum_{i=1}^{n} \frac{1}{|\alpha_i|}. \tag{2.9}$$

Using the condition (b). and (2.9) we obtain

$$\frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| \leq 1 \tag{2.10}$$

for all $z \in \mathcal{U}$.
From (2.7) and (2.10) we obtain that the function $F_{\alpha_1,\alpha_2,\cdots,\alpha_n,\beta}(z)$ defined by (1.1) is in the class $\mathcal{S}$.

□

Setting $n = 1$ in Theorem 2.1 we have

Corollary 2.2. *Let $f \in \mathcal{A}$ and α, β be complex numbers with $\mathrm{Re}(\beta) \geq 0$. If*

$$(a). \quad \frac{4}{|\alpha|} \leq \mathrm{Re}(\beta), \quad \mathrm{Re}(\beta) \in (0,1)$$

or

$$(b). \quad \frac{1}{|\alpha|} \leq \frac{1}{4}, \quad \mathrm{Re}(\beta) \in [1,2]$$

then the function

$$F(z) = \left(\beta \int_0^z t^{\beta-1} \left(\frac{f(t)}{t} \right)^{\frac{1}{\alpha}} dt \right)^{\frac{1}{\beta}}$$

is in the class $\mathcal{S}$.

Theorem 2.3. *Let the functions* $f_i \in \mathcal{A}\left(i \in \{1, 2, \cdots, n\}\right)$, β, α_i *be complex numbers for all* $i \in \{1, 2, \cdots, n\}$, $\beta = \left(\sum_{i=1}^{n} (\alpha_i - 1) + 1\right)$ *and* $\mathrm{Re}(\beta) \geq 0$. *If*

$$(a). \quad 4\sum_{i=1}^{n} |\alpha_i - 1| \leq \mathrm{Re}(\beta), \quad \mathrm{Re}(\beta) \in (0, 1)$$

or

$$(b). \quad \sum_{i=1}^{n} |\alpha_i - 1| \leq \frac{1}{4}, \quad \mathrm{Re}(\beta) \in [1, \infty)$$

then the function $G_{\alpha_1, \alpha_2}, \cdots, {}_{\alpha_n}(z)$ *defined by* 1.2 *is in the class* $\mathcal{S}$.

Proof. Defining the function $h(z)$ by

$$h(z) = \int_0^z \prod_{i=1}^{n} \left(\frac{f_i(t)}{t}\right)^{\alpha_i - 1} dt$$

we take the same steps as in the proof. of Theorem 2.1.
Then, we obtain that

$$\begin{aligned} \frac{1 - |z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left|\frac{zh''(z)}{h'(z)}\right| &= \frac{1 - |z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left|\sum_{i=1}^{n} (\alpha_i - 1) \left(\frac{zf_i'(z)}{f_i(z)} - 1\right)\right| \\ &\leq \frac{1 - |z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i - 1| \left|\frac{zf_i'(z)}{f_i(z)} - 1\right| \\ &\leq \frac{1 - |z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i - 1| \left(\left|\frac{zf_i'(z)}{f_i(z)}\right| + 1\right). \end{aligned} \tag{2.11}$$

From (2.2) and (2.11) we obtain

$$\begin{aligned} \frac{1 - |z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left|\frac{zh''(z)}{h'(z)}\right| &\leq \frac{1 - |z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i - 1| \left(\frac{1 + |z|}{1 - |z|} + 1\right) \\ &\leq \frac{1 - |z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i - 1| \left(\frac{2}{1 - |z|}\right) \\ &\leq \frac{1 - |z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left(\frac{2}{1 - |z|}\right) \sum_{i=1}^{n} |\alpha_i - 1|. \end{aligned} \tag{2.12}$$

Now, we consider the cases

$$i1). \quad 0 < \mathrm{Re}(\beta) < 1.$$

We have

$$1 - |z|^{2\mathrm{Re}(\beta)} \leq 1 - |z|^2 \tag{2.13}$$

for all $z \in \mathcal{U}$.
From (2.12) and (2.13), we have

$$\frac{1 - |z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \left|\frac{zh''(z)}{h'(z)}\right| \leq \frac{1 - |z|^2}{\mathrm{Re}(\beta)} \left(\frac{2}{1 - |z|}\right) \sum_{i=1}^{n} |\alpha_i - 1|$$

$$\leq \frac{4}{\operatorname{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i - 1| \,. \tag{2.14}$$

Using the condition (a). and (2.14) we have

$$\frac{1 - |z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| \leq 1 \tag{2.15}$$

for all $z \in \mathcal{U}$.

$$i2). \quad \operatorname{Re}(\beta) \geq 1.$$

We have

$$\frac{1 - |z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \leq 1 - |z|^2 \tag{2.16}$$

for all $z \in \mathcal{U}$.

From (2.12) and (2.16) we have

$$\frac{1 - |z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| \leq \left(1 - |z|^2\right) \left(\frac{2}{(1 - |z|)} \right) \sum_{i=1}^{n} |\alpha_i - 1|$$

$$\leq 4 \sum_{i=1}^{n} |\alpha_i - 1| \,. \tag{2.17}$$

Using the condition (b). and (2.17) we obtain

$$\frac{1 - |z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| \leq 1 \tag{2.18}$$

for all $z \in \mathcal{U}$.

From (2.15) and (2.18) we obtain that the function $G_{\alpha_1, \alpha_2, \cdots, \alpha_n}(z)$ defined by (1.2) is in the class $\mathcal{S}$.

□

Setting $n = 1$ in Theorem 2.3 we have

Corollary 2.4. *Let $f \in \mathcal{A}$ and α be complex number with $\operatorname{Re}(\alpha) \geq 0$. If*

$$(a). \quad 4\,|\alpha - 1| \leq \operatorname{Re}(\alpha), \quad \operatorname{Re}(\alpha) \in (0, 1)$$

or

$$(b). \quad |\alpha - 1| \leq \frac{1}{4}, \quad \operatorname{Re}(\alpha) \in [1, 2]$$

then the function

$$G(z) = \left(\alpha \int_0^z (f(t))^{\alpha - 1} \, dt \right)^{\frac{1}{\alpha}}$$

is in the class $\mathcal{S}$.

Theorem 2.5. *Let the functions $f_i \in \mathcal{A}, (i \in \{1, \cdots, n\})$ and α_i, β be complex numbers with $\operatorname{Re}(\beta) \geq 0$ for all $i \in \{1, 2, \cdots, n\}$. If*

$$(a). \quad 4 \sum_{i=1}^{n} |\alpha_i| \leq \operatorname{Re}(\beta), \quad \operatorname{Re}(\beta) \in (0, 1)$$

or

$$(b). \quad \sum_{i=1}^{n} |\alpha_i| \leq \frac{1}{4}, \quad \operatorname{Re}(\beta) \in [1, \infty)$$

then the function $H_{\alpha_1,\alpha_2,\cdots,\alpha_n,\beta}(z)$ defined by (1.3) is in the class $\mathcal{S}$.

Proof. Defining the function $h(z)$ by

$$h(z) = \int_0^z \prod_{i=1}^{n} \left(\frac{f_i(t)}{t} \right)^{\alpha_i} dt.$$

we take the same steps as in the proof. of Theorem 2.1.
Then we obtain that

$$\begin{aligned} \frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| &= \frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \left| \sum_{i=1}^{n} \alpha_i \left(\frac{zf_i'(z)}{f_i(z)} - 1 \right) \right| \\ &\leq \frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i| \left| \frac{zf_i'(z)}{f_i(z)} - 1 \right| \\ &\leq \frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i| \left(\left| \frac{zf_i'(z)}{f_i(z)} \right| + 1 \right). \end{aligned} \tag{2.19}$$

From (2.2) and (2.19) we obtain

$$\begin{aligned} \frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| &\leq \frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i| \left(\frac{1+|z|}{1-|z|} + 1 \right) \\ &\leq \frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i| \left(\frac{2}{1-|z|} \right) \\ &\leq \frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \left(\frac{2}{1-|z|} \right) \sum_{i=1}^{n} |\alpha_i|. \end{aligned} \tag{2.20}$$

Now, we consider the cases

$$i1). \quad 0 < \operatorname{Re}(\beta) < 1.$$

We have

$$1 - |z|^{2\operatorname{Re}(\beta)} \leq 1 - |z|^2 \tag{2.21}$$

for all $z \in \mathcal{U}$.
From (2.20) and (2.21), we have

$$\begin{aligned} \frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| &\leq \frac{1-|z|^2}{\operatorname{Re}(\beta)} \left(\frac{2}{1-|z|} \right) \sum_{i=1}^{n} |\alpha_i| \\ &\leq \frac{4}{\operatorname{Re}(\beta)} \sum_{i=1}^{n} |\alpha_i|. \end{aligned} \tag{2.22}$$

Using the condition (a). and (2.22) we have

$$\frac{1-|z|^{2\operatorname{Re}(\beta)}}{\operatorname{Re}(\beta)} \left| \frac{zh''(z)}{h'(z)} \right| \leq 1 \tag{2.23}$$

for all $z \in \mathcal{U}$.

$$i2). \quad \mathrm{Re}(\beta) \geq 1.$$

We have

$$\frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)} \leq 1-|z|^2 \tag{2.24}$$

for all $z \in \mathcal{U}$.
From (2.24) and (2.20) we have

$$\frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)}\left|\frac{zh''(z)}{h'(z)}\right| \leq \left(1-|z|^2\right)\left(\frac{2}{(1-|z|)}\right)\sum_{i=1}^{n}|\alpha_i|$$

$$\leq 4\sum_{i=1}^{n}|\alpha_i|. \tag{2.25}$$

Using the condition (b). and (2.25) we obtain

$$\frac{1-|z|^{2\mathrm{Re}(\beta)}}{\mathrm{Re}(\beta)}\left|\frac{zh''(z)}{h'(z)}\right| \leq 1 \tag{2.26}$$

for all $z \in \mathcal{U}$.
From (2.23) and (2.26) we obtain that the function $H_{\alpha_1,\alpha_2,\cdots,\alpha_n,\beta}(z)$ defined by (1.3) is in the class $\mathcal{S}$. $\square$

Setting $n = 1$ in Theorem 2.5 we have

Corollary 2.6. *Let $f \in \mathcal{A}$ and α, β be complex numbers with $\mathrm{Re}(\beta) \geq 0$. If*

$$(a). \quad |\alpha| \leq \frac{\mathrm{Re}(\beta)}{4}, \quad \mathrm{Re}(\beta) \in (0,1)$$

or

$$(b). \quad |\alpha| \leq \frac{1}{4}, \quad \mathrm{Re}(\beta) \in [1,2]$$

then the function

$$H(z) = \left(\beta\int_0^z t^{\beta-1}\left(\frac{f(t)}{t}\right)^{\frac{1}{\alpha}} dt\right)^{\frac{1}{\beta}}$$

is in the class $\mathcal{S}$.

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