Indefinite trans-Sasakian manifold with semi-symmetric metric connection

Rajendra Prasad[1] and Sushil Kumar[2]

Department of Mathematics and Astronomy, Lucknow Univesity
E-mail: [1]rp.manpur@rediffmail.com, [2]sushilmath20@gmail.com

Abstract

The objective of the present paper is to study of indefinite trans-Sasakian manifold with a semi-symmetric metric connection. We have found the relations between curvature tensors, Ricci curvature tensors and scalar curvature of indefinite trans-Sasakian manifolds with semi-symmetric metric connection and with metric connection. Also, we have proved some results on quasi-projectively flat and $\varphi-$projectively flat manifolds with respect to semi-symmetric metric connection. It is shown that the manifold satisfying $\bar{R}.\bar{S} = 0$ is an $\eta-$Einstein manifold if $\alpha = 0$ and $\beta = constant$. It is also proved that the manifold satisfying $\bar{P}.\bar{S} = 0$ is an $\eta-$Einstein manifold if $\alpha = 0$ and $\beta = constant$. Finally, we have obtained the conditions for the manifold with semi-symmetric metric connection to be conformally flat and $\xi-$conformally flat.

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1 Introduction

The notion of semi-symmetric linear connection on a differentiable manifold was introduced by Friedman and Schouten in 1924 [8] and metric connection with a torsion on a Riemannain manifold was studied by Hayden in 1932 [9]. Semi-symmetric metric connection on a Riemannain manifold had been given by Yano in 1970 [17] and later studied by K.S.Amur and S.S.Pujar [11], C.S.Bagewadi [1], U. C. De et al [7], Sharafuddin and Hussain [10] and others. U.C.De [7] and C.S.Bagewadi *et al.* [2, 3, 14] previously provided some results on the conservativeness of Projective, Pseudo projective, Conformal, Concircular, Quasi conformal curvature tensors on K-contact, Kenmotsu and trans-Sasakian manifolds.

A new class of $n-$dimensional almost contact manifold namely trans-Sasakian manifold was introduced by J. A. Oubina in 1985 [15] and further study about the local structures of trans-Sasakian manifolds was carried by J.C.Marrero [13]. As a natural generalization of both Sasakian and Kenmotsu manifolds, the notion of trans-Sasakian manifolds, which are closely related to the locally conformal Kahler manifolds introduced by $Oubi\widetilde{na}$ [15]. Trans-Sasskian manifold of type $(0, 0)$, $(\alpha, 0)$, and $(0, \beta)$ are, respectively, called the cosymplectic, α-Sasakian, and β-Kenmotsu manifolds, with α, β being scalar functions. In particular, if $\alpha = 0$, $\beta = 1$; $\alpha = 1$, $\beta = 0$; then a trans-Sasakian manifold becomes Kenmotsu and Sasakian manifolds respectively. Hence trans -Saskian structures give a large class of generalized Quasi- Sasakian structures. The concept of indefinite Sasakian manifolds was introduced by A. Bejancu and K. L. Duggal [4] and further study was taken up by Xufend and Xiaoli[18], Rakesh Kumar et al. [12]. De and Sarkar [6].

In this present paper we shall introduce relation between metric connection and semi-symmetric metric connection in an $n-$dimensional indefinite trans Sasakian manifold M^n. Also, we have

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proved some results on curvature tensors, Ricci curvature tensors, scalar curvatures, quasi projectively flat, $\varphi-$projectively flat, $\bar{R}.\bar{S}=0$, $\bar{P}.\bar{S}=0$, Weyl conformally flat, Weyl $\xi-$conformally flat respectively in $n-$dimensional indefinite trans-Sasakian manifold M^n with a semi-symmetric metric connection.

2 Preliminaries

An $n-$dimensional smooth manifold (M^n, g) is said to be an indefinite almost contact metric manifold [5], if it admits a $(1,1)$ tensor field φ, a structure vector field ξ, a $1-$form η and an indefinite metric g such that

$$\varphi^2 X = -X + \eta(X)\xi,\ \varphi\xi = 0,\ \eta(\varphi X) = 0, \tag{2.1}$$

$$\eta(\xi) = 1, \tag{2.2}$$

$$g(\xi, \xi) = \varepsilon, \tag{2.3}$$

$$\eta(X) = \varepsilon g(X, \xi), \tag{2.4}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \varepsilon\eta(X)\eta(Y), \tag{2.5}$$

for all vector fields X, Y on M^n, where ε is 1 or -1 according as ξ is space like or time like vector field and rank φ is $n-1$. If

$$d\eta(X, Y) = g(X, \varphi Y), \tag{2.6}$$

then $M^n(\varphi, \xi, \eta, g)$ is called an indefinite contact metric manifold. An indefinite almost contact metric manifold is called an indefinite trans-Sasakian manifold if

$$(\nabla_X\varphi)Y = \alpha\{g(X, Y)\xi - \varepsilon\eta(Y)X\} + \beta\{g(\varphi X, Y)\xi - \varepsilon\eta(Y)\varphi X\}, \tag{2.7}$$

for any $X, Y \in \Gamma(TM^n)$, where ∇ is metric connection of indefinite metric g, α and β are smooth functions on M^n.
From equations (2.1), (2.2), (2.3), (2.4), (2.5) and (2.7), we have

$$\nabla_X\xi = \varepsilon\{-\alpha\varphi X + \beta(X - \eta(X)\xi)\}, \tag{2.8}$$

$$(\nabla_X\eta)Y = -\alpha g(\varphi X, Y) + \beta\{g(X, Y) - \varepsilon\eta(X)\eta(Y)\}. \tag{2.9}$$

Moreover, on such a indefinite trans-Sasakian manifold M^n of dimension n with structure $M^n(\varphi, \xi, \eta, g)$ the following relations holds:

$$\begin{aligned} R(X, Y)\xi &= (\alpha^2 - \beta^2)\{\eta(Y)X - \eta(X)Y\} \\ &\quad +2\alpha\beta\{\eta(Y)\varphi X - \eta(X)\varphi Y\} + \varepsilon\{(Y\alpha)\varphi X \\ &\quad -(X\alpha)\varphi Y + (Y\beta)\varphi^2 X - (X\beta)\varphi^2 Y\}, \end{aligned} \tag{2.10}$$

$$\begin{aligned} R(\xi, Y)Z &= (\alpha^2 - \beta^2)\{\varepsilon g(Y, Z)\xi - \eta(Z)Y\} + 2\alpha\beta\{\varepsilon g(Y, \varphi Z)\xi \\ &\quad +\eta(Z)\varphi Y\} + \varepsilon(Z\alpha)\varphi Y + \varepsilon g(Y, \varphi Z)(grad\alpha) \\ &\quad -\varepsilon g(\varphi Y, \varphi Z)(grad\beta) + \varepsilon(Z\beta)\{Y - \eta(Z)\xi\}, \end{aligned} \tag{2.11}$$

$$S(Z,\xi) = \{(n-1)(\alpha^2-\beta^2) - \varepsilon(\xi\beta)\}\eta(Z) - \varepsilon(\varphi Z)\alpha - \varepsilon(n-2)(Z\beta), \tag{2.12}$$

$$S(\xi,\xi) = (n-1)(\alpha^2-\beta^2) - \varepsilon(n-1)(\xi\beta), \tag{2.13}$$

$$Q\xi = \varepsilon(n-1)(\alpha^2-\beta^2)\xi - (\xi\beta)\xi + \varepsilon\varphi(grad\alpha) - \varepsilon(n-2)(grad\beta), \tag{2.14}$$

$$\begin{aligned} C(X,Y)Z \quad = \quad & R(X,Y)Z - \frac{1}{(n-2)}[S(Y,Z)X - S(X,Z)Y \\ & + g(Y,Z)QX - g(X,Z)QY] + \frac{r}{(n-1)(n-2)} \\ & [g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{2.15}$$

$$\begin{aligned} g(R(\xi,Y)Z,\xi) \quad = \quad & [(\alpha^2-\beta^2) - \varepsilon(\xi\beta)]g(Y,Z) \\ & + [(\xi\beta) - \varepsilon(\alpha^2-\beta^2)]\eta(Y)\eta(Z) \\ & - [2\alpha\beta + \varepsilon(\xi\alpha)]g(\varphi Y,Z), \end{aligned} \tag{2.16}$$

$$r = S(e_i,e_i) = \sum_{i=1}^{n} \varepsilon_i\, {}'R(e_i,e_i,e_i,e_i). \tag{2.17}$$

3 Semi-symmetric metric connection

A affine connection $\bar{\nabla}$ in M^n is called semi-symmetric connection [10], if its torsion tensor

$$\bar{T}(X,Y) = \bar{\nabla}_X Y - \bar{\nabla}_Y X - [X,Y], \tag{3.1}$$

satisfies

$$\bar{T}(X,Y) = \eta(X)Y - \eta(Y)X. \tag{3.2}$$

Moreover, a semi-symmetric connection is called semi-symmetric metric connection if

$$(\bar{\nabla}_X g)(Y,Z) = 0. \tag{3.3}$$

If ∇ is metric connection and $\bar{\nabla}$ is the semi-symmetric metric with non-vanishing torsion tensor T in M^n, then we have

$$T(X,Y) = \eta(Y)X - \eta(X)Y, \tag{3.4}$$

$$\bar{\nabla}_X Y - \nabla_X Y = \frac{1}{2}[T(X,Y) + T^{|}(X,Y) + T^{|}(X,Y)], \tag{3.5}$$

where

$$g(T(Z,X),Y) = g(T^{|}(X,Y),Z). \tag{3.6}$$

By using equations (2.4), (3.4) and (3.6), we get

$$g(T^{|}(X,Y),Z) = g(\eta(X)Z - \eta(Z)X, Y),$$

$$g(T^{|}(X,Y),Z) = \eta(X)g(Z,Y) - \varepsilon g(X,Y)g(\xi,Z),$$

$$T^{|}(X,Y) = \eta(X)Y - \varepsilon g(X,Y)\xi, \tag{3.7}$$

$$T^{|}(Y,X) = \eta(Y)X - \varepsilon g(X,Y)\xi, \tag{3.8}$$

From equations (3.4), (3.5), (3.7) and (3.8), we get

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)X - \varepsilon g(X,Y)\xi.$$

Let M^n be an $n-$dimensional indefinite trans-Sasakian manifold and ∇ be the metric connection on M^n. The relation between the semi-symmetric metric connection $\bar{\nabla}$ and the metric connection ∇ on M^n is given by

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)X - \varepsilon g(X,Y)\xi. \tag{3.9}$$

4 Curvature tensor on indefinite trans-Sasakian manifold with semi-symmetric metric connection

Let M^n be an $n-$dimensional indefinite trans-Sasakian manifold. The curvature tensor $\bar{R}$ of M^n with respect to the semi-symmetric metric connection $\bar{\nabla}$ is defined by

$$\bar{R}(X,Y)Z = \bar{\nabla}_X \bar{\nabla}_Y Z - \bar{\nabla}_Y \bar{\nabla}_X Z - \bar{\nabla}_{[X,Y]} Z. \tag{4.1}$$

By using equations (2.2), (2.4), (3.9) and (4.1), we get

$$\begin{aligned} \bar{R}(X,Y)Z &= R(X,Y)Z + (2\beta+\varepsilon)[g(X,Z)Y - g(Y,Z)X\ ] \\ &\quad +(\beta+\varepsilon)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi \\ &\quad +(1+\beta\varepsilon)\eta(Z)[\eta(Y)X - \eta(X)Y\ ] \\ &\quad -\alpha[g(\varphi X,Z)Y - g(\varphi Y,Z)X - g(Y,Z)\varphi X + g(X,Z)\varphi Y], \end{aligned} \tag{4.2}$$

where

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z,$$

is the Riemannian curvature tensor of metric connection ∇ .

Taking inner product of equation (4.2) with U and using equation (2.4), we have

$$\begin{aligned} &{}'\bar{R}(X,Y,Z,U) \\ = &{}'R(X,Y,Z,U) + (\varepsilon+2\beta)[g(X,Z)g(Y,U) - g(Y,Z)g(X,U)] \\ &+(1+\varepsilon\beta)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\eta(U) \\ &+(1+\varepsilon\beta)[\eta(Y)g(X,U) - \eta(X)g(Y,U,)]\eta(Z) \\ &-\alpha[g(\varphi X,Z)g(Y,U) - g(\varphi Y,Z)g(X,U) - g(Y,Z)g(\varphi X,U) \\ &+g(X,Z)g(\varphi Y,U)], \end{aligned} \tag{4.3}$$

where ${}'\bar{R}(X,Y,Z,U) = g(\bar{R}(X,Y)Z,U)$ and ${}'R(X,Y,Z,U) = g(R(X,Y)Z,U)$.

Let $\{e_1, e_2, ..., e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields on indefinite trans-Sasakian manifold M^n, then $\{\varphi e_1, \varphi e_2, ..., \varphi e_{n-1}, \xi\}$ is a also local orthonormal basis of vector fields on indefinite trans-Sasakian manifold M^n. Now, putting $X = U = e_i$ in the equation (4.3) and taking summation over $i, 1 \leq i \leq n-1$, we get

$$\sum_{i=1}^{n-1} {}'\bar{R}(e_i, Y, Z, e_i) \tag{4.4}$$
$$= \sum_{i=1}^{n-1} {}'R(e_i, Y, Z, e_i) + (2\beta + \varepsilon) \sum_{i=1}^{n-1} [g(e_i, Z)g(Y, e_i) - g(Y, Z)g(e_i, e_i)]$$
$$+(\beta + \varepsilon) \sum_{i=1}^{n-1} [\varepsilon g(Y, Z)g(e_i, \xi)g(\xi, e_i) - \eta(Y)g(e_i, Z)g(\xi, e_i)]$$
$$+(\beta\varepsilon + 1) \sum_{i=1}^{n-1} [\eta(Z)\eta(Y)g(e_i \ , e_i \ ) - \varepsilon\eta(Z)g(e_i, \xi)g(Y, e_i) \ ]$$
$$-\alpha \sum_{i=1}^{n-1} [g(e_i\varphi, Z)g(Y, e_i) - g(\varphi Y, Z)g(e_i, e_i) - g(Y, Z)g(\varphi e_i, e_i)$$
$$+g(e_i, Z)g(\varphi Y, e_i)].$$

We have using

$$S(Y, Z) = \sum_{i=1}^{n} \varepsilon_i \ {}'R(e_i, Y, Z, e_i), \ \ g(Y, Z) = \sum_{i=1}^{n} \varepsilon_i g(Y, e_i)g(e_i \ , Z), \tag{4.5}$$

$$\bar{S}(Y, Z) = \sum_{i=1}^{n} \varepsilon_i \ {}'\bar{R}(e_i, Y, Z, e_i \ ). \tag{4.6}$$

Also

$$\sum_{i=1}^{n-1} {}'\bar{R}(e_i \ , Y, Z, e_i \ ) = \bar{S}(Y, Z) - \varepsilon \ {}'\bar{R}(\xi, Y, Z, \xi), \tag{4.7}$$

$$\sum_{i=1}^{n-1} {}'R(e_i \ , Y, Z, e_i) = S(Y, Z) - \varepsilon \ {}'R(\xi, Y, Z, \xi), \tag{4.8}$$

$$\sum_{i=1}^{n-1} g(e_i, Z)g(Y, e_i) = g(Y, Z) - \varepsilon g(\xi, Z)g(Y, \xi \ ), \tag{4.9}$$

$$\sum_{i=1}^{n-1} g(e_i, e_i) = n - 1, \tag{4.10}$$

$$\sum_{i=1}^{n-1} g(e_i, \xi)g(\xi, e_i) = 0, \tag{4.11}$$

$$\sum_{i=1}^{n-1} g(e_i, Z)g(\xi, e_i) = 0, \tag{4.12}$$

$$\sum_{i=1}^{n-1} g(\varphi e_i, Z)g(Y, e_i) = g(\varphi Y, Z), \tag{4.13}$$

$$\sum_{i=1}^{n-1} g(\varphi e_i, e_i) = 0, \tag{4.14}$$

$$\sum_{i=1}^{n-1} g(\varphi e_i, \varphi e_i) = n-1, \tag{4.15}$$

$$\sum_{i=1}^{n-1} g(\varphi e_i, Z)g(Y, \varphi e_i \ ) = g(Y,Z) - \varepsilon g(\xi, Z)g(Y,\xi). \tag{4.16}$$

Hence, by virtue of equations (4.4) and (4.7) – (4.16), we get

$$\begin{aligned} \bar{S}(Y,Z) \quad &= \quad S(Y,Z) - [(2\beta+\varepsilon)(n-2)+\beta]g(Y,Z) \\ &\quad +(1+\varepsilon\beta)(n-2)\eta(Z)\eta(Y) + \alpha(n-2)g(\varphi Y, Z), \end{aligned} \tag{4.17}$$

where $\bar{S}$ and S are the Ricci tensor of connection $\bar{\nabla}$ and ∇, respectively in M^n .

$$\bar{Q}Y = QY - [(2\beta+\varepsilon)(n-2)+\beta]Y + (\varepsilon+\beta)(n-2)\eta(Y)\xi + \alpha(n-2)\varphi Y, \tag{4.18}$$

where $\bar{Q}$ and Q are Ricci operator with respect to the semi-symmetric metric connection and metric connection respectively and are define as $g(\bar{Q}Y,Z) = \bar{S}(Y,Z)$ and $g(QY,Z) = S(Y,Z)$ respectively.

Replace $Y = \xi$ in (4.18) and using (2.14), we get

$$\begin{aligned} \bar{Q}\xi \quad &= \quad \varepsilon(n-1)(\alpha^2-\beta^2)\xi - (\xi\beta)\xi + \varepsilon\varphi(grad\alpha) \\ &\quad -\varepsilon(n-2)(grad\beta) - \beta(n-1)\xi. \end{aligned} \tag{4.19}$$

Putting $Y = Z = e_i$ and taking summation over i, $1 \le i \le n-1$ in equation (4.17), using equations (2.13), (2.17), (4.10), (4.11) and (4.14), we get

$$\bar{r} \ = r - (n-1)[(2\beta+\varepsilon)(n-2)+2\beta], \tag{4.20}$$

where $\bar{r}$ and r are the scalar curvatures of the connection $\bar{\nabla}$ and ∇, respectively.

Replace $Y \to \varphi Y$ in equation (4.17), we get

$$\begin{aligned} \bar{S}(\varphi Y,Z) \quad &= \quad S(\varphi Y,Z) - [(2\beta+\varepsilon)(n-2)+\beta]g(\varphi Y,Z) \\ &\quad -\alpha(n-2)g(Y,Z) + \varepsilon\alpha(n-2)\eta(Z)\eta(Y). \end{aligned} \tag{4.21}$$

Using equations (2.1), (2.2), (2.4) and (2.12) in equation (4.21), we get

$$\bar{S}(\varphi Y,\xi) = S(\varphi Y,\xi) = -\varepsilon(\varphi^2 Y)\alpha - \varepsilon(n-2)(\varphi Y)\beta. \tag{4.22}$$

Using equations (2.1), (2.2), (2.4), (2.12), (2.13) and (4.17), we get

$$\begin{aligned} \bar{S}(Y,\xi) \quad &= \quad [(n-1)(\alpha^2-\beta^2) - \varepsilon(\xi\beta) - \varepsilon\beta(n-1)]\eta(Y) \\ &\quad -\varepsilon(\varphi Y)\alpha - \varepsilon(n-2)(Y\beta), \end{aligned} \tag{4.23}$$

$$\bar{S}(\xi,\xi) = (n-1)(\alpha^2-\beta^2) - \varepsilon\beta(n-1) - \varepsilon(n-1)(\xi\beta), \tag{4.24}$$

$$\begin{aligned}\bar{S}(grad\alpha,\xi) &= \varepsilon(n-1)(\alpha^2-\beta^2)(\xi\alpha) - \beta(n-1)(\xi\alpha) - (\xi\alpha)(\xi\beta) \\ &\quad -\varepsilon(\varphi grad\alpha)\alpha - \varepsilon(n-2)g(grad\alpha, grad\beta),\end{aligned} \tag{4.25}$$

$$\begin{aligned}\bar{S}(grad\beta,\xi) &= \varepsilon(n-1)(\alpha^2-\beta^2)(\xi\beta) - \beta(n-1)(\xi\beta) - (\xi\beta)^2 \\ &\quad -\varepsilon(\varphi grad\beta)\alpha - \varepsilon(n-2)(grad\beta)^2.\end{aligned} \tag{4.26}$$

From equation (4.2) by the cyclic permutations of X, Y and Z, we get

$$\begin{aligned}&\bar{R}(Y,Z)X \\ =\ & R(Y,Z)X + (\varepsilon+2\beta)[g(Y,X)Z - g(Z,X)Y\ ] \\ & +(\varepsilon+\beta)[\eta(Y)g(Z,X) - \eta(Z)g(Y,X)]\xi \\ & +(1+\beta\varepsilon)\eta(X)[\eta(Z)Y - \eta(Y)Z\ ] \\ & -\alpha[g(\varphi Y,X)Z - g(\varphi Z,X)Y - g(Z,X)\varphi Y + g(Y,X)\varphi Z],\end{aligned} \tag{4.27}$$

$$\begin{aligned}&\bar{R}(Z,X)Y \\ =\ & R(Z,X)Y + (2\beta+\varepsilon)[g(Z,Y)X - g(X,Y)Z\ ] \\ & +(\varepsilon+\beta)[\eta(Y)[\eta(Z)g(X,Y) - \eta(X)g(Y,Z)]\xi \\ & +(1+\varepsilon\beta)\eta(Y)[\eta(X)Z - \eta(Z)X\ ] \\ & -\alpha[g(\varphi Z,Y)X - g(\varphi X,Y)Z - g(X,Y)\varphi Z + g(Z,Y)\varphi X].\end{aligned} \tag{4.28}$$

Adding equations (4.2), (4.27) and (4.28), using the Bianchi' first identity, we get

$$\begin{aligned}&\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y \\ =\ & 2\alpha[g(\varphi Y,Z)X + g(\varphi Z,X)Y + g(\varphi X,Y)Z].\end{aligned} \tag{4.29}$$

If $\alpha = 0$ in equation (4.29), we have

$$\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y = 0.$$

Theorem 4.1. Let M^n be an $n-$dimensional indefinite trans-Sasakian manifold with semi-symmetric metric connection $\bar{\nabla}$ and curvature tensor $\bar{R}$ satisfies Bianchi first identity if and only if $\alpha = 0$, i.e. M^n is a $\beta-$Kenmotsu manifold.

Now, from equation (4.3) interchanging X and Y, we get

$$\begin{aligned}&{}'\bar{R}(Y,X,Z,U) \\ =\ & {}'R(Y,X,Z,U) + (\varepsilon+2\beta)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)] \\ & +(1+\varepsilon\beta)[\eta(Y)g(X,Z) - \eta(X)g(Y,Z)]\eta(U) \\ & +(1+\varepsilon\beta)[\eta(X)g(Y,U) - \eta(Y)g(X,U,)]\eta(Z) \\ & -\alpha[g(\varphi Y,Z)g(X,U) - g(\varphi X,Z)g(Y,U) - g(X,Z)g(\varphi Y,U) \\ & +g(Y,Z)g(\varphi X,U)].\end{aligned} \tag{4.30}$$

From equations (4.3) and (4.30), we get

$$'\bar{R}(X,Y,Z,U) = -'\bar{R}(Y,X,Z,U), \tag{4.31}$$

where

$$'R(X,Y,Z,U) = -'R(Y,X,Z,U).$$

Again from equation (4.3) interchanging Z and U, we get

$$\begin{aligned} &'\bar{R}(X,Y,U,Z) \\ = \;& 'R(X,Y,U,Z) + (2\beta+\varepsilon)[g(X,U)g(Y,Z) - g(Y,U)g(X,Z)] \\ &+(1+\varepsilon\beta)[\eta(X)g(Y,U) - \eta(Y)g(X,U)]\eta(Z) \\ &+(1+\varepsilon\beta)[\eta(Y)g(X,Z) - \eta(X)g(Y,Z)]\eta(U) \\ &-\alpha[g(\varphi X,U)g(Y,Z) - g(\varphi Y,U)g(X,Z) - g(Y,U)g(\varphi X,Z) \\ &+g(X,U)g(\varphi Y,Z)]. \end{aligned} \tag{4.32}$$

From equations (4.3) and (4.32), we get

$$'\bar{R}(X,Y,Z,U) = -'\bar{R}(X,Y,U,Z), \tag{4.33}$$

where

$$'R(X,Y,Z,U) = -'R(X,Y,U,Z).$$

Again from equation (4.3) interchanging pair of slots, we get

$$\begin{aligned} &'\bar{R}(Z,U,X,Y) \\ = \;& 'R(Z,U,X,Y) + (2\beta+\varepsilon)[g(Z,X)g(U,Y) - g(U,X)g(Z,Y)] \\ &+(1+\varepsilon\beta)[\eta(Z)g(U,X) - \eta(U)g(Z,X)]\eta(Y) \\ &+(1+\varepsilon\beta)[\eta(U)g(Z,Y) - \eta(Z)g(U,Y)]\eta(X) \\ &-\alpha[g(\varphi Z,X)g(U,Y) - g(\varphi U,X)g(Z,Y) - g(\varphi Z,Y)g(U,X) \\ &+g(\varphi U,Y)g(Z,X)]. \end{aligned} \tag{4.34}$$

From equations (4.3) and (4.34), we get

$$\begin{aligned} &'\bar{R}(X,Y,Z,U) \\ = \;& '\bar{R}(Z,U,X,Y) + 2\alpha[g(\varphi Z,X)g(U,Y) \\ &+g(\varphi Y,Z)g(X,U) + g(\varphi X,U)g(Y,Z) \\ &+g(\varphi U,Y)g(Z,X)]. \end{aligned} \tag{4.35}$$

If $\alpha = 0$ in equation (4.35), we have

$$'\bar{R}(X,Y,Z,U) =' \bar{R}(Z,U,X,Y)$$

Theorem 4.2. The curvature tensor $\bar{R}$ of type $(0,4)$ of semi-symmetric metric connection $\bar{\nabla}$ is an indefinite trans-Sasakian manifolds is

(i) Skew symmetric in first two slots.
(ii) Skew symmetric in last two slots.
(iii) Symmetric in pair of slots if and only if $\alpha = 0$ in equation (4.35), then M^n is a $\beta-$Kenmotsu manifold.

Now, let $\bar{R}(X,Y)Z = 0$ in equation (4.2), we get

$$\begin{aligned} R(X,Y)Z \quad &= \quad (\varepsilon+2\beta)[g(Y,Z)X - \ g(X,Z)Y] \\ &+(\varepsilon+\beta)[\eta(Y)g(X,Z)-\eta(X)g(Y,Z)]\xi \\ &+(1+\varepsilon\beta)\eta(Z)[\eta(X)Y-\eta(Y)X\ ] \\ &+\alpha[g(\varphi X,Z)Y-g(\varphi Y,Z)X-g(Y,Z)\varphi X \\ &+g(X,Z)\varphi Y], \end{aligned} \tag{4.36}$$

Taking the inner product of equation (4.36) with ξ and using (2.4), we get

$$\begin{aligned} \varepsilon\eta(R(X,Y)Z) \quad &= \quad \varepsilon\beta[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)] \\ &+\varepsilon\alpha[g(\varphi X,Z)\eta(Y)-g(\varphi Y,Z)\eta(X)]. \end{aligned} \tag{4.37}$$

Using equation (2.4) in equation (4.37), we get

$$R(X,Y)Z = \beta[g(Y,Z)X-g(X,Z)Y]+\alpha[g(\varphi X,Z)Y-g(\varphi Y,Z)X]. \tag{4.38}$$

Theorem 4.3. If the curvature tensor $\bar{R}$ of a semi-symmetric metric connection in an indefinite trans-Sasakian manifold M^n vanishes, then the indefinite trans sasakian manifold is of constant curvature if $\alpha = 0$. i.e. M^n is $\beta-$Kenmotsu manifold.

Now, in equation (4.2) putting $Z = \xi$, using equations (2.1), (2.2), (2.4) and (2.10), we get

$$\begin{aligned} \bar{R}(X,Y)\xi \quad &= \quad (\alpha^2-\beta^2-\varepsilon\beta)[\eta(Y)X-\eta(X)Y] \\ &+(2\alpha\beta+\varepsilon\alpha)[\eta(Y)\varphi X-\eta(X)\varphi Y] \\ &+\varepsilon[(Y\alpha)\varphi X-(X\alpha)\varphi Y+(Y\beta)\varphi^2 X-(X\beta)\varphi^2 Y]. \end{aligned} \tag{4.39}$$

Replace $Y = \xi$ in equation (4.39), using equations (2.1), (2.2) and (2.4), we get

$$\begin{aligned} \bar{R}(X,\xi)\xi \quad &= \quad (\alpha^2-\beta^2-\varepsilon\beta)[X-\eta(X)\xi] \\ &+[2\alpha\beta+\varepsilon\alpha+\varepsilon(\xi\alpha)]\varphi X+\varepsilon(\xi\beta)\varphi^2 X]. \end{aligned} \tag{4.40}$$

Now, again replace $X = \xi$ in equation (4.39), using equations (2.1), (2.2) and (2.4), we get

$$\begin{aligned} \bar{R}(\xi,Y)\xi \quad &= \quad (\alpha^2-\beta^2-\varepsilon\beta)[\eta(Y)\xi-Y]-[2\alpha\beta+\varepsilon\alpha \\ &+\varepsilon(\xi\alpha)]\varphi Y-\varepsilon(\xi\beta)\varphi^2 Y. \end{aligned} \tag{4.41}$$

Replace $Y = X$ in equation (4.41), we get

$$\begin{aligned} \bar{R}(\xi,X)\xi \quad &= \quad -(\alpha^2-\beta^2-\varepsilon\beta)[X-\eta(X)\xi]-[2\alpha\beta+\varepsilon\alpha \\ &-\varepsilon(\xi\alpha)]\varphi X-\varepsilon(\xi\beta)\varphi^2 X. \end{aligned} \tag{4.42}$$

From equations (4.40) and (4.42), we get

$$\bar{R}(X,\xi)\xi = -\bar{R}(\xi,X)\xi. \tag{4.43}$$

Theorem 4.4. Let M^n be an $n-$dimensional indefinite trans-Sasakian manifold with a semi-symmetric metric connection, then

$$\begin{aligned}\bar{R}(X,Y)\xi &= (\alpha^2-\beta^2-\varepsilon\beta)[\eta(Y)X-\eta(X)Y] \\ &+(2\alpha\beta+\varepsilon\alpha)[\eta(Y)\varphi X-\eta(X)\varphi Y] \\ &+\varepsilon[(Y\alpha)\varphi X-(X\alpha)\varphi Y+(Y\beta)\varphi^2X-(X\beta)\varphi^2Y].\end{aligned} \tag{4.44}$$

Lemma 4.5. Let M^n be an $n-$dimensional indefinite trans-Sasakian manifold with a semi-symmetric metric connection, then

$$(\bar{\nabla}_X\eta)(Y)=(\beta+\varepsilon)g(X,Y)-(1+\varepsilon\beta)\eta(X)\eta(Y)-\alpha g(\varphi X,Y), \tag{4.45}$$

$$\bar{\nabla}_X\xi=(1+\varepsilon\beta)X-(1+\varepsilon\beta)\eta(X)\xi-\varepsilon\alpha(\varphi X). \tag{4.46}$$

Proof. By the covariant differentiation of $\eta(Y)$ with respect X, we have

$$\begin{aligned}\bar{\nabla}_X\eta(Y) &= (\bar{\nabla}_X\eta)(Y)+\eta(\bar{\nabla}_XY), \\ (\bar{\nabla}_X\eta)(Y) &= \bar{\nabla}_X\eta(Y)-\eta(\bar{\nabla}_XY).\end{aligned}$$

By using equations (2.4) and (3.3), we get

$$(\bar{\nabla}_X\eta)(Y)=\varepsilon g(Y,\bar{\nabla}_X\xi). \tag{4.47}$$

On putting $Y=\xi$ in equation (3.9), we have

$$\bar{\nabla}_X\xi=\nabla_X\ \xi+\eta(\xi)X-\varepsilon g(X,\xi)\xi.$$

By using equation (2.4), we get

$$\bar{\nabla}_X\xi=\nabla_X\ \xi+X-\eta(X)\xi, \tag{4.48}$$

From equation (4.47), using equations (2.4), (2.8) and (4.48), we get

$$(\bar{\nabla}_X\eta)(Y)=(\beta+\varepsilon)g(X,Y)-(1+\varepsilon\beta)\eta(X)\eta(Y)-\alpha g(\varphi X,Y).$$

From equations (2.8) and (4.6), we get

$$\bar{\nabla}_X\xi=(1+\varepsilon\beta)X-(1+\varepsilon\beta)\eta(X)\xi-\varepsilon\alpha(\varphi X).$$

Q.E.D.

5 Quasi-projectively flat indefinite trans-Sasakian manifold with respect to semi-symmetric metric connection

Let M^n be an $n-$dimensional indefinite trans-Sasakian manifold. If there exists a one to one correspondence between each co-ordinate neighbourhood of M^n and a domain in Euclidean space such that any geodesic of the indefinite trans-Sasakian manifold corresponds to a straight line in the Euclidean space, then M^n is said to be locally projectively flat. The projective curvature tensor $\bar{P}$ with respect to semi-symmetric metric connection is defined by

$$\bar{P}(X,Y)Z = \bar{R}(X,Y)Z - \frac{1}{(n-1)}[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y]. \tag{5.1}$$

Definition 5.1. An indefinite trans-Sasakian manifold M^n is said to be quasi-projectively flat with respect to semi-symmetric metric connection, if

$$g(\bar{P}(\varphi X,Y)Z,\varphi U) = 0, \tag{5.2}$$

where $\bar{P}$ is the projective curvature tensor with respect to semi-symmetric metric connection. From equation (5.1) taking inner product with U, we get

$$\begin{aligned} g(\bar{P}(X,Y)Z,U) &= g(\bar{R}(X,Y)Z,U) - \frac{1}{(n-1)} \\ &\quad [\bar{S}(Y,Z)g(X,U) - \bar{S}(X,Z)g(Y,U)]. \end{aligned} \tag{5.3}$$

Replace $X = \varphi X$ and $U = \varphi U$ in equation (5.3), we get

$$\begin{aligned} g(\bar{P}(\varphi X,Y)Z,\varphi U) &= g(\bar{R}(\varphi X,Y)Z,\varphi U) - \frac{1}{(n-1)} \\ &\quad [\bar{S}(Y,Z)g(\varphi X,\varphi U) - \bar{S}(\varphi X,Z)g(Y,\varphi U)]. \end{aligned} \tag{5.4}$$

From equations (5.2) and (5.4), we have

$$g(\bar{R}(\varphi X,Y)Z,\varphi U) = \frac{1}{(n-1)}[\bar{S}(Y,Z)g(\varphi X,\varphi U) - \bar{S}(\varphi X,Z)g(Y,\varphi U)]. \tag{5.5}$$

Now, using equations $(2,1),(2.4)$, (4.17) and (4.21) in equation (5.5), we have

$$\begin{aligned} & g(R(\varphi X,Y)Z,\varphi U) \\ = & \frac{1}{(n-1)}[S(Y,Z)g(\varphi X,\varphi U) - S(\varphi X,Z)g(Y,\varphi U)] \\ & -\frac{(\beta+\varepsilon)}{(n-1)}g(\varphi X,Z)g(Y,\varphi U) + \frac{(\beta+\varepsilon)}{(n-1)}g(Y,Z)g(\varphi X,\varphi U) \\ & -\frac{(1+\varepsilon\beta)}{(n-1)}\eta(Y)\eta(Z)g(\varphi X,\varphi U) + \frac{\varepsilon\alpha}{(n-1)}\eta(X)\eta(Z)g(Y,\varphi U) \\ & -\frac{\alpha}{(n-1)}g(X,Z)g(Y,\varphi U) - \frac{\alpha}{(n-1)}g(\varphi Y,Z)g(\varphi X,\varphi U) \\ & +\alpha g(Y,Z)g(X,\varphi U) + \alpha g(\varphi X,Z)g(\varphi Y,\varphi U). \end{aligned} \tag{5.6}$$

Let $\{e_1, e_2, ..., e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields on indefinite trans-Sasakian manifold M^n, then $\{\varphi e_1, \varphi e_2, ..., \varphi e_{n-1}, \xi\}$ is a also local orthonormal basis of vector fields on indefinite trans-Sasakian manifold M^n. Now, putting $X = U = e_i$ in equation (5.6) and taking summation over $i, 1 \leq i \leq n-1$, we have

$$\begin{aligned}
&\sum_{i=1}^{n-1} g(R(\varphi e_i, Y)Z, \varphi e_i) \qquad (5.7)\\
= \;& \frac{1}{(n-1)}\sum_{i=1}^{n-1}[S(Y,Z)g(\varphi e_i,\varphi e_i) - S(\varphi e_i, Z)g(Y,\varphi e_i)]\\
&-\frac{(\beta+\varepsilon)}{(n-1)}\sum_{i=1}^{n-1} g(\varphi e_i, Z)g(Y,\varphi e_i) + \frac{(\beta+\varepsilon)}{(n-1)}\sum_{i=1}^{n-1} g(Y,Z)g(\varphi e_i,\varphi e_i)\\
&-\frac{(1+\varepsilon\beta)}{(n-1)}\sum_{i=1}^{n-1}\eta(Y)\eta(Z)g(\varphi e_i,\varphi e_i) + \frac{\varepsilon\alpha}{(n-1)}\sum_{i=1}^{n-1}\eta(e_i)\eta(Z)g(Y,\varphi e_i)\\
&-\frac{\alpha}{(n-1)}\sum_{i=1}^{n-1} g(e_i, Z)g(Y,\varphi e_i) - \frac{\alpha}{(n-1)}\sum_{i=1}^{n-1} g(\varphi Y, Z)g(\varphi e_i,\varphi e_i)\\
&+\alpha\sum_{i=1}^{n-1} g(Y,Z)g(e_i,\varphi e_i) + \alpha\sum_{i=1}^{n-1} g(\varphi e_i, Z)g(\varphi Y,\varphi e_i).
\end{aligned}$$

We have also

$$\sum_{i=1}^{n-1} S(\varphi e_i, Z)g(Y,\varphi e_i) = S(Y,Z) - \varepsilon g(\xi, Z)S(Y,\xi). \qquad (5.8)$$

Now, using equations $(2.1), (2.2), (2.4), (2.16), (4.8) - (4.16)$ and (5.8) in equation (5.7), we have

$$\begin{aligned}
S(Y,Z) \;=\; & [(n-2)(\varepsilon+\beta) + \varepsilon(n-1)(\alpha^2-\beta^2) - (n-1)(\xi\beta)]g(Y,Z) \qquad (5.9)\\
&+[\varepsilon(n-2)(\xi\beta) - (n-2)(1+\varepsilon\beta)]\eta(Y)\eta(Z)\\
&-[2\varepsilon(n-1)\alpha\beta + (n-1)(\xi\alpha) - \alpha]g(\varphi Y, Z)\\
&-\varepsilon\eta(Y)(\varphi Z)\alpha - \varepsilon(n-2)(\xi\beta)\eta(Y).
\end{aligned}$$

If $\alpha = 0$ and $\beta = constant$ in equation (5.9), we get

$$S(Y,Z) = [(n-2)(\varepsilon+\beta) - (n-1)\varepsilon\beta^2]g(Y,Z) + (2-n)(1+\varepsilon\beta)\eta(Y)\eta(Z). \qquad (5.10)$$

Therefore,

$$S(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z),$$

where $a = (n-2)(\varepsilon+\beta) - (n-1)\varepsilon\beta^2$ and $b = (2-n)(1+\varepsilon\beta)$.
This result shows that the manifold under the consideration is an $\eta-$ Einstein manifold. Thus, we can state the following theorem:

Theorem 5.2. An $n-$dimensional quasi projectively flat indefinite trans-Sasakin manifold M^n with respect to a semi-symmetric metric connection is $\eta-$Einstein manifold if $\alpha = 0$ and $\beta =$ *constant*.

6 $\varphi-$Projectively flat indefinite trans Sasakian manifold with respect to a semi-symmetric metric connection

An M^n indefinite trans-Sasakian manifold with respect to a semi-symmetric metric connection is said be $\varphi-$projectively flat if

$$\varphi^2(\bar{P}(\varphi X, \varphi Y)\varphi Z) = 0, \tag{6.1}$$

where $\bar{P}$ is the projective curvature tensor of the M^n indefinite trans-Sasakian manifold with respect to a semi-symmetric metric connection. Suppose M^n be a $\varphi-$projectively flat indefinite trans-Sasakian manifold with respect to a semi-symmetric metric connection . It is known that $\varphi^2(\bar{P}(\varphi X, \varphi Y)\varphi Z) = 0$ holds if and only if

$$g(\bar{P}(\varphi X, \varphi Y)\varphi Z), \varphi U) = 0, \tag{6.2}$$

for any $X, Y, Z, U \in TM^n$. Replace $Y = \varphi Y$ and $U = \varphi U$ in the equation (5.4), we have

$$\begin{aligned} g(\bar{P}(\varphi X, Y\varphi)\varphi Z, \varphi U) &= g(\bar{R}(\varphi X, \varphi Y)\varphi Z, \varphi U) - \frac{1}{(n-1)} \\ & [\bar{S}(\varphi Y, \varphi Z)g(\varphi X, \varphi U) - \bar{S}(\varphi X, \varphi Z)g(\varphi Y, \varphi U)]. \end{aligned} \tag{6.3}$$

From equations (6.2) and (6.3), we have

$$\begin{aligned} g(\bar{R}(\varphi X, \varphi Y)\varphi Z, \varphi U) &= \frac{1}{(n-1)}[\bar{S}(\varphi Y\varphi, Z)g(\varphi X, \varphi U) \\ & -\bar{S}(\varphi X, \varphi Z)g(\varphi Y, \varphi U)]. \end{aligned} \tag{6.4}$$

Using equations $(2.1), (2.2), (2.4), (2.5), (4.2)$ and (4.17) in equation (6.4), we have

$$\begin{aligned} & g(R(\varphi X, \varphi Y)\varphi Z, \varphi U) \\ = & \frac{1}{(n-1)}[S(\varphi Y, \varphi Z)g(\varphi X, \varphi U) - S(\varphi X, \varphi Z)g(\varphi Y, \varphi U)] \\ & +\frac{(\varepsilon+\beta)}{(n-1)}g(\varphi Y, \varphi Z)g(\varphi X, \varphi U) - \frac{(\varepsilon+\beta)}{(n-1)}g(\varphi X, \varphi Z)g(\varphi Y, \varphi U) \\ & +\frac{\alpha}{(n-1)}g(Y, \varphi Z)g(\varphi X, \varphi U) - \frac{\alpha}{(n-1)}g(X, \varphi Z)g(\varphi Y, \varphi U) \\ & +\alpha g(\varphi Y, \varphi Z)g(X, \varphi U) - \alpha g(\varphi X, \varphi Z)g(Y, \varphi U). \end{aligned} \tag{6.5}$$

Let $\{e_1, e_2, ..., e_{-1}, \xi\}$ be a local orthonormal basis of vector fields on indefinite trans-Sasakian manifold M^n, then $\{\varphi e_1, \varphi e_2, ..., \varphi e_{n-1}, \xi\}$ is a also local orthonormal basis of vector fields on indefinite trans-Sasakian manifold M^n. Now, replace $X = U = e_i$ in equation (6.5) and taking summation over $i, 1 \le i \le n-1$, we get

$$\begin{aligned} &\sum_{i=1}^{n-1} g(R(\varphi e_i, \varphi Y)\varphi Z, \varphi e_i) \qquad (6.6)\\ = \quad &\frac{1}{(n-1)} \sum_{i=1}^{n-1} [S(\varphi Y, \varphi Z) g(\varphi e_i, \varphi e_i) - S(\varphi e_i, \varphi Z) g(\varphi Y, \varphi e_i)] \\ &+\frac{(\varepsilon+\beta)}{(n-1)} \sum_{i=1}^{n-1} g(\varphi Y, \varphi Z) g(\varphi e_i, \varphi e_i) - \frac{(\varepsilon+\beta)}{(n-1)} \sum_{i=1}^{n-1} g(\varphi e_i, \varphi Z) g(\varphi Y, \varphi e_i) \\ &+\frac{\alpha}{(n-1)} \sum_{i=1}^{n-1} g(Y, \varphi Z) g(\varphi e_i, \varphi e_i) - \frac{\alpha}{(n-1)} \sum_{i=1}^{n-1} g(e_i, \varphi Z) g(\varphi Y, \varphi e_i) \\ &+\alpha \sum_{i=1}^{n-1} g(\varphi Y, \varphi Z) g(e_i, \varphi e_i) - \alpha \sum_{i=1}^{n-1} g(\varphi e_i, \varphi Z) g(Y, \varphi e_i) \end{aligned}$$

Now, using equations $(2.1), (2.2), (2.4), (2.5), (2.16), (4.8)-(4.16)$ and (5.8) in (6.6), we have

$$\begin{aligned} &S(Y,Z) \qquad (6.7)\\ = \quad &[(n-2)(\varepsilon+\beta) + \varepsilon(n-1)(\alpha^2-\beta^2) - (n-1)(\xi\beta)]g(Y,Z) \\ &+[2\varepsilon(n-2)(\xi\beta) - (n-2)(1+\varepsilon\beta)]\eta(Y)\eta(Z) \\ &+[\alpha - 2\varepsilon\alpha\beta(n-1) - (n-1)(\xi\alpha)]g(\varphi Y, Z) \\ &-[\varepsilon(\varphi Z)\alpha + \varepsilon(n-2)(Z\beta)]\eta(Y) - [\varepsilon(\varphi Y)\alpha + \varepsilon(n-2)(Y\beta)]\eta(Z), \end{aligned}$$

If $\alpha = 0$ and $\beta = constant$ in equation (6.7), we get

$$S(Y,Z) = [(n-2)(\varepsilon+\beta) - (n-1)\varepsilon\beta^2]g(Y,Z) + (2-n)(1+\varepsilon\beta)\eta(Y)\eta(Z). \qquad (6.8)$$

Therefore

$$S(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z),$$

where $a = (n-2)(\varepsilon+\beta) - (n-1)\varepsilon\beta^2$ and $b = (2-n)(1+\varepsilon\beta)$.
This result shows that the manifold under the consideration is an $\eta-$ Einstein manifold. Thus, we can state the following theorem:

Theorem 6.1. An $n-$dimensional φ-projectively flat indefinite trans-Sasakin manifold M^n with semi-symmetric metric connection is $\eta-$Einstein manifold if $\alpha = 0$ and $\beta = constant$.

7 Indefinite trans-Sasakian manifold with a semi-symmetric metric connection satisfying $\bar{R}.\bar{S} = 0$

Now, suppose that an M^n be $n-$dimensional indefinite trans-Sasakian manifold with a semi-symmetric metric connection $\bar{\nabla}$ satisfying the condition

$$\bar{R}(X,Y).\bar{S} = 0. \qquad (7.1)$$

Then, we have

$$\bar{S}(\bar{R}(X,Y)Z,U)+\bar{S}(Z,\bar{R}(X,Y)U)=0 \tag{7.2}$$

Now, replace $X=\xi$ in equation (7.2), using equations (2.11) and (4.2), we have

$$\begin{aligned}
&\varepsilon(\alpha^2-\beta^2)g(Y,Z)\bar{S}(\xi,U)-(\alpha^2-\beta^2)\eta(Z)\bar{S}(Y,U)-2\varepsilon\alpha\beta g(\varphi Y,Z)\bar{S}(\xi,U) \qquad (7.3)\\
&+2\alpha\beta\eta(Z)\bar{S}(\varphi Y,U)+\varepsilon(Z\alpha)\bar{S}(\varphi Y,U)-\varepsilon g(\varphi Y,Z)\bar{S}(gard\alpha,U)\\
&-\varepsilon g(\varphi Y,\varphi Z)\bar{S}(gard\beta,U)+\varepsilon(Z\beta)\bar{S}(Y,U)-\varepsilon(Z\beta)\eta(Y)\bar{S}(\xi,U)\\
&-\beta g(Y,Z)\bar{S}(\xi,U)+\varepsilon\beta\eta(Z)\bar{S}(Y,U)+\alpha g(\varphi Y,Z)\bar{S}(\xi,U)-\varepsilon\alpha\eta(Z)\bar{S}(\varphi Y,U)\\
&+\varepsilon(\alpha^2-\beta^2)g(Y,U)\bar{S}(\xi,Z)-(\alpha^2-\beta^2)\eta(U)\bar{S}(Y,Z)-2\varepsilon\alpha\beta g(\varphi Y,U)\bar{S}(\xi,Z)\\
&+2\alpha\beta\eta(U)\bar{S}(\varphi Y,Z)+\varepsilon(U\alpha)\bar{S}(\varphi Y,Z)-\varepsilon g(\varphi Y,U)\bar{S}(gard\alpha,Z)\\
&-\varepsilon g(\varphi Y,\varphi U)\bar{S}(gard\beta,Z)+\varepsilon(U\beta)\bar{S}(Y,Z)-\varepsilon(U\beta)\eta(Y)\bar{S}(\xi,Z)\\
&-\beta g(Y,U)\bar{S}(\xi,Z)+\varepsilon\beta\eta(U)\bar{S}(Y,Z)+\alpha g(\varphi Y,U)\bar{S}(\xi,Z)-\varepsilon\alpha\eta(U)\bar{S}(\varphi Y,Z)\\
=\;&0.
\end{aligned}$$

Using equations $(2.1)-(2.5),(2.12),(2.13),(4.17)$ and $(4.21)-(4.26)$ in equation (7.3), we get

$$\begin{aligned}
&[(\alpha^2-\beta^2)-\varepsilon(\xi\beta)-\varepsilon\beta]S(Y,Z) \qquad (7.4)\\
=\;&[\varepsilon(n-1)(\alpha^2-\beta^2)^2-2\beta(n-1)(\alpha^2-\beta^2)-2(n-1)(\alpha^2-\beta^2)(\xi\beta)\\
&+2\varepsilon\beta(n-1)(\xi\beta)+\varepsilon(\xi\beta)^2+(\varphi gard\beta)\alpha+(n-2)(grad\beta)^2\\
&+\varepsilon\beta^2(n-2)+(2\beta+\varepsilon)(n-2)(\alpha^2-\beta^2)+\beta(\alpha^2-\beta^2)\\
&-2\alpha^2\beta(n-2)-\varepsilon\alpha(\xi\alpha)-(2\varepsilon\beta+1)(n-2)(\xi\beta)-\varepsilon\beta(\xi\beta)\\
&-\varepsilon\beta(2\beta+\varepsilon)(n-2)+\varepsilon\alpha^2(n-2)]g(Y,Z)+[-\varepsilon(\varphi gard\beta)\alpha\\
&-\varepsilon(n-2)(grad\beta)^2-(n-2)(1+\varepsilon\beta)(\alpha^2-\beta^2)\\
&+2\varepsilon\alpha^2\beta(n-2)+\alpha(n-2)(\xi\alpha)+(\varepsilon+\beta)(n-2)(\xi\beta)\\
&+\beta(\varepsilon+\beta)(n-2)-\alpha^2(n-2)]\eta(Y)\eta(Z)+[-2\varepsilon\alpha\beta(n-1)(\alpha^2-\beta^2)\\
&+2(n-2)\alpha\beta^2+2\alpha\beta(n-1)(\xi\beta)-(n-1)(\alpha^2-\beta^2)(\xi\alpha)\\
&+\varepsilon\beta(n-2)(\xi\alpha)+\varepsilon(\xi\alpha)(\xi\beta)+(\varphi grad\alpha)\alpha+(n-2)\\
&g(grad\alpha,grad\beta)+\alpha(\alpha^2-\beta^2)-\varepsilon\alpha(\xi\beta)-2\alpha\beta(n-2)(\varepsilon+2\beta)\\
&-(1+2\varepsilon\beta)(n-2)(\varepsilon\alpha)+\varepsilon\alpha(\varepsilon+2\beta)(n-2)]g(\varphi Y,Z)\\
&+[\varepsilon(\xi\alpha)+2\alpha\beta-\varepsilon\alpha]S(\varphi Y,Z)+[(n-2)(\xi\beta)(Z\beta)\\
&-\varepsilon(\alpha^2-\beta^2)(\varphi Z)\alpha-\varepsilon(n-2)(\alpha^2-\beta^2)(Z\beta)+(\xi\beta)(\varphi Z)\alpha\\
&+\beta(\varphi Z)\alpha+\beta(n-2)(Z\beta)]\eta(Y)+[\varepsilon(\alpha^2-\beta^2)(\varphi Y)\alpha\\
&+\varepsilon(n-2)(\alpha^2-\beta^2)(Y\beta)-2\varepsilon\alpha\beta(\varphi^2Y)\alpha-2\varepsilon\alpha\beta(n-2)(\varphi Y\beta)\\
&-\beta(\varphi Y)\alpha-\beta(n-2)(Y\beta)+\alpha(\varphi^2Y)\alpha+\alpha(n-2)(\varphi Y\beta)]\eta(Z)\\
&-(Z\alpha)(\varphi^2Y)\alpha-(n-2)(Z\alpha)(\varphi Y\beta)-(Z\beta)(\varphi Y)\alpha\\
&-(n-2)(Y\beta)(Z\beta)+(n-2)(Z\beta)(\xi\beta).
\end{aligned}$$

If $\alpha = 0$ and $\beta = constant$ in equation (6.7), we get

$$S(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z), \tag{7.5}$$

where $a = -[\frac{(n-1)\varepsilon\beta^4+(n-2)(gard\beta)^2+(n-2)\varepsilon\beta^2-(n-2)(\varepsilon+2\beta)\beta^2-(n-2)\varepsilon\beta(\varepsilon+2\beta)+(2n-3)\beta^3}{\beta(\varepsilon+\beta)}]$
and $b = -[\frac{(n-2)(1+\varepsilon\beta)\beta^2+(n-2)(\varepsilon+\beta)\beta-(n-2)\varepsilon(grad\beta)^2}{\beta(\varepsilon+\beta)}]$.
This shows that M^n is an $\eta-$Einstein manifold. Thus, we can state the following:

Theorem 7.1. If an indefinite trans-Sasakian manifold with a semi symmetric metric connection $\bar{\nabla}$ satisfies $\bar{R}.\bar{S} = 0$, then the indefinite trans-Sasakian manifold is an $\eta-$ Einstein manifold if $\alpha = 0$ and $\beta = constant$.

8 Indefinite trans-Sasakian manifold with respect to a semi-symmetric metric connection satisfies $\bar{P}.\bar{S} = 0$

Now, we consider an indefinite trans-Sasakian manifold with a semi-symmetric connection $\bar{\nabla}$ satisfying

$$(\bar{P}(X,Y).\bar{S})(Z,U) = 0 \tag{8.1}$$

where $\bar{P}$ is the projective curvature tensor and $\bar{S}$ is the Ricci tensor with a semi-symmetric metric connection . Then, we have

$$\bar{S}(\bar{P}(X,Y)Z,U) + \bar{S}(Z,\bar{P}(X,Y)U) = 0 \tag{8.2}$$

Replace $X = \xi$ in the equation (8.2), we get

$$\bar{S}(\bar{P}(\xi,Y)Z,U) + \bar{S}(Z,\bar{P}(\xi,Y)U) = 0 \tag{8.3}$$

Putting $X = \xi$ in equation (5.1), we get

$$\bar{P}(\xi,Y)Z = \bar{R}(\xi,Y)Z - \frac{1}{(n-1)}[\bar{S}(Y,Z)\xi - \bar{S}(\xi,Z)Y \tag{8.4}$$

Using equations $(2.1),(2.2),(2.4),(2.11),(4.2),(4.17),(4.23)$ and (8.4) in equation (8.3), we get

$$
\begin{aligned}
& \frac{\varepsilon(n-1)(\alpha^2-\beta^2)+(n-2)(\varepsilon+\beta)}{(n-1)} g(Y,Z)\bar{S}(\xi,U) - \frac{1}{(n-1)} S(Y,Z)\bar{S}(\xi,U) \qquad (8.5)\\
& -\frac{(n-2)}{(n-1)}(1+\varepsilon\beta)\eta(Y)\eta(Z)\bar{S}(\xi,U) + \frac{\alpha-2(n-1)\varepsilon\alpha\beta}{(n-1)} g(\varphi Y,Z)\bar{S}(\xi,U)\\
& -\varepsilon g(\varphi Y,Z)\bar{S}(grad\alpha,U) - \varepsilon g(\varphi Y,\varphi Z)\bar{S}(grad\beta,U) + 2\alpha\beta\eta(Z)\bar{S}(\varphi Y,U)\\
& +\varepsilon(Z\alpha)\bar{S}(\varphi Y,U) + \varepsilon(Z\beta)\bar{S}(Y,U) - \varepsilon(Z\beta)\eta(Y)\bar{S}(\xi,U) - \varepsilon\alpha\eta(Z)\bar{S}(\varphi Y,U)\\
& -\frac{1}{(n-1)}\varepsilon(\xi\beta)\eta(Z)\bar{S}(Y,U) - \frac{(n-2)}{(n-1)}\varepsilon(Z\beta)\bar{S}(Y,U) - \frac{1}{(n-1)}\varepsilon(\varphi Z)\alpha\bar{S}(Y,U)\\
& +\frac{\varepsilon(n-1)(\alpha^2-\beta^2)+(n-2)(\varepsilon+\beta)}{(n-1)} g(Y,U)\bar{S}(\xi,Z) - \frac{1}{(n-1)} S(Y,U)\bar{S}(\xi,Z)\\
& -\frac{(n-2)}{(n-1)}(1+\varepsilon\beta)\eta(Y)\eta(U)\bar{S}(\xi,Z) + \frac{\alpha-2(n-1)\varepsilon\alpha\beta}{(n-1)} g(\varphi Y,U)\bar{S}(\xi,Z)\\
& -\varepsilon g(\varphi Y,U)\bar{S}(grad\alpha,Z) - \varepsilon g(\varphi Y,\varphi U)\bar{S}(grad\beta,Z) + 2\alpha\beta\eta(U)\bar{S}(\varphi Y,Z)\\
& +\varepsilon(U\alpha)\bar{S}(\varphi Y,Z) + \varepsilon(U\beta)\bar{S}(Y,Z) - \varepsilon(U\beta)\eta(Y)\bar{S}(\xi,Z) - \varepsilon\alpha\eta(U)\bar{S}(\varphi Y,Z)\\
& -\frac{1}{(n-1)}\varepsilon(\xi\beta)\eta(U)\bar{S}(Y,Z) - \frac{(n-2)}{(n-1)}\varepsilon(U\beta)\bar{S}(Y,Z) - \frac{1}{(n-1)}\varepsilon(\varphi U)\alpha\bar{S}(Y,Z)\\
= \; & 0.
\end{aligned}
$$

Putting $U=\xi$ and using equations $(2.1)-(2.5), (4.17)$ and $(4.21)-(4.26)$ in equation (8.5), we get

$$
\begin{aligned}
& [(\alpha^2-\beta^2)-\varepsilon(\xi\beta)-\varepsilon\beta]S(Y,Z) \qquad (8.6)\\
= \; & [\varepsilon(n-1)(\alpha^2-\beta^2)^2+(n-2)(\varepsilon+\beta)(\alpha^2-\beta^2)-\beta(n-1)(\alpha^2-\beta^2)\\
& -\beta(n-2)(1+\varepsilon\beta)-2(n-1)(\xi\beta)(\alpha^2-\beta^2)-(n-2)(1+\varepsilon\beta)(\xi\beta)\\
& -2\alpha^2\beta(n-2)-\varepsilon\alpha(n-2)(\xi\alpha)+\varepsilon\alpha^2(n-2)+\varepsilon\beta(n-1)+\varepsilon(\xi\beta)^2\\
& +(\varphi grad\alpha)\alpha+(n-2)(grad\beta)^2]g(Y,Z)+[(n-2)\beta(\varepsilon+\beta)-(n-2)(\alpha^2-\beta^2)\\
& +2(n-2)\varepsilon\alpha^2\beta+\alpha(n-2)(\xi\alpha)+(n-2)(\varepsilon+\beta)(\xi\beta)-\alpha^2(n-2)\\
& -\varepsilon(\varphi grad\beta)\alpha-\varepsilon(n-2)(grad\beta)^2]\eta(Y)\eta(Z)+[\alpha(\alpha^2-\beta^2)\\
& -2(n-1)\varepsilon\alpha\beta(\alpha^2-\beta^2)-2n\alpha\beta^2-\varepsilon\alpha(\xi\beta)-\varepsilon\beta(\xi\alpha)+2\alpha\beta(\xi\beta)\\
& -2(n-2)\alpha\beta(2\beta+\varepsilon)-(n-2)(1+2\varepsilon\beta)(\xi\alpha)+\alpha(n-2)(2\varepsilon\beta+1)\\
& -(n-1)(\alpha^2-\beta^2)(\xi\alpha)+(n-1)\varepsilon\beta(\xi\alpha)+\varepsilon(\xi\alpha)(\xi\beta)+(\varphi grad\alpha)\alpha\\
& +(n-2)g(grad\alpha,grad\beta)]g(\varphi Y,Z)+[2\alpha\beta+\varepsilon(\xi\alpha)-\varepsilon\alpha]S(\varphi Y,Z)\\
& +[\varepsilon(n+3)(\alpha^2-\beta^2)(Z\beta)+\beta(n-2)(Z\beta)-\varepsilon(\alpha^2-\beta^2)(\varphi Z)\alpha\\
& +(n-1)\beta(\varphi Z)\alpha+(\xi\beta)(\varphi Z)\alpha]\eta(Y)+[-2\varepsilon\alpha\beta(\varphi^2 Y)\alpha-2\varepsilon\alpha\beta(n-2)(\varphi Y\beta)\\
& +\alpha(\varphi^2 Y)\alpha+\alpha(n-2)(\varphi Y\beta)+\varepsilon(\alpha^2-\beta^2)(\varphi Y)\alpha+\varepsilon(n-2)(\alpha^2-\beta^2)(Y\beta)\\
& -\beta(\varphi Y)\alpha-\beta(n-2)(Y\beta)]\eta(Z)-(Z\alpha)(\varphi^2 Y)\alpha-(n-2)(Z\beta)(\varphi Y\beta)\\
& -(Z\beta)(\varphi Y)\alpha-\beta(n-2)(Y\beta).
\end{aligned}
$$

If $\alpha = 0$ and $\beta = constant$ in equation (8.6), we get

$$S(Y, Z) = ag(Y, Z) + b\eta(Y)\eta(Z), \tag{8.7}$$

where $a = -[\frac{(n-1)\varepsilon\beta^4-(n-2)\beta^2(\varepsilon+\beta)+(n-1)\beta^3-(n-2)\beta(1+\varepsilon\beta+(n-1)\varepsilon\beta+(n-2)(grad\beta)^2}{\beta(\varepsilon+\beta)}]$
and $b = -[\frac{(n-2)\beta(\varepsilon+\beta)+(n-2)\beta^2-(n-2)\varepsilon(grad\beta)^2}{\beta(\varepsilon+\beta)}]$.
This result shows that the manifold under the consideration is an $\eta-$Einstein manifold. Thus,we can state the following theorem:

Theorem 8.1. If an indefinite trans-Sasakian manifold with a semi symmetric metric connection $\bar{\nabla}$ satisfies $\bar{P}.\bar{S} = 0$, then the indefinite trans-Sasakian manifold is an $\eta-$ Einstein manifold if $\alpha = 0$ and $\beta = constant$.

9 Weyl conformal curvature tensor on indefinite trans-Sasakian manifold with a semi-symmetric metric connection

The weyl conformal curvature tensor $\bar{C}$ of type $(1, 3)$ of M^n an $n-$dimensional indefinite trans-Sasakian manifold with semi-symmtric metric connection $\bar{\nabla}$ is given by [16]

$$\begin{aligned}\bar{C}(X, Y)Z \quad = \quad & \bar{R}(X, Y)Z - \frac{1}{(n-2)}[\bar{S}(Y, Z)X - \bar{S}(X, Z)Y + g(Y, Z)\bar{Q}X \\ & -g(X, Z)\bar{Q}Y] + \frac{\bar{r}}{(n-1)(n-2)}[g(Y, Z)X - g(X, Z)Y],\end{aligned} \tag{9.1}$$

where $\bar{Q}$ is the Ricci operator with respect to the semi-symmetric metric connection $\bar{\nabla}$. Let M^n be an $n-$dimensional indefinite trans-Sasakian manifold . The Weyl conformal curvature tensor $\bar{C}$ of M^n with respect to the semi-symmetric metric connection $\bar{\nabla}$ is defined in equation (9.1).

Now, taking inner product with U in equation (9.1), we get

$$\begin{aligned}& g(\bar{C}(X, Y)Z, U) \\ = \quad & g(\bar{R}(X, Y)Z, U) - \frac{1}{(n-2)}[\bar{S}(Y, Z)g(X, U) - \bar{S}(X, Z)g(Y, U) \\ & +g(Y, Z)g(\bar{Q}X, U) - g(X, Z)g(\bar{Q}Y, U)] + \frac{\bar{r}}{(n-1)(n-2)} \\ & [g(Y, Z)g(X, U) - g(X, Z)g(Y, U)].\end{aligned} \tag{9.2}$$

Using equations (2.4), (4.2), (4.17), (4.18) and (4.20) in equation (9.2), we get

$$\begin{aligned}& \bar{C}(X, Y, Z, U) \\ = \quad & g(R(X, Y)Z, U) - \frac{1}{(n-2)}[S(Y, Z)g(X, U) - S(X, Z)g(Y, U) \\ & +g(Y, Z)g(QX, U) - g(X, Z)g(QY, U)] + \frac{r}{(n-1)(n-2)} \\ & [g(Y, Z)g(X, U) - g(X, Z)g(Y, U)]\end{aligned} \tag{9.3}$$

where $g(\bar{C}(X,Y)Z,U) = \bar{C}(X,Y,Z,U)$ and $g(R(X,Y)Z,U) = C(X,Y,Z,U)$ are Weyl curvature tensor with respect to semi-symmetric metric connection and metric connection respectively. we have

$$\bar{C}(X,Y,Z,U) = C(X,Y,Z,U), \tag{9.4}$$

where

$$\begin{aligned} & C(X,Y,Z,U) \\ = \; & g(R(X,Y)Z,U) - \frac{1}{(n-2)}[S(Y,Z)g(X,U) - S(X,Z)g(Y,Z) \\ & +g(Y,Z)g(QX,U) - g(X,Z)g(QY,U)] + \frac{r}{(n-1)(n-2)} \\ & [g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]. \end{aligned} \tag{9.5}$$

Theorem 9.1. The Weyl conformal curvature tensor of an indefinite trans-Sasakian manifold M^n with respect to a metric connection is equal to the weyl conformal curvature of with respect to semi-symmetric metric connection.

10 Indefinite trans-Sasakian manifold with weyl conformal flat conditions with a semi-symmetric metric connection

Let us consider that the indefinite trans-Sasakian manifold M^n with respect to the semi-symmetric metric connection is Weyl conformally flat, that is $\bar{C} = 0$. Then from equation (9.1), we get

$$\begin{aligned} \bar{R}(X,Y)Z \;\; = \;\; & \frac{1}{(n-2)}[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y + g(Y,Z)\bar{Q}X \\ & -g(X,Z)\bar{Q}Y] - \frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{10.1}$$

Now, taking the inner product of equation (10.1) with U . Then, we get

$$\begin{aligned} g(\bar{R}(X,Y)Z,U) \;\; = \;\; & \frac{1}{(n-2)}[\bar{S}(Y,Z)g(X,U) - \bar{S}(X,Z)g(Y,U) \\ & +g(Y,Z)g(\bar{Q}X,U) - g(X,Z)g(\bar{Q}Y,U)] \\ & -\frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)g(X,U) \\ & -g(X,Z)g(Y,U)]. \end{aligned} \tag{10.2}$$

Using equations $(2.4), (4.2), (4.17), (4.18)$ and (4.20) in equation (10.2), we get

$$\begin{aligned} g(R(X,Y)Z,U) \;\; = \;\; & \frac{1}{(n-2)}[S(Y,Z)g(X,U) - S(X,Z)g(Y,U) \\ & +g(Y,Z)g(QX,U) - g(X,Z)g(QY,U)] \\ & -\frac{r}{(n-1)(n-2)}[g(Y,Z)g(X,U) \\ & -g(X,Z)g(Y,U)]. \end{aligned} \tag{10.3}$$

Putting $X = U = \xi$ in equation (10.3) and using equations (2.2), (2.3) and (2.4), we get

$$\begin{aligned} & g(R(\xi, Y)Z, \xi) \\ = & \frac{1}{(n-2)}[\varepsilon S(Y, Z) - \varepsilon\eta(Y)S(\xi, Z) + g(Y, Z)S(\xi, \xi) \\ & -\varepsilon\eta(Z)S(Y, \xi)] - \frac{r}{(n-1)(n-2)}[\varepsilon g(Y, Z) - \eta(Y)\eta(Z)], \end{aligned} \tag{10.4}$$

Where $g(QY, Z) = S(Y, Z)$.

Now, using equations (2.12), (2.13) and (2.16), we get

$$\begin{aligned} S(Y, Z) = & [(\xi\beta) - \varepsilon(\alpha^2 - \beta^2) + \frac{r}{(n-1)}]g(Y, Z) + [\varepsilon(n-4)(\xi\beta) \\ & +n(\alpha^2 - \beta^2) - \frac{\varepsilon r}{(n-1)}]\eta(Y)\eta(Z) - [2\varepsilon(n-2)\alpha\beta \\ & +(n-2)(\xi\alpha)]g(\varphi Y, Z) - [\varepsilon(\varphi Z)\alpha + \varepsilon(Z\beta)(n-2)]\eta(Y) \\ & -[\varepsilon(\varphi Y)\alpha + \varepsilon(n-2)(Y\beta)]\eta(Z). \end{aligned} \tag{10.5}$$

If $\alpha = 0$ and $\beta = constant$ in equation (8.6), we get

$$S(Y, Z) = [\varepsilon\beta^2 + \frac{r}{(n-1)}]g(Y, Z) + [-n\beta^2 - \frac{\varepsilon r}{(n-1)}]\eta(Y)\eta(Z). \tag{10.6}$$

Therefore

$$S(Y, Z) = ag(Y, Z) + b\eta(Y)\eta(Z)$$

where $a = [\varepsilon\beta^2 + \frac{r}{(n-1)}]$ and $b = [-n\beta^2 - \frac{\varepsilon r}{(n-1)}]$.
This shows that M^n is an $\eta-$Einstein manifold. Thus, we can state as follows:

Theorem 10.1. Let M^n be a $n-$dimensional Weyl conformally flat indefinite trans-Sasakian manifold with respect to semi symmetric metric connection $\bar{\nabla}$ is an $\eta-$Einstein manifold if $\alpha = 0$ and $\beta = constant$.

Now, taking equation (9.1), we have

$$\begin{aligned} \bar{C}(X, Y)Z = & \bar{R}(X, Y)Z - \frac{1}{(n-2)}[\bar{S}(Y, Z)X - \bar{S}(X, Z)Y \\ & +g(Y, Z)\bar{Q}X - g(X, Z)\bar{Q}Y] \\ & +\frac{\bar{r}}{(n-1)(n-2)}[g(Y, Z)X - g(X, Z)Y]. \end{aligned} \tag{10.7}$$

Using equations (2.15), (4.2)(4.17), (4.18) and (4.20) in equation (10.7), we get

$$\begin{aligned} &\bar{C}(X,Y)Z \qquad (10.8)\\ = \ &C(X,Y)Z + (2\beta+\varepsilon)[g(X,Z)Y - g(Y,Z)X\ ]\\ &+(\varepsilon+\beta)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi\\ &+(1+\varepsilon\beta)\eta(Z)[\eta(Y)X - \eta(X)Y\ ] - \alpha[g(\varphi X,Z)Y\\ &-g(\varphi Y,Z)X - g(Y,Z)\varphi X\ + g(X,Z)\varphi Y] - \frac{1}{(n-2)}\\ &[(1+\varepsilon\beta)(n-2)\eta(Y)\eta(Z) - ((2\beta+\varepsilon)(n-2)+\beta)g(Y,Z)X\\ &+\alpha(n-2)g(\varphi Y,Z)X + ((2\beta+\varepsilon)(n-2)+\beta)g(X,Z)Y\\ &-(1+\varepsilon\beta)(n-2)\eta(X)\eta(Z)Y - \alpha(n-2)g(\varphi X,Z)Y\\ &-((2\beta+\varepsilon)(n-2)+\beta)g(Y,Z)X + (\varepsilon+\beta)(n-2)g(Y,Z)\eta(X)\xi\\ &+\alpha(n-2)g(Y,Z)\varphi X + ((2\beta+\varepsilon)(n-2)+\beta)g(X,Z)Y\\ &-(\varepsilon+\beta)(n-2)g(X,Z)\eta(Y)\xi - \alpha(n-2)g(X,Z)\varphi Y]\\ &-\frac{2\beta+(2\beta+\varepsilon)(n-2)}{(n-2)}[g(Y,Z)X - g(X,Z)Y]. \end{aligned}$$

Let X and Y are orthogonal basis to ξ . Putting $Z = \xi$ and using equations (2.1), (2.2) and (2.4) in equation (10.8), we get

$$\bar{C}(X,Y)\xi = C(X,Y)\xi. \qquad (10.9)$$

Theorem 10.2. A $n-$dimensional indefinite trans-Sasakian manifold M^n is Weyl $\xi-$conformally flat with respect to the semi-symmetric metric connection if and only if the manifold is also weyl $\xi-$conformally flat with respect to the metric connection provided that the vector fields are horizontal vector fields.

Example 10.3. Let us consider a $3-$dimensional manifold $M = \{(x,y,z) \in R^3 : z \neq 0\}$, where (x,y,z) are the standard co-ordinates in R^3. we choose the vector fields

$$e_1 = -z(\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}),\ e_2 = -z\frac{\partial}{\partial y},\ e_3 = \frac{\partial}{\partial z}$$

which are linearly independent at each point of M. Let g be the Riemannian metric defined by

$$\begin{aligned} g(e_1,e_2) &= g(e_2,e_3) = g(e_3,e_1) = 0,\\ g(e_1,e_1) &= g(e_2,e_2) = g(e_3,e_3) = \varepsilon. \end{aligned}$$

Let η be the $1-$form defined by $\eta(X) = \varepsilon g(X,\xi)$ for any $X \in \chi(M)$ and φ be the $(1,1)$ tensor field defined by

$$\varphi(e_1) = e_2,\ \varphi(e_2) = -e_1,\ \varphi(e_3) = 0.$$

Then using the linearity of φ and g, we have

$$\begin{aligned} \varphi^2 X &= -X + \eta(X)e_3,\ \eta(e_3) = 1,\\ g(\varphi X,\varphi Y) &= g(X,Y) - \varepsilon\eta(X)\eta(Y), \end{aligned}$$

where $e_3 = \xi$, the structure (φ, ξ, η, g) defines an indefinite Trans-Sasakian structure on M. Let ∇ the Levi-Civita connection with metric g, then we have

$$[e_1,e_2] = \varepsilon(-ye_2 - z^2 e_3),\ [e_1,e_3] = \frac{1}{z}\varepsilon e_1,\ [e_2,e_3] = \frac{1}{z}\varepsilon e_2.$$

The Riemannian connection ∇ of the metric g is given by

$$\begin{aligned} 2g(\nabla_X Y, Z) &= Xg(Y,Z) + Yg(Z,X) - Zg(X,Y) + g([X,Y],Z) \\ &\quad -g([Y,Z],X) + g([Z,X],Y), \end{aligned}$$

which known as Koszul's formula Taking $e_3 = \xi$ and using Koszul's formula, we have

$$\begin{aligned} \nabla_{e_1} e_1 &= -\frac{1}{z}\varepsilon e_3,\ \nabla_{e_1} e_2 = -\frac{1}{2}\varepsilon z^2 e_3, \nabla_{e_1} e_3 = \frac{1}{z}\varepsilon e_1 + \frac{1}{2}\varepsilon z^2 e_2, \\ \nabla_{e_2} e_1 &= \frac{1}{2}\varepsilon z^2 e_3 + \varepsilon y e_2, \nabla_{e_2} e_2 = \varepsilon(-ye_1 - \frac{1}{z}e_3), \nabla_{e_3} e_3 = 0, \\ \nabla_{e_2} e_3 &= \frac{1}{z}\varepsilon e_2 - \frac{1}{2}\varepsilon z^2 e_1,\ \nabla_{e_3} e_1 = \frac{1}{2}\varepsilon z^2 e_2,\ \nabla_{e_3} e_2 = -\frac{1}{2}\varepsilon z^2 e_1. \end{aligned}$$

Therefore, the semi-symmetric metric connection on M is given by

$$\begin{aligned} \bar{\nabla}_{e_1} e_1 &= -\frac{1}{z}\varepsilon e_3 - \varepsilon e_3,\ \bar{\nabla}_{e_1} e_2 = -\frac{1}{2}\varepsilon z^2 e_3,\ \bar{\nabla}_{e_3} e_3 = 0 \\ \bar{\nabla}_{e_1} e_3 &= \frac{1}{z}\varepsilon e_1 + \frac{1}{2}\varepsilon z^2 e_2 + \varepsilon e_1,\ \bar{\nabla}_{e_3} e_2 = -\frac{1}{2}\varepsilon z^2 e_1 \\ \bar{\nabla}_{e_2} e_1 &= \frac{1}{2}\varepsilon z^2 e_3 + \varepsilon y e_2,\ \bar{\nabla}_{e_2} e_2 = \varepsilon(-ye_1 - \frac{1}{z}e_3 - e_3), \\ \bar{\nabla}_{e_2} e_3 &= \frac{1}{z}\varepsilon e_2 - \frac{1}{2}\varepsilon z^2 e_1 + \varepsilon e_2,\ \bar{\nabla}_{e_3} e_1 = \frac{1}{2}\varepsilon z^2 e_2. \end{aligned}$$

Now, for $\xi = e_3$, above results satisfy

$$\bar{\nabla}_X \xi = -\varepsilon\alpha(\varphi X) + (1 + \varepsilon\beta)(X - \eta(X)\xi),$$

with $\alpha = -\frac{1}{2}z^2$ and $\beta = \frac{1}{z}$. Consequently $M(\varphi, \xi, \eta, g, \varepsilon)$ is a indefinite trans Sasakian manifold with semi-symmetric connection.

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