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S3.02 = S3.03

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Sobociński [1] asks whether S3.03 properly contains S3.02. To answer in the negative, it is enough to show that $C\mathfrak{t}1\mathfrak{t}2$ is a thesis of S3. Suppose for *reductio* that it is not. Then there is a Kripke model $\mathfrak{A} = \langle W, R, N\rangle$ for S3 and a valuation V on $\mathfrak{A}$ such that

$$V(C\mathfrak{C}\mathfrak{C}\mathfrak{C}pLppCLMLpp\mathfrak{C}\mathfrak{C}\mathfrak{C}pLpp\mathfrak{C}LMLpp, w) = \mathbf{F}$$

for some normal world w of $\mathfrak{A}$. Hence

$$V(\mathfrak{C}\mathfrak{C}\mathfrak{C}pLppCLMLpp, w) = \mathbf{T} \tag{1}$$

$$V(\mathfrak{C}\mathfrak{C}\mathfrak{C}pLpp\mathfrak{C}LMLpp, w) = \mathbf{F}. \tag{2}$$

From (2) and the fact that w is normal, it follows that

$$V(\mathfrak{C}\mathfrak{C}pLpp, x) = \mathbf{T} \tag{3}$$

$$V(\mathfrak{C}LMLpp, x) = \mathbf{F} \tag{4}$$

for some world x of $\mathfrak{A}$ where wRx. In light of (3), we know that x is normal. Thus (4) yields

$$V(CLMLpp, u) = \mathbf{F} \tag{5}$$

for some world u of $\mathfrak{A}$ where xRu. But now wRu by the transitivity of R, and so from (1) and the fact that w is normal we obtain

$$V(C\mathfrak{C}\mathfrak{C}pLppCLMLpp, u) = \mathbf{T},$$

whence, by (5), it follows that

$$V(\mathfrak{C}\mathfrak{C}pLpp, u) = \mathbf{F}.$$

We know that u is normal since (5) also entails that $V(LMLp, u) = \mathbf{T}$. Therefore

$$V(C\mathfrak{C}pLpp, z) = \mathbf{F}$$

for some world z of $\mathfrak{A}$ where uRz. However xRu and so by the transitivity of R we have xRz. Consequently, by (3) and the fact that x is normal,

$$V(C\mathfrak{C}pLpp, z) = \mathbf{T}$$

and we have a contradiction.

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REFERENCE

[1] Sobociński, B., "Modal system S3 and the proper axioms of S4.02 and S4.04," *Notre Dame Journal of Formal Logic*, vol. 14 (1973), pp. 415-418.

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