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CHANNEL IDENTIFICATION UNDER DOPPLER AND TIME SHIFTS USING MIXED TRAINING SIGNALS

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Communicated by Charles Groetsch

This paper is dedicated with affection to Professor Zuhair Nashed.

ABSTRACT. Channel identification in the presence of Doppler is not as well studied as the one free from Doppler due to the difficulty caused from the time-varying characteristics of the channel. In this paper, we present a method to identify channels with both Doppler and time shifts using mixed training signals. The training signals we use consist of two parts, where one part is a constant and the other part is a conventional training signal, such as a pseudo-random signal or a chirp signal. These two parts in a training signal may be separated either in the time domain or in the frequency domain. The constant signal part is used to identify the Doppler shifts and the other part is used to identify the time shifts. We provide a necessary and sufficient condition on the channel identifiability in terms of the time and Doppler shifts when mixed training signals are used. It can be shown that the condition holds almost surely in most cases of interest in practice.

1. Introduction. Doppler and time shifts (or delays or spread) usually occur in wireless mobile communication systems with high speed transmission, which often causes problems of channel impairments. Due to the Doppler shifts of moving vehicles, the channel is usually modeled as a time variant linear system and is not as well studied as a time-invariant linear channel is. There has been a tremendous amount of research on time-invariant linear system identification with both blind

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and non-blind (using training signals) methods[1] This is, however, not equally the case for time-variant linear system identification. Some researches on this topic have appeared, such as [1-10], and increasing attention has being paid mainly because of the need of wireless high speed data communications.

In this paper, we focus on the problem of the channel identification in the presence of both Doppler and time shifts by using training signals. Specifically, the following channel model studied in [1] is used. Let $x(t)$ and $y(t)$ be transmitted and received signals, respectively. Then

$$y(t) = \sum_{k=1}^{N_p} \alpha_k x(t-\tau_k) e^{j\omega_k t} + n(t), \tag{1.1}$$

where α_k, τ_k, and ω_k are the path coefficient (complex-valued), the time shift (real-valued), and the Doppler shift (real-valued) of the kth multipath component in the channel, respectively, and N_p is the number of the total multipath components, and $n(t)$ is the channel additive noise. The Doppler shifts $\omega_k \approx 2v\omega_c/c$ with the carrier frequency ω_c, the velocity v of the moving object, and the velocity c of light. The channel identification here is to estimate the unknown parameters $\{(\alpha_k, \tau_k, \omega_k),\ 1 \le k \le N_p\}$ through knowledge of the transmitted and the received signals $x(t)$ and $y(t)$ in (1.1). Most of the existing methods for the channel identification are based on single type training signals called pilot signals.

In this paper, we propose to use mixed training signals in the above channel identification, which have two parts separated either in the time domain or in the frequency domain. One part of the training signal is a constant and the other part is a pseudo-random signal or other type of linear time-invariant (LTI) channel identification training signals, such as chirps. The constant part is used to identify the Doppler shifts ω_k and the other part is used to identify the time shifts τ_k. The corresponding multipath coefficients α_k are identified using both parts. Note that not all channels in (1.1) can be identified with this approach. A necessary and sufficient condition in terms of the Doppler and time shifts on the channel identifiability is given. It turns out that almost all channels (1.1) are identifiable with the approach proposed in this paper in most cases of practical interest.

[1] Since these methods are not used in this paper, they are not cited here.

This paper is organized as follows. In Section 2, we present channel analysis and mixed training signal analysis. In Section 3, we present a necessary and sufficient condition for the identifiability. In Section 4, numerical simulations are presented.

2. Channel and Mixed Training Signal Analyses. Let us first analyze the received signal $y(t)$ in (1.1). To analyze the identifiability, for convenience we assume the additive noise $n(t)$ in the model (1.1) does not appear, i.e., $n(t) = 0$. Suppose the transmitted training signal $x(t)$ is a constant, say 1. Then (1.1) becomes

$$y(t) = \sum_{k=1}^{N_p} \alpha_k e^{j\omega_k t}. \tag{2.1}$$

If all Doppler shifts ω_k, $k = 1, 2, ..., N_p$, are distinct, then, by taking a discrete Fourier transform of a certain length for a segment of the received signal $y(t)$, all Doppler shifts ω_k and multipath coefficients α_k may be detected. If there are duplications of the Doppler shifts ω_k, all the (distinct) Doppler shifts can still be detected with the method above, but not all the coefficients α_k. For instance, assume $\omega_1 = \omega_2$ and it is not equal to other ω_k. Equation (2.1) becomes

$$y(t) = (\alpha_1 + \alpha_2)e^{j\omega_1 t} + \sum_{k=3}^{N_p} \alpha_k e^{j\omega_k t}. \tag{2.2}$$

In this case, only the sum $\alpha_1 + \alpha_2$ of the two coefficients α_1 and α_2 can be detected, which is not enough to detect their individual values α_1 and α_2. However, the Doppler frequencies $\{\omega_1, \omega_2, ..., \omega_{N_p}\}$ are still detectable.

Similarly the time shifts τ_k can be detected in the frequency domain of (1.1) as follows. Taking the Fourier transform of (1.1) we have

$$Y(e^{j\omega}) = \sum_{k=1}^{N_p} \alpha_k X(\omega - \omega_k) e^{-i\tau_k(\omega - \omega_k)}, \tag{2.3}$$

where $Y(e^{j\omega})$ and $X(e^{j\omega})$ are the Fourier transforms of $y(t)$ and $x(t)$, respectively. Suppose $X(e^{j\omega})$ is a constant, say 1. Then

$$Y(e^{j\omega}) = \sum_{k=1}^{N_p} \alpha_k e^{-i\tau_k(\omega-\omega_k)}. \tag{2.4}$$

If all time shifts τ_k, $k = 1, 2, ..., N_p$, are distinct, all these time shifts τ_k and the coefficients α_k can be detected by taking an inverse discrete Fourier transform of (2.4). Similar to the previous time domain analysis, it is not possible to detect all the coefficients α_k when not all the time shifts τ_k are distinct. Consider a training signal $x(t)$ that has two parts either separated in the time domain or in the frequency domain.

When $x(t)$ has two parts separated in the time domain, it has the following form:

$$x(t) = \begin{cases} x_0, & T_0 < t < T_1, \\ x_1(t), & T_1 < t < T_2, \end{cases} \tag{2.5}$$

where x_0 is a nonzero constant and $x_1(t)$ is a conventional pseudo-random signal or the delta pulse, i.e., its Fourier transform $X_1(e^{j\omega})$ is a constant (flat). In the detection, these two parts are processed separately.

When $x(t)$ has two parts separated in the frequency domain, it has the following form:

$$x(t) = x_0 + x_1(t)e^{j\omega_0 t}, \tag{2.6}$$

where x_0 and $x_1(t)$ are as in (2.5) and ω_0 is a frequency shift. In this case, the received signal $y(t)$ is first filtered by using a lowpass filter to extract the constant part x_0 and a bandpass filter to extract $x_1(t)$. We, however, do not want to filter out other information in $y(t)$. Therefore, the frequency ω_0 needs to be at least greater than all the Doppler shifts ω_k, i.e.

$$|\omega_0| > |\omega_k|, \quad k = 1, 2, ..., N_p. \tag{2.7}$$

In what follows, for convenience we use the training signal in (2.5), i.e., the two parts are separated in the time domain. Without loss of generality, for simplicity we assume that the two part information is available at the same time interval, for example, $[0, T]$, and

$$y_1(t) = \sum_{k=1}^{N_p} \alpha_k e^{j\omega_k t} x_0, \quad t \in [0, T], \tag{2.8}$$

and

$$y_2(t) = \sum_{k=1}^{N_p} \alpha_k e^{j\omega_k t} x_1(t - \tau_k), \quad t \in [0, T]. \tag{2.9}$$

The goal here is to identify the unknown parameters $\{(\alpha_k, \tau_k, \omega_k), 1 \leq k \leq N_p\}$ from the above equations (2.8) and (2.9). In the following, we also assume that the sampling interval length of the received signals $y_1(t)$ and $y_2(t)$ is small enough so that all the distinct Doppler shifts ω_k in (2.1) and the distinct time shifts τ_k in (2.4) can be detected by using the discrete Fourier transform (DFT) and the inverse discrete Fourier transform (IDFT) as discussed above. Since these two sets of distinct Doppler shifts and time shifts are obtained from two different DFT and IDFT, their orders and their corresponding coefficients α_k may be different, which may cause an identifiability problem. To study this problem, we first have the following straightforward result.

Theorem 1. *Let $x(t)$ be a training signal with the two parts as described above. If either all the Doppler shifts ω_k, $1 \leq k \leq N_p$, or all the time shifts τ_k, $1 \leq k \leq N_p$, are distinct, then the unknown parameters $\{(\alpha_k, \tau_k, \omega_k), 1 \leq k \leq N_p\}$ are detectable by applying the DFT in the time domain and the IDFT in the frequency domain to the two parts of the received data corresponding to the two parts of the training signal, respectively.*

Proof: Without loss of generality, we assume ω_k, $1 \leq k \leq N_p$, are distinct. Thus, α_k, ω_k, $1 \leq k \leq N_p$, can be detected with the correct order. Since τ_k, $1 \leq k \leq N_p$, may have repetitions, only the linear combinations of α_k from the IDFT of (2.9) and the corresponding τ_k can be detected. Form the detected α_k, $1 \leq k \leq N_p$, from (2.8), the corresponding τ_k with the same order of α_k and ω_k can be determined by checking the linear combinations of the detected α_k. □

In the following section, we want to present a necessary and sufficient condition on the identifiability of $\{\alpha_k, \omega_k, \tau_k\}$.

3. Necessary and Sufficient Condition on the Identifiability. In this section, we first present a necessary and sufficient condition and then study the probability that the condition holds.

3.1 *Necessary and Sufficient Condition.* Since the Doppler shifts ω_k, $1 \leq k \leq N_p$, and the time shifts τ_k, $1 \leq k \leq N_p$, only take finite possible values, such as in Hz and μs, respectively, they may have repetitions. In other words, ω_{k_1} (or τ_{k_1}) may be equal to ω_{k_2} (or τ_{k_2}) for $k_1 \neq k_2$. Therefore the identifiability problem now arises from the possible duplications of the Doppler shifts ω_k and the time shifts τ_k as discussed in (2.2).

Although individuals of ω_k and τ_k may have repetitions, the pairs (ω_k, τ_k), $1 \leq k \leq N_p$, do not have a repetition and otherwise the duplicated pairs (or multiple paths) may be grouped together into a single term (path) in (1.1). Therefore, in what follows we always assume that all the pairs (ω_k, τ_k), $1 \leq k \leq N_p$, are *distinct*, i.e., if $\omega_{k_1} = \omega_{k_2}$, then $\tau_{k_1} \neq \tau_{k_2}$; and if $\tau_{k_1} = \tau_{k_2}$, then $\omega_{k_1} \neq \omega_{k_2}$.

A general setting of the repetitions of ω_k and τ_k is as follows. Let I_1,..., I_f be a *partition* of the integer set

$$\mathcal{I} \triangleq \{1, 2, ..., N_p\}$$

such that all the Doppler shifts ω_k for $k \in I_l$ for any fixed l are equal, i.e.,

$$\omega_k = \tilde{\omega}_l \quad \text{for all} \quad k \in I_l, \tag{3.1}$$

but ω_k in different I_l are different, where "partition" means any two sets I_{l_1} and I_{l_2} for $l_1 \neq l_2$ do not intersect, i.e., $I_{l_1} \cap I_{l_2} = \phi$ for $l_1 \neq l_2$, and the union of all I_l is the integer set $\mathcal{I}$, i.e.,

$$\bigcup_{l=1}^{f} I_l = \mathcal{I},$$

and each set I_l is not empty. Let J_1,..., J_g be another *partition* of the integer set $\mathcal{I}$ such that all the time shifts τ_k for $k \in I_l$ for any fixed l are equal, i.e.,

$$\tau_k = \tilde{\tau}_l \quad \text{for all} \quad k \in J_l. \tag{3.2}$$

We use $|S|$ to denote the cardinality of a set S. Since all pairs (ω_k, τ_k), $k = 1, 2, ..., N_p$, are distinct as assumed before, the intersection of any set I_{l_1} and any set J_{l_2} has at most one element, i.e.,

$$|I_{l_1} \cap J_{l_2}| \leq 1, \tag{3.3}$$

but τ_k in different J_l are different,

Similar to the discussion in (2.2), the following summations can only be detected from the DFT of $y_1(t)$ in (2.8) and the IDFT of the Fourier transform $Y_2(e^{j\omega})$ of $y_2(t)$ in (2.9):

$$\sum_{k \in I_l} \alpha_k = \beta_l, \quad 1 \leq l \leq f, \tag{3.4}$$

$$\sum_{k \in J_l} \alpha_k = \gamma_l, \quad 1 \leq l \leq g, \tag{3.5}$$

where β_l and γ_l are the detected values, and the two partitions $\{I_{l_1}\}$ and $\{J_{l_2}\}$ are not known. The identifiability of $\{(\alpha_k, \tau_k, \omega_k), 1 \leq k \leq N_p\}$ is then reduced to the solvability of these equations (3.4)-(3.5) with unknown partitions $\{I_{l_1}\}$ and $\{J_{l_2}\}$. The following is a necessary and sufficient condition for the identifiability.

Theorem 2. *The parameters $\{(\alpha_k, \tau_k, \omega_k), 1 \leq k \leq N_p\}$ are uniquely determined using the above mixed training signal model if and only if there does not exist any path k, $1 \leq k \leq N_p$, such that both the kth Doppler shift ω_k and the kth time shift τ_k have their repetitions.*

A necessary and sufficient condition in Theorem 2 can be stated in the following mathematical way: there do not exist three distinct integers k, k_1, and k_2 such that $\omega_k = \omega_{k_1}$ and $\tau_k = \tau_{k_2}$. The proof of the necessary part of Theorem 2 is in the Appendix. To prove the sufficiency part, we need the following result.

Lemma 1. *There does not exist any path k, $1 \leq k \leq N_p$, such that both the kth Doppler shift ω_k and the kth time shift τ_k have their repetitions if and only if there are two subsets I and J of $\{1, 2, ..., N_p\}$ such that their union is $\mathcal{I}$, that is, $I \cup J = \mathcal{I} = \{1, 2, ..., N_p\}$, and all the Doppler shifts ω_k for $k \in I$ are distinct in the set of all ω_k for $k \in \{1, 2, ..., N_p\}$ and all the time shifts τ_k for $k \in J$ are distinct in the set of all τ_k for $k \in \{1, 2, ..., N_p\}$.*

Proof : We first prove the necessity part. Let $F = \{l_1 : 1 \leq l_1 \leq f \text{ and } |I_{l_1}| > 1\}$ and $I_c = \cup_{l_1 \in F} I_{l_1}$ and $G = \{l_2 : 1 \leq l_2 \leq g \text{ and } |J_{l_2}| > 1\}$ and $J_c = \cup_{l_2 \in G} J_{l_2}$. By the condition, I_c and J_c don't intersect, i.e., $I_c \cap J_c = \phi$. Let I and J be the complementary sets of I_c and J_c of $\{1, 2, ..., N_p\}$, respectively. Since all ω_k, for $k \in I$, and all τ_k, for $k \in J$, are distinct, by $I_c \cap J_c = \phi$ we have $J_c \subset I$ and $I_c \subset J$. Therefore, $I \cup J = \{1, 2, ..., N_p\}$.

We now prove the sufficiency part. Assume that there exist three distinct integers k, k_1, and k_2 such that $\omega_k = \omega_{k_1}$ and $\tau_k = \tau_{k_2}$. Then, the integer k will not be in any integer sets I and J as described in the lemma, i.e., $I \cup J \neq \{1, 2, ..., N_p\}$. □

Lemma 2. *If there are two subsets I and J of $\{1, 2, ..., N_p\}$ such that their union is $I \cup J = \{1, 2, ..., N_p\}$, and all the Doppler shifts ω_k for $k \in I$ are distinct in the set of all ω_k for $k \in \{1, 2, ..., N_p\}$ and all the time shifts τ_k for $k \in J$ are distinct in the set of all τ_k for $k \in \{1, 2, ..., N_p\}$, then $\{\alpha_k, \omega_k, \tau_k\}$, $k \in \{1, 2, ..., N_p\}$, can be uniquely determined.*

Proof : Two sets of coefficients β_k from (2.8) and the DFT and γ_k from (2.9) and the IDFT can be solved. By the condition in the lemma, it is known that $\alpha_k = \beta_k$ for $k \in I$ and $\alpha_k = \gamma_k$ for $k \in J$. Since $I \cup J = \{1, 2, ..., N_p\}$, all the α_k can be detected. The rest is the same as the proof of Theorem 1 by checking the linear combinations of α_k for the order determinations of ω_k and τ_k. □

The sufficiency part in Theorem 2 is a consequence of Lemma 1 and Lemma 2. From Theorem 2 and Lemma 1, it is immediate that the condition in Lemma 2 is also necessary, i.e., the following corollary holds.

Corollary 1. *The parameters* $\{\alpha_k, \omega_k, \tau_k\}$*,* $k \in \{1, 2, ..., N_p\}$*, can be uniquely determined using the mixed training signal model if and only if there are two subsets* I *and* J *of* $\{1, 2, ..., N_p\}$ *such that their union is* $I \cup J = \{1, 2, ..., N_p\}$*, and all the Doppler shifts* ω_k *for* $k \in I$ *are distinct in the set of all* ω_k *for* $k \in \{1, 2, ..., N_p\}$ *and all the time shifts* τ_k *for* $k \in J$ *are distinct in the set of all* τ_k *for* $k \in \{1, 2, ..., N_p\}$*.*

3.2 *Probability Analysis.* In the following, we access the probability of channel identifiability in the case of four paths, i.e., for the conditions in Theorems 1-2 to hold, when $N_p = 4$. The condition is in terms of the Doppler and time shifts ω_k and τ_k. Since in practical digital processing, these Doppler and time shifts are quantized to finite values. For convenience, we assume that there are total M_d possible different values of the Doppler shifts and total M_t possible values for the time shifts. In other words, each ω_k may take one of M_d different values

$$D_{range} = \{v_{d,1}, v_{d,2}, ..., v_{d,M_d}\}, \tag{3.6}$$

and each τ_k may take one of M_t different values

$$T_{range} = \{v_{t,1}, v_{t,2}, ..., v_{t,M_t}\}. \tag{3.7}$$

For example, $D_{range} = \{-50\text{Hz}, -49\text{Hz}, ..., 50\text{Hz}\}$ and $T_{range} = \{0\mu\text{s}, 1\mu\text{s}, ..., 100\mu\text{s}\}$. The two numbers M_d and M_t can be determined when the Doppler spread width f_m and the rms time spread width σ_τ are known for a given channel.

As we mentioned earlier, we have a sufficient condition in Theorem 1 and a necessary and sufficient condition in Theorem 2. These two conditions coincide for $N_p = 1, 2, 3$. Although the probability expressions for the conditions in Theorems 1-2 to hold for a general N_p are complicated, in the following we calculate them when $N_p = 4$.

Clearly, the total number of all distinct N_p pairs of (ω_k, τ_k) is

$$P_{total} = \binom{M_d M_t}{N_p}. \tag{3.8}$$

We first consider the condition in Theorem 1, i.e., either all ω_k for $1 \le k \le N_p$ or all τ_k for $1 \le k \le N_p$ are distinct. By some calculations, it is not hard to see that the total number of such N_p pairs (ω_k, τ_k) is

$$
\begin{aligned}
(3.9)\quad P_1 = &\binom{M_d}{4}\binom{M_t}{1} + \binom{M_d}{4}\binom{M_t}{2}\left[\frac{4!}{1!3!} + \frac{4!}{2!2!} + \frac{4!}{1!3!}\right] \\
&+ \binom{M_d}{4}\binom{M_t}{3}\binom{3}{1}\frac{4!}{1!1!2!} \\
&+ \binom{M_t}{4}\binom{M_d}{1} + \binom{M_t}{4}\binom{M_d}{2}\left[\frac{4!}{1!3!} + \frac{4!}{2!2!} + \frac{4!}{1!3!}\right] \\
&+ \binom{M_t}{4}\binom{M_d}{3}\binom{3}{1}\frac{4!}{1!1!2!} + \binom{M_t}{4}\binom{M_d}{4}4!.
\end{aligned}
$$

Therefore, the probability is

$$
\begin{aligned}
(3.10)\quad &\text{Probability}\,(\text{either all } \omega_k \text{ for} 1 \le k \le N_p \text{ or all } \tau_k \\
&\qquad \text{for } 1 \le k \le N_p \text{ are distinct}) = \frac{P_1}{P_{total}},
\end{aligned}
$$

where P_{total} is defined in (3.8) with $N_p = 4$ and P_1 is defined in (3.9).

The total number of pairs (ω_k, τ_k) that satisfy the necessary and sufficient condition in Theorem 2 is

$$
\begin{aligned}
(3.11)\quad P_2 = &\binom{M_d}{4} M_t^4 + \binom{M_d}{3}\binom{3}{1}\binom{M_t}{2} M_t^2 \\
&+ \binom{M_d}{2}\left[\binom{M_t}{2}\binom{M_t - 2}{2} + \binom{2}{1}\binom{M_t}{3} M_t\right].
\end{aligned}
$$

Therefore, the probability for the condition in Theorem 2 is

$$
\begin{aligned}
(3.12)\quad &\text{Probability}\,(\text{the necessary and sufficient} \\
&\qquad \text{condition in Theorem 2 holds}) = \frac{P_2}{P_{total}},
\end{aligned}
$$

where P_{total} is defined in (4.3) with $N_p = 4$ and P_2 is defined in (4.6).

To compare the above two probabilities, their corresponding curves are plotted in Fig. 1, where we set $M_d = M_t$ and the x-axis indicates the variable M_d, which is from 4 to 101. One can clearly see that the probability for the necessary and sufficient condition in Theorem 2 is above the one for sufficient condition in Theorem 1. When the total numbers M_d and M_t of the possible Doppler and time shifts are large relative to the total number N_p of multipath components in a channel, the necessary and sufficient condition in Theorem 2 holds almost surely, i.e., the probability is very close to 1.

4. Numerical Simulations. In the following simulations, we use $N_p = 4$, and

$$\frac{\omega_k}{2\pi} \in D_{range} = \{-50\text{Hz}, -49\text{Hz}, ..., 50\text{Hz}\}, \quad k = 1, 2, 3, 4,$$

and

$$\tau_k \in T_{range} = \{0\mu\text{s}, 1\mu\text{s}, ..., 100\mu\text{s}\}, \quad k = 1, 2, 3, 4.$$

The Doppler and time shifts ω_k and τ_k for $k = 1, 2, 3, 4$ are randomly chosen from the above sets D_{range} and T_{range}, respectively, such that all pairs (ω_k, τ_k) for $k = 1, 2, 3, 4$ are distinct. The multipath coefficients α_k for $k = 1, 2, 3, 4$ are randomly chosen from Gaussian random processes with all possible real values.

For the first piece $y_1(t)$ of data, the sampling rate is chosen as $1/T = 128$, i.e.,

$$y_1[l] = y_1(l/128) = \sum_{k=1}^{N_p} \alpha_k e^{jl\omega_k/128} + n_1(l/128), \quad -63 \leq l \leq 64. \tag{4.1}$$

For the second piece $Y_2(\omega)$ of data, the sampling rate is chosen $1/T = 128/(2\pi)$, i.e.,

$$Y_2[l] = Y_2(2\pi l/128) = \sum_{k=1}^{N_p} \alpha_k e^{jl2\pi\tau_k/128} + n_2(2\pi l/128), \quad 0 \leq l \leq 127. \tag{4.2}$$

Fig. 2 shows the curves of the ratios of the mean square errors (MSE) of the true ω_k, τ_k, and α_k, and their detected values over their mean powers. The x-axis is the ratios of the mean powers of the multipath coefficients α_k over the variance of the additive noise n_i, $i = 1, 2$, in (4.1-4.2). In Fig. 2, 10000 Monte Carlo simulations are implemented.

5. Conclusions. In this paper, we proposed a channel identification algorithm using a mixed training signal, where the channel has both the Doppler shifts and time shifts. The mixed training signals consist of two parts with one part constant and the other part a conventional training signal, such as a pseudo-random signal. These two parts of the signals may be separated either in the time domain or in the frequency domain. The constant part can be used to detect the Doppler shifts while the other part can be used to detect the time shifts. Both parts are used to detect the corresponding coefficients and the synchronization between the detected Doppler and time shifts. A necessary and sufficient condition was given for the channel identifiability based on the mixed training signal approach. A probability analysis for the identifiability was presented. It turns out that almost all channels of practical interests are identifiable. Finally, some simple numerical examples were presented.

Appendix: The Necessity Proof of Theorem 2 The linear equations in (3.4 - 3.5) can be expressed by the following matrix form:

$$A \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_{N_p} \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_f \\ \gamma_1 \\ \vdots \\ \gamma_g \end{bmatrix} \tag{5.1}$$

where A is an $f + g$ by N_p matrix with only 0 and 1 entries as follows

$$A = \begin{bmatrix} \mathbf{a}_1 \\ \vdots \\ \mathbf{a}_f \\ \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_g \end{bmatrix},$$

where $\mathbf{a}_l = (a_l(k))_{1 \le k \le N_p}$, $1 \le l \le f$, with

$$a_l(k) = \begin{cases} 1, \text{if } k \in I_l, \\ 0, \text{otherwise}, \end{cases}$$

$\mathbf{b}_l = (b_l(k))_{1 \le k \le N_p}$, $1 \le l \le g$, with

$$b_l(k) = \begin{cases} 1, \text{if } k \in J_l, \\ 0, \text{otherwise}, \end{cases}$$

where $|I_{l_1} \cap I_{l_2}| \le 1$ for $1 \le l_1 \le f$ and $1 \le l_2 \le g$. Clearly the identifiability or the solvability of $\{\alpha_k, \omega_k, \tau_k\}$ implies that the expression (5.1) of the vector $(\beta_1, \dots, \beta_f, \gamma_1, \dots, \gamma_g)^T$ for (3.4 - 3.5) is unique. Thus, to prove the necessity in Theorem 2, we need to prove that the constraint $|I_{l_1} \cap I_{l_2}| \le 1$ and the uniqueness of the expression (5.1) for (3.4 - 3.5) implies the condition in Theorem 2.

Suppose the condition in Theorem 2 does not hold. In other words, there are three distinct integers k, k_1, k_2 such that $\omega_k = \omega_{k_1}$ and $\tau_k = \tau_{k_2}$. In the following, we prove that the expression (5.1) for (3.4 - 3.5) is not unique. Without loss of generality, we assume $k < k_1 < k_2$. Then there exists the following submatrix A_0 in the matrix A:

$$A_0 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Consider the subsystem of (5.1):

$$A_0 \begin{bmatrix} \alpha_k \\ \alpha_{k_1} \\ \alpha_{k_2} \end{bmatrix} = \begin{bmatrix} \beta_{f_1} \\ \beta_{f_2} \\ \gamma_{g_1} \\ \gamma_{g_2} \end{bmatrix}.$$

Consider

$$U = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad U^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Let

$$\bar{A}_0 = A_0 U = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\begin{bmatrix} \bar{\alpha}_k \\ \bar{\alpha}_{k_1} \\ \bar{\alpha}_{k_2} \end{bmatrix} = U^{-1} \begin{bmatrix} \alpha_k \\ \alpha_{k_1} \\ \alpha_{k_2} \end{bmatrix}.$$

Then,

$$\bar{A}_0 \begin{bmatrix} \bar{\alpha}_k \\ \bar{\alpha}_{k_1} \\ \bar{\alpha}_{k_2} \end{bmatrix} = A_0 \begin{bmatrix} \alpha_k \\ \alpha_{k_1} \\ \alpha_{k_2} \end{bmatrix} = \begin{bmatrix} \beta_{f_1} \\ \beta_{f_2} \\ \gamma_{g_1} \\ \gamma_{g_2} \end{bmatrix},$$

where $\omega_{k_1} = \omega_{k_2}$ and $\tau_k = \tau_{k_2}$. This gives a second distinct expression of (5.1) for (3.4 - 3.5). Thus, the necessity is proved. □

Since all pairs (ω_k, τ_k) for $1 \le k \le N_p$ are distinct, each product set $I_{l_1} \times J_{l_2}$ has at most one pair. Since I_l, $1 \le l \le N_p$, and J_l, $1 \le l \le N_p$, are two partitions of $\mathcal{I}$, it is clear that

$$\cup_{k=1}^{N_p} \{(\omega_k, \tau_k)\} \subset \cup_{l_1=1}^{f} \cup_{l_2=1}^{g} I_{l_1} \times J_{l_2}.$$

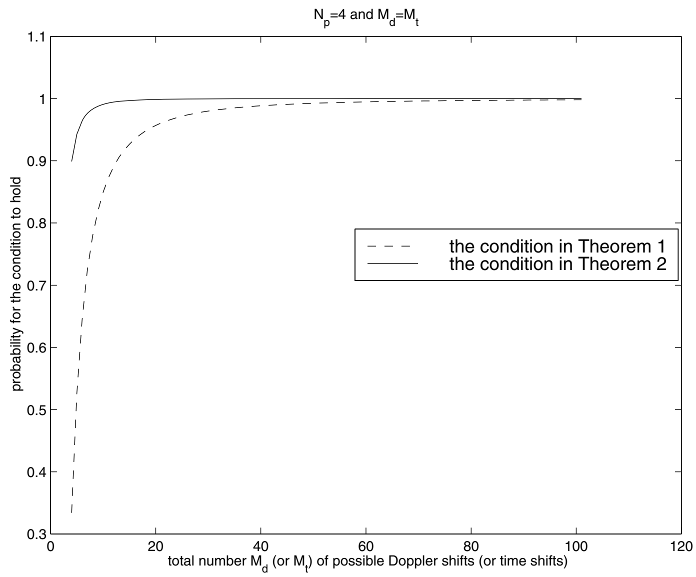

FIGURE 1. Probabilities for the conditions in Theorem 1-2 to hold when $N_p = 4$.

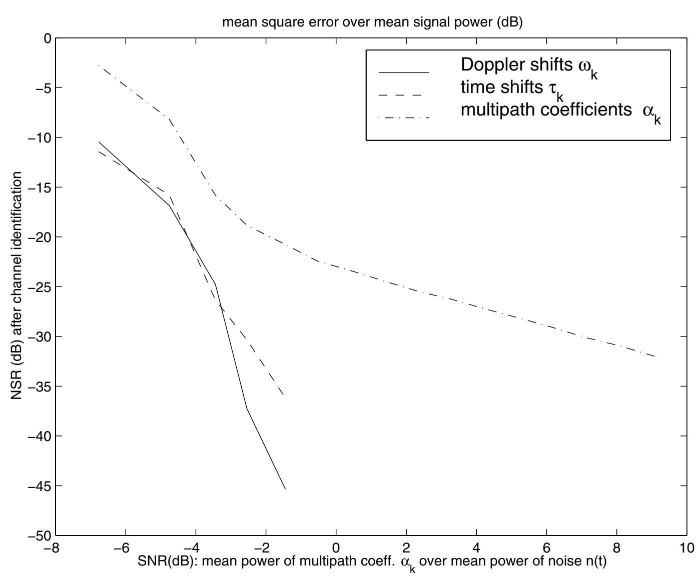

FIGURE 2. MSE for the detected ω_k, τ_k vs. the SNR.

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