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SOME RECENT WORK ON THE ASSERTORIC SYLLOGISTIC

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Over the last few decades there have been many attempts to approach the Aristotelian syllogistic by utilizing the techniques of contemporary formal logic. The aim of this paper is to examine the most significant of these attempts and evaluate their fidelity to and consistency with Aristotle's own basic exposition of the syllogistic as expressed in the *Prior Analytics* (Book I, 1-2; 4-6).

Two major approaches to the formalization of the assertoric syllogistic can be distinguished in the literature. The first and older approach construes the syllogistic as an axiomatic system, while the second and more recent approach considers the syllogistic as a natural deduction system. Since many of the attempts of the first sort fail to be mentioned in current discussion, this paper will try to summarize them and only make a concluding reference to the second approach which is readily accessible in the more recent publications.

There are two main issues which must be confronted in the case of each attempt to present Aristotle's assertoric syllogistic as an axiomatic system: first, whether the method of representation, i.e., the logical alphabet and the well-formed formulas of the system, conforms to Aristotle's own approach; second, whether the specific formulas chosen as axioms and definitions, the rules of inference, and the manner of proof, etc., are faithful to or at least consistent with Aristotle's writings. Although it might appear that the first issue, a discussion of the logical symbols employed, is not of any real value, one must remember that Aristotle's logic seems tied to some basic philosophical or, better, metaphysical presuppositions. That there can be a close link between certain symbolical representations and some ontological positions is clear in the case of some other philosophers. One instance in the twentieth century is G. Bergmann whose espousal of a bare particularist theory of individuation is linked to his employment of a type of Russelian formal language [1].

Attempts at presenting the syllogistic in a formal way have proceeded along four lines: first, the attempt to present the syllogistic by means of the first-order predicate calculus; second, the classic attempt of Łukasiewicz

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to develop the syllogistic; third, the attempt to present the syllogistic as a theory of classes; fourth, Lejewski's attempt to relate the syllogistic to Leśniewski's ontology. Each of these attempts will be treated below in light of the two issues raised above.

1 *Predicate calculus* The use of the first-order predicate calculus to transcribe syllogistic laws is a practice that is quite widespread and can be found in most elementary logic textbooks. It is also found in some of the authors who have shown a special interest in the syllogism: Patzig employed this approach ([23], esp. ch. 2) and Smiley once advocated the adoption of many-sorted quantification in representing the syllogistic [28].

Three elements come into play in this representation of a syllogistic law: the explicit use of quantifiers, the use of indexicals, and the use of the formal implication sign to represent the relation of subject and predicate. This is clear in the standard representation of a syllogism like Barbara: $(\forall x(Mx \rightarrow Px) \wedge \forall x(Sx \rightarrow Mx)) \rightarrow \forall x(Sx \rightarrow Px)$. An interest in the historical accuracy of such a transcription demands that we examine Aristotle's own presentation in the *Prior Analytics* with respect to each of these elements.

Statements which today are represented with either the universal or existential quantifier were expressed by Aristotle without them. His most usual manner for expressing a universal affirmative statement (known to traditional logic as an A-statement: 'All A is B') is 'B belongs to all A' while the particular affirmative (I-statement: 'Some A is B') is written as 'B belongs to some A' (25a19-20 and 14-26; 26a23-27; 26a33-37; etc.) where 'belongs' translates the verb *hyparchein*. Aristotle does employ some other expressions for predication: 'B is in the whole of A' (alternatively 'B is in part of A') (*An. Pr.* 24b27; 25b32-34), 'B is predicated of all of A' (*An. Pr.* 25b37-39; 27a18-21; 27b4-5; etc.), and even uses simply the copula (*An. Pr.* 25a6-13). None of these alternate expressions seems to have the currency of *hyparchein*. There is no special term in Aristotle for the concept of quantifier; such an expression occurs only later in Theophrastus ([3], p. 44; [13], p. 46 and 65). Further, the universal quantifier's occurrence in a proposition is not incompatible with the occurrence of nonreferential terms in the same proposition, and hence the A-propositions transcribed into the language of the predicate calculus have no existential import—something not true of the propositions occurring in Aristotle's syllogistic ([24], pp. 164-170).

It may seem that beyond the aforementioned objections, the introduction of quantifiers presents no significant problems for use in the Aristotelian syllogistic. However, the employment of quantifiers necessitates the use of indexicals; quantification is made over a "dummy subject" rather than the predicates themselves. Thus, whereas Aristotle would write 'Animal is predicated of all of man' (*An. Pr.* 25a25), and thereby employ only general names in his syllogistic, the representation of the first order calculus will also employ the indexical, $\forall x(Mx \rightarrow Ax)$, where M stands for 'man' and A for 'animal'. The Kneale's have noted that Aristotle's use of general terms in this case is the result of his Platonic background and is no mere accident ([15], pp. 67-8; [30], pp. 54-56). However, it has been suggested by some and argued in detail by K. Gyekye that, since for Aristotle only individuals are truly substances, only individual names or name-variables can stand in the subject position of a natural predication and, as a result, the above translation using the first-order predicate calculus is quite fitting [14]. Thus in the case of an Aristotelian proposition, the subject term must be seen as merely an "apparent subject", i.e., it must be construed as a predicate term and an incomplete symbol. A proposition such as 'Piety is virtue' should be translated as $(x)\ (Px \rightarrow Vx)$ or even better as $\exists x(Px \wedge Vx)$ where P stands for 'piety' and V for 'virtue'. Such an employment of the quantifiers and indexicals, it is argued, nicely corresponds to Aristotle's own theory of substance and allows the syllogistic to reflect most accurately Aristotle's ontology as reflected in the *Categories* and *Metaphysics*.

However, some serious reservations must be expressed with regard to this proposal. First of all, there is the difficulty of such a reductive program in general. The translation of "All man is animal" to "Every man is an animal" or the translation of "Piety is virtue" to "Something pious is virtuous" is not a move that can be easily made in all cases. The example cited by Gyekye, 'Piety is virtue' is especially interesting in this regard since it is reminiscent of a passage in Quine which shows some difficulties in trying to reduce all statements containing abstract singular terms to those which do not ([25], pp. 122-23). Aristotle makes no clear distinction between abstract singular terms and concrete general terms and there is no positive evidence to indicate that he ever tried to reduce statements containing either kinds of terms to statements making reference only to individuals. Second, in support of his contention that all statements containing apparent reference to second substances are fittingly translated into statements making reference—which reference is best displayed in the predicate calculus notation—only to first substances, Gyekye cites passages not only from the *Metaphysics* but also from the *Categories*, the work in which the first-second substance distinction is clearly formulated. This latter work, however, seems to militate against the position he adopts, for in that work Aristotle makes provision for different kinds of predication, each of which, however, would presumably be symbolized in a different way. This is indeed a point that is developed by Surdu who elaborated a system of symbols for the various kinds of predication Aristotle envisioned [31]. Vuillemin also noted that the different kinds of predication demand different symbolic representations ([35], pp. 44-125). Third, even if the representations proposed for the propositions, i.e., concerning $\forall x(Mx \rightarrow Ax)$ and $\exists x(Mx \wedge Ax)$, were admitted the question arises what might be the referent of the indexical 'x'? Aristotle's own presentation leads one to believe that he is dealing with relations between what is directly represented by the general terms whereas in the functional representation the original subject and predicate terms of an Aristotelian proposition only indirectly fall under reference. A realistic construal of the terms, i.e., "For all things such that they possess the *property* man, they also possess the *property* animal," is not wholly appropriate to Aristotle's approach. Such a translation seems to speak of "something" being a substance of this or that sort, of possessing this or that property. It seems to reflect a Lockean notion of substance as a "something I know not what" underlying the accidents or qualities that are said to inhere in it. However, the Aristotelian notion of substance as a kinded thing [18] is not that of Locke. Although a Lockean theory might well be represented by the above symbolism, an Aristotelian one is not. Bochenski has argued that this functional mode of representation not only is compatible with but also neatly expressive of the Aristotelian ontology. The union of an indeterminate matter and form which results in a substance is fitly portrayed by the juxtaposition of an incomplete predicate symbol and a variable—which is a genuine Aristotelian predication ([3], pp. 48-51). Yet, it must be remembered that for Aristotle prime matter cannot itself be the subject of a predication, since it is not itself an actuality (*Metaph.* VII, 1029a12ff). Similarly to suppose that the designate of the indexical is an instance of matter determined by certain quantitative dimensions does not obviate the difficulty. According to Aristotle, to say that "such and such a thing is a man" is merely to express a nonnatural predication (*An. Post.* 83a1ff., esp. a14-16).

The final element in this consideration of the translation of Aristotelian propositions is the use of the implication sign as a copula, at least in the case of universal propositions. This conditional statement of the universal affirmative was first proposed by Bradley in the last century in the hope of preventing a reduction of a universal to its instances ([4], c. 2 sec. 44). Its use along with quantifiers of the predicate calculus, however, tends (as noted above) to a reductive approach. Prior noted that although many laws employing formal implication seem to resemble laws of the Aristotelian syllogistic, the paradoxes which would arise from formal implication (analogous to those which arise in material implication) have no place in Aristotle's system [24], pp. 80-81. He also noted the lack of existential import in such propositions as another factor which, from an Aristotelian perspective, renders such a representation questionable ([24], p. 170). Finally, it seems that the connection between subject and predicate was seen by Aristotle as atomic and that the use of an implication sign can lead one to construe the subject-predicate relationship and the premiss-conclusion relationship as the same since both are signified by the same connective.[1] Such a construal is foreign to the text of Aristotle.

The above criticisms of the use of the predicate calculus in representing Aristotelian propositions must be tempered by some historical evidence which might support their use. Theophrastus, Aristotle's successor, developed the so-called "prosleptic premisses" or premisses *kata proslepsin* which seem to resemble the universally quantified formulas of the predicate calculus ([3], pp. 50-51). The resemblance of the "third figure" of these premisses to the modern formulation is especially striking: "to whatever entity *b* belongs in every instance, *a* belongs to it in every instance" ([5], 189b43). Of course, were the occurrences of such formulas limited to the work of Theophrastus, one would have no certain evidence that Aristotle would have considered them as appropriate. However, some passages in Aristotle indicate that he himself was aware of these so-called "prosleptic premisses" and even noted their equivalence to his more usual formulation of statements (*An. Pr.* 32b29). Nonetheless, it is important to note that these formulas frequently occur in a modal context, or in Book II of the *Prior Analytics*, and not in the basic exposition of the assertoric syllogistic itself. Due to their relatively infrequent appearance, any textual support based on such passages for the use of the functional calculus must be seen as minimal.[2]

Thus far it seems that no one has presented a set of axioms, rules, and definitions formulated in the language of the first-order calculus and with these made derivations of the various valid syllogistic moods. Vieru has attempted to embed the assertoric syllogistic into the predicate calculus, but has shown interest more in the metalogical questions involved than in a detailed presentation of the syllogistic itself [34].

2 *The Łukasiewicz school* A second approach to a formalized presentation of the syllogistic can be found in a number of logicians employing the notation of the Polish school, and whose formalization follows the lines laid down by J. Łukasiewicz best expressed in his significant work *Aristotle's Syllogistic*.[3] Included among these logicians are I. Bochenski [2] and I. Thomas [33]. In some ways this approach has come to be considered standard, probably (and justly) due to the wealth of detailed and critical observations contained in Łukasiewicz' work which makes numerous references to the texts of Aristotle himself.

Łukasiewicz presents Aristotle's syllogistic as an axiomatic system of laws. These laws are stated as implications, since, according to Łukasiewicz, a syllogism is not to be understood as an inference rule in the way the tradition formulated it. This notion he tries to support both by reference to Aristotle's statement of various moods of the syllogism and by deducing all of the moods of the syllogism traditionally recognized as valid within the system he sets forth.

The logical alphabet employed in this syllogistic makes use of the Polish notation (save that 'Q' is employed for equivalence to avoid confusion with the syllogistic operator 'E') ([19], p. 108). Small Latin letters are used as name variables whose substituends are to be general names. The large Latin letters, *A, E, I, O* are binary operators in the formation of the four basic kinds of syllogistic premisses. The language includes both propositional variables and logical connectives from the standard propositional calculus.

The system itself is composed in the following way. Two definitions (stated as rules) of operators E and O are given in terms of the primitives A and I. There are two rules of inference: substitution and detachment. Four axioms are noted as specific to this theory of syllogistic:

A1 *Aaa*
A2 *Iaa*
A3 *CKAbcAabAac* (Barbara)
A4 *CKAbcIbaIac* (Datisi)

In addition to these, Łukasiewicz employs as axiomatic the following laws from the propositional calculus:

I. *CpCqp*
II. *CCqrCCpqCpr*
III. *CCpCqrCqCpr*
IV. *CpCNpq*
V. *CCNppp*
VI. *CCpqCNqNp*
VII. *CCKpqrCpCqr*
VIII. *CpCCKpqrCqr*
IX. *CCspCCKpqrCKsqr*
X. *CCKpqrCCsqCKpsr*
XI. *CCrsCCKpqrCKqps*
XII. *CCKpqrCKpNrNq*
XIII. *CCKpqrCKNrqNp*
XIV. *CCKpNqNrCKprq*.

Within such a system Łukasiewicz is able to deduce all the valid moods of the Aristotelian syllogistic. Bochenski's system differs only in minor details from that of Łukasiewicz, e.g., two rules of substitution, one rule of definition, and one rule of derivation, includes Ferio instead of Datisi among its basic axioms and includes a different set of laws from the propositional calculus ([2], pp. 20-22). Thomas' system differs somewhat more significantly since he employed an operator for term negation and was thereby able to construct the same system using only one syllogistic operator ([33], p. 40).[4]

The question now arises how exact a correspondence this system has with the syllogistic as developed by Aristotle. By employing the four syllogistic operators given above, Łukasiewicz seems to avoid the problems encountered by the translation into the predicate calculus which uses quantifiers. It also does not commit him to an extensionalist account of the syllogistic, although his treatment of necessity makes it clear that he endorses such an account ([19], pp. 10-12). His employment of the two laws of identity, *Aaa* and *Iaa*, among the axioms seems not to accord with Aristotle's practice nor, as von Wright noted, with his basic conception of logic.[5] Similarly, the use of only two of the first figure moods as axiomatic rather than all four marks a departure from Aristotle's method. The most striking discrepancy, however, can be noted in the large number of laws of the propositional calculus that are needed to complete the system. Although it is true that knowledge of the laws of the propositional calculus was not limited to the Stoics, Aristotle was certainly not cognizant of all of the laws listed above. Only VI, XII, and XIII can be found in some explicit fashion in the Aristotelian text: VI occurs in *An. Pr.* II, 57b1-17 and also seemingly in the *Topics* (113b17ff.; 124b7ff.); XII and XIII seem to be mentioned in *An. Pr.* II, 57a37-40. None of these, however, are cited by Aristotle in his exposition of the assertoric syllogistic.

Łukasiewicz' divergence from Aristotle's method becomes more obvious in the actual derivation of syllogistic moods. Consider the proof he presents for *Disamis*. Łukasiewicz first invokes the following law, a theorem of his system:

$$CCKpqIbaCKqpIab$$

which after the appropriate substitutions (p/Aba, q/Ibc, b/c) yields:

$$CCKAbaIbcIcaCKIbcAbaIac.$$

Then, by Axiom 4 given above (Datisi) and one application of *modus ponens* one arrives at the thesis:

$$CKIbcAbaIac$$

which is the valid syllogistic mood known as Disamis. This proof is valid, of course, but it is certainly far more complex than the one used by Aristotle at *Pr. An.* I, 28b7ff. Aristotle proves Disamis by a reduction to *Darii*. Reduction is one of the common techniques used by Aristotle to establish the validity of a mood not in the first figure. It consists basically in transforming the mood in question into a first figure mood that has already been recognized as valid. The reduction of Datisi to Darii can be portrayed as follows:[6]

$$\begin{aligned}&Aba,\ Ibc - \text{(by Datisi)} \rightarrow Ica\\ &Aba,\ Ibc - \text{(simple conversion)} \rightarrow Aba,\ Icb\\ &\qquad\qquad - \text{(by Darii)} \rightarrow Ica.\end{aligned}$$

According to Aristotle, then, all that need be done in order to establish the validity of this mood is to perform a conversion on the minor premiss. This conversion transforms the syllogism into a valid mood of the first figure. Since the first figure moods can be accepted as intuitively obvious, the proof is complete. It should be noted that in some cases Aristotle can (and in the case of Datisi does) make use of two alternative techniques to establish the validity of a syllogism: *reductio ad impossibile* and *ecthesis*.[7] In the former technique, Aristotle assumes the contradictory of the original conclusion and also one of the premisses of the original syllogism thereby to arrive at a conclusion which contradicts the other premiss known to be true. The proof of Datisi by *reductio ad impossibile* is as follows:

$$\begin{aligned}&Aba,\ Ibc - \text{(by Datisi)} \rightarrow Ica\\ \text{is broken up into } &Ibc,\ NIca - \text{(by reductio)} \rightarrow Ibc,\ Eca\\ &\qquad - \text{(by Ferio)} \rightarrow Oba\\ &\qquad - \text{(equivalent)} \rightarrow NAba\\ &\text{ergo, } Aba,\ Ibc,\ NAba \text{ is an impossible combination}\\ &\text{ergo, } Aba,\ Ibc \rightarrow Ica.\end{aligned}$$

In other words, the assumption of the contradictory of the conclusion ($NIca$) together with the premiss Ibc yields a conclusion (Oba) which contradicts the other premiss (Aba). This alternative method of proof Łukasiewicz finds objectionable since in his opinion a syllogistic mood is not an inference but a proposition, and therefore the contradictory of the whole proposition and not merely a part of it must be assumed as false ([19], p. 56). Mignucci has followed Łukasiewicz on these points and has noted that in both the case of reduction and *reductio*, some use must be made of the propositional calculus ([22], p. 55 and p. 57), and he goes on to give a presentation of these methods which differs only in slight detail from Łukasiewicz' own method ([22], p. 231 and p. 234). However, it is this discrepancy between Łukasiewicz' account and that of Aristotle that led some to believe that perhaps Łukasiewicz' system was really not a faithful translation of Aristotle's syllogistic [8-12, 29].

3 *The class theoretic approach* The third major approach to the syllogistic which has appeared in the literature employs a theory of classes. Wedberg's was the first sophisticated endeavor which provided an axiomatic base to which the ambiguity in such statements as 'Man is animal' (whose subject and predicate terms could be read as making reference either to the properties man and animal or to the classes whose members had those properties) Wedberg's was the first sophisticated endeaver which provided an axiomatic base to resolve the ambiguity in such statements as 'Man is animal' (whose terms could be read as making reference either to the properties man and animal or to the classes whose members had those properties) in favor of classes [36]. Wedberg's main interest was not in interpreting the texts of Aristotle or even providing a derivation of the valid syllogistic moods, but rather centered on the foundations of the system itself and some of metalogical questions (consistency and completeness) concerning it. Menne also adopted a class theoretic approach but, in contradistinction to Wedberg, has a keen awareness of other attempts made to formalize the syllogistic as it is found both in Aristotle and the tradition, and provides derivations for the various syllogistic moods [20 and 21]. These later derivations make Menne's system of greater interest for the purposes of this paper, but Wedberg raised some issues that are also of interest.

The language employed by Wedberg bears some resemblance to that used by Łukasiewicz. The large Latin letters A, E, I, and O are used as operators which represent relations between classes designated by small Roman letters. The symbol x' is used to designate the complement of the class x. In addition to these special symbols, Wedberg employed any number of the connectives of the propositional calculus. Thus a mood like Barbara is presented as $(Aab \;\&\; Abc) \rightarrow Aac$, with '&' standing for conjunction and '$\rightarrow$' for implication. Thus it becomes clear that for Wedberg the syllogistic moods are theses and not rules of inference.

In the system constructed with this language, Wedberg employs A as the only primitive operator, a move followed later by Thomas [33]. Wedberg finds it possible to do this because of his use of class complementation. *Modus ponens* and substitution are the rules of inference. As did Łukasiewicz, Wedberg included within his system the needed valid formulas of the propositional calculus. The inclusion of theses of the propositional calculus puts Wedberg among the many authors who believe the syllogistic presupposes an underlying logic ([7], pp. 57-8)—a position criticized by those who consider Aristotle's logic independent of the propositional calculus [8-12, 29]. The special set of axioms needed for the system are the following:

A1 Aaa''
A2 $Aa''a$
A3 $Aab \rightarrow Ab'a'$
A4 $(Aab \; \& \; Abc) \rightarrow Aac$
A5 $Aab \rightarrow \text{-}Aab'$.

Since x' stands for the complement of a class, Axioms 1 and 2 are axioms of double complementation; Axiom 3 is merely one of contraposition; Axiom 4 is the transitivity of class inclusion (or on this model of the syllogistic, Barbara). Axiom 5 ensures that the Aristotelian classes treated by the system are not empty ($\text{-}Aab'$ is equivalent to Iab) and is thus a form of the law of subalternation.

Wedberg did not go on to develop the various valid syllogistic moods from these axioms, but rather considered some of the properties of the system, as noted earlier. Our criticism of his system must be limited, therefore, to the points raised above. Wedberg's use of a single operator as primitive differs from Aristotle's presentation which seems to employ all four types of propositions as basic although one must concede that Aristotle's admission that all valid moods can be reduced to the two universal moods of the first figure (*An. Pr.* 29b1) might be a justification for trying to make the system as elegant as possible. Although it is true that in chapter x of the *Peri Hermeneias* Aristotle makes use of alpha-privative predicates, e.g., nonwhite, nonjust, etc., he does not employ them in the systematic development of syllogistic to reduce the number of operators, nor would such predicates be considered as class complements. Wedberg's use of Axioms 1 and 2 is subject to the same criticism von Wright raised with respect to Łukasiewicz' axioms of identity ([37], p. 21); Aristotle did develop his syllogistic without using them. Finally, the use of the propositional calculus in the system is not supported by Aristotle's practice.

Although Wedberg was the first to initiate an axiomatized set theorectic approach to the syllogistic, it was Menne who developed the derivations for the valid moods of the syllogistic within a theory of classes. Menne's work is valuable, moreover, because it includes a historical summary of the algebraic approach to the syllogism (Boole, Venn, Schröder) besides some other nineteenth-century logicians, and sets out several alternative class approaches to the syllogism and discusses some of their interrelationships, although he does not raise the metalogical issues discussed by Wedberg. Menne aims to provide a formalization of the syllogistic which will avoid the problems it has met in other authors—problems arising mostly from the existential presupposition involved in some of the modes and in the rules of subalternation and conversion. His system is able to accommodate the laws of the square of opposition, the rules of conversion, and the valid moods of the syllogism.

Menne employs Latin capitals (S, P, M) as nonlogical symbols to designate classes and initially also as variables in propositions having the syllogistic operators designated by small Latin a, e, i, and o. The symbol P' is used to designate the complement of a class. In addition to some connectives from the propositional calculus ($\rightarrow$ for implication, $\wedge$ for conjunction, $\overline{p}$ for negation-p), Menne uses the symbols $\Subset$ and $\pitchfork$ to designate definite inclusion and definite intersection respectively.[8] He also employs variables of the propositional calculus.

The system he establishes is built upon the following elements. The three axioms used are (Menne's numbering):

15.011 $S\ a\ P \leftrightarrow P'\ a\ S'$
15.012 $S\ a\ P \rightarrow \overline{S\ a\ P'}$
15.013 $M\ a\ P \wedge S\ a\ M \rightarrow S\ a\ P$

and the three definitions:

15.014 $S\ e\ P = S\ a\ P'$
15.015 $S\ i\ P = \overline{S\ a\ P'}$
15.016 $S\ o\ P = \overline{S\ a\ P}$.

A number of rules such as double negation, commutation, *modus ponens*, two rules of substitution are employed in addition to the following theses from the propositional calculus:

15.0191 If $p \wedge q \rightarrow r$, then $p \wedge \overline{r} \rightarrow \overline{q}$
15.0192 If $p \leftrightarrow q$ and $q \rightarrow r$, then $p \rightarrow r$
15.0193 If $p \wedge q \rightarrow r$ and $s \rightarrow q$, then $p \wedge s \rightarrow r$
15.0194 If $p \rightarrow q$ and $q \rightarrow r$, then $p \rightarrow r$.

By using the two following symbols for class relations:

$$S \Subset P \text{ for } S\ a\ P$$
$$S \pitchfork P \text{ for } S\ i\ P$$

and the notion of class complement he can derive the valid moods of the syllogistic. The following are cited to show his procedure (the Polish deduction method is used).

15.411 $M \Subset P \wedge S \Subset M \rightarrow S \Subset P$ (Barbara)—assumed

15.411 P/P' yields
15.412 $M \Subset P' \wedge S \Subset M \rightarrow S \Subset P'$ (Celarent)

15.413 $M \Subset P \wedge S \pitchfork M \rightarrow S \pitchfork P$ (Darii)—assumed

15.413 P/P' yields
15.414 $M \Subset P' \wedge S \pitchfork M \rightarrow S \pitchfork P'$ (Ferio)

15.412 $I \times 15.31$ (i.e., $S \Subset P' \leftrightarrow P \Subset S'$) yields
15.421 $P \Subset M' \wedge S \Subset M \rightarrow S \Subset P'$ (Cesare).

Menne notes that one of the advantages of his axiom system over that of Łukasiewicz and others is that it avoids the difficulties associated with the reflexive laws ([26], p. 143).

While one can admit that Menne's book contains a wealth of information and detail, his approach allows for criticism in several points. The attempt to give an extensionalist account of the syllogistic has been criticized above at different points, and there is no need to repeat that criticism here. Of more importance, however, is a closer inspection of the derivations of the valid moods. In this regard three remarks can be made. First, Menne employs only Barbara and Datisi as axiomatic whereas Aristotle considers the other two moods of the first figure, Celarent and Ferio, also as basic, as "perfect" moods. Second, the notion of class complement which allows Menne to do without either of the negative moods mentioned is not employed as such in the Prior Analytics. His proof of Cesare cited above, for instance, depends on 15.31 which is, at most, analogous to Aristotle's law of E-conversion. Thirdly, his method of proof resembles that of Łukasiewicz and not that of Aristotle; rather than reducing Cesare to Celarent as does Aristotle, he derives Cesare from Celarent with a "converted" premiss (see above). In a similar fashion he derives the other moods. His use of theses of the propositional calculus, of course, also distinguishes his approach from that of Aristotle.

4 *Approaches based on Leśniewski* Only a few remarks can be made with regard to this last variety of approaches to the syllogistic, since very little work has been done on the syllogistic which utilizes the investigations of Leśniewski. However, Lejewski did publish a paper which aimed at extending the syllogistic to arrive at the basic ontology of Leśniewski [17]. Although he employs much of Leśniewski's symbolism, he does not proceed to derive the valid syllogistic moods themselves but rather accepts Łukasiewicz' system (with alternative symbolism) and thus uses the same four axioms (two laws of identity, two moods Barbara and Datisi), definitions of E and O propositions, rules of substitution and detachment, and the relevant theses of the propositional calculus. He also includes the requirement of existential import for the basic Aristotelian propositions. Lejewski then expands this system to include quantifiers, noting that the range of the value of the variables must be referential names. Since a system with quantifiers of such a restricted interpretation poses problems, he proposes a system where the quantifiers have an unrestricted interpretation, and subsequently proceeds to extend the syllogistic to include universal affirmative propositions without existential import. He sets up two such systems, shows that they are inferentially equivalent and then proceeds to extend them into a basic Boolean algebra and finally basic ontology.

As is clear from this account, Lejewski's work does not deal primarily with providing a formal model of syllogistic, but is rather concerned with relating the syllogistic to ontology. Insofar as he follows Łukasiewicz, his presentation is subject to similar criticisms, e.g., his use of Barbara and Datisi as the two basic axioms is a departure from Aristotle's practice (as is his use of a single axiom in one of the extensions).

One further aspect of Lesniewski's work is of interest with regard to the syllogistic. Vuillemin has discussed the possibility of using mereology as an approach to the syllogistic. Indeed, it initially seems that a theory of parts might well correspond to some of the intuitions of Aristotle whose terminology for I-propositions and A-propositions is occasionally 'being in a part' (*en merei einai*) and 'being in a whole' (*en to holo einai*). However, there seems to be no such formalization in the literature as yet.[9]

5 *Conclusion* The works noted above include the classic formalizations of the syllogistic as an axiomatic system. Recently, however, another approach seems to be gaining acceptance, that which considers the syllogistic as a natural deduction system. Smiley [29] and Corcoran [8-12] independently developed such an approach. They argue that such an approach recommends itself; first, the syllogistic moods are no longer conditionals but deductions; second, the development of the syllogistic does not rest on the use of the propositional calculus—it needs no "underlying logic"; third, Aristotle's methods of reduction and *reductio* are employed unaltered. The authors argue that in each of these points their approach is much more faithful to the text of Aristotle and they offer deductions of various moods to illustrate this. The motivation behind the work of these two authors, i.e., an accurate rendering of the Aristotelian text, is of course commendable, and of all interpretations theirs takes the most concern to follow Aristotle's word and procedure. However, it is important to bear in mind Mignucci's remarks that contemporary logic can only serve as a partial and incomplete model of Aristotle's logic and can merely be a heuristic device for better understanding the text. Models can be both revealing and misleading—one model rejecting parts of a system that can be accepted in another ([22], p. 47). Each model, one might add therefore, has its value in throwing some light on Aristotle's work. The very ambiguity of Aristotle's language and his failure to be more explicit about his logical enterprise have invited the many recent attempts to give a formal presentation of the syllogistic. Each of these has been shown to have its failings, nonetheless each has contributed to a better understanding of Aristotle's syllogistic and more generally of his logic.

NOTES

1. Patzig identifies them, see [23], pp. 43-47. D. Burrell's construal of the syllogism as in some way exhibiting the *Dictum de omni et nullo* principle would also prepare the way for the identification of the two types of necessity; see [6]. For an explanation of the *Dictum de omni et nullo* principle, see [24], pp. 111-112.

2. Moreover it should be noted that Lejewski does not translate this into the functional calculus in the way that Bocheński does and rejects Bocheński's interpretation. See [16], esp. p. 169. See also Prior's discussion [24], p. 121.

3. See [19]. Work by Łukasiewicz and some other Polish logicians had been done on this topic before 1951—references in J. Słupecki [27] and R. Suszko [32].

4. J. C. Shepherdson in [26], p. 140, seems to categorize Thomas' approach as a class-theoretic axiomatization which it is not.

5. See [37], p. 21: "It seems to me much more to the point to say that *the notion of logical truth is unknown to Aristotle* [author's italics]."

6. Adopted from [24], pp. 115-116.

7. Ecthesis is a mode of proof which utilizes references to an individual instance to guarantee the validity of a particular mood.

8. These are operations on classes which are distinct from the null and universal classes.

9. In the opinion of Vuillemin [35], Leśniewski's mereology is closer in spirit than his ontology to Aristotle's work (p. 124), but notes that the mereology would suppress the distinction of first and second substance (p. 58).

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