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A coupling of infinite particle systems

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In this note we extend a coupling technique introduced in Mountford (1993) to a large class of interacting particle systems (IPSs) on the one dimensional lattice. A one dimensional IPS is a Markov process on state space D^Z where Z is the integers and (in this paper) D is some finite set of possible spin values. The generator for this process can be written as

$$\Omega f(\eta)=\sum_{T}\sum_{\nu\in D^T}(f(\nu\eta)-f(\eta))c_T(\eta,\nu)$$

where the first sum is over finite subsets of Z, T and where $\nu\eta$ denotes the configuration with $\nu\eta(y)$ equal to $\nu(y)$ if $y\in T$ and equal to $\eta(y)$ otherwise. The function $c_T(\nu,\eta)$ can be assumed to be zero if $\nu(y)=\eta(y)$ for some y in T. In this case for ν different from η on T, we should think of the process as satisfying

$$P[\eta_{t+dt}=\nu \text{ on } T\,|\eta_t]=c_T(\eta_t,\nu)dt+o(dt).$$

See Liggett (1985), especially section 1.3, for a discussion of existence questions. Throughout this paper we will assume that the process

is of *finite range* : there exists an $R<\infty$ so that $c_T(\,,\,)$ is zero if T has length greater than R and such that for any x in Z and T containing x of length at most R, $c_T(\nu,\eta)$ depends only on the spins $\eta(x-R)$, $\eta(x-R+1)$,,,,, $\eta(x)$,....$\eta(x+R)$.

and

has *bounded flip rates* : for each site x, $\sum_{x\in T}\sum_{\nu\in D^T}c_T(\nu,\eta)<1$. The bound of 1 is arbitrary, any bound can be reduced to 1 by rescaling time.

Given these hypotheses, there exists a unique Markov semigroup S(t). corresponding to operator Ω. It should be noted that if the "flip" functions c are translation invariant, then (perhaps after rescaling time) the bounded flip rates hypothesis is guaranteed once the finite range hypothesis is satisfied.

A probability measure v on D^Z is invariant for the process if for each f continuous on D^Z and for each $t>0$

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$$\int f(\omega)v(d\omega) = \int S(t)f(\omega)v(d\omega)$$

where S is the semi-group for the process. To check that a measure is invariant it is only necessary to verify the above equations for cylinder functions f.

Our principal result is

Theorem One. *Let η_t be a one dimensional particle of finite range and with bounded flip rates.*

If t_n is a sequence of times tending to infinity such that $\eta_{t_n} \to v$ in distribution, then v is an invariant measure for the process.

Remark 1. the hypotheses of the theorem are clearly not the most general and could be easily weakened but we feel that our simple assumptions enhance the clarity of the paper.

Remark 2. This result improves the result obtained in Mountford (1993) in that we no longer require the initial configuration to be "finite" and we no longer require δ_0 to be an invariant measure.

Remark 3. Given assumption (ii) there is no problem with existence of our process.

We wish to explain our motivation for the result. It is well known (see e.g. Liggett (1985), Proposition 1.8 page 10) that if a particle system if Feller (which is certainly the case for particle systems satisfying conditions (i) and (ii) of Theorem One) and t_n is a sequence of times tending to infinity such that $\frac{1}{T}\int_0^{t_n} \mu S(t)\,dt$ converges in distribution to measure v then v must be invariant. Thus if our particle system η_t admits only one invariant measure, v, it must be the case that as T tends to infinity

$$\frac{1}{T}\int_0^T \mu S(t)\,dt \to v$$

in distribution for any initial probability μ. Theorem One enables us to make the following immediate extension

Theorem Two. *A finite range one dimensional interacting particle system η_t, with bounded flip rates which has a unique invariant measure is ergodic.*

This result is relevant to question 4, chapter one of Liggett (1985). It is only a partial answer as it says nothing of particle systems with long range behaviour or of higher dimensions; the method given here is strictly one dimensional. It seems more than plausible that the conclusion of Theorem Two is true in higher dimensions. Theorem One is, I hope, of interest, but a problem which limits its application is that the main criterion for the uniqueness of invariant measures, the (ε, M) criterion (see Liggett (1985) or for recent work Maes and Schlossman (1993)), also establishes ergodicity. Indeed exponential convergence is deduced, a much stronger result than we show.

The paper is planned as follows: in §1 some elementary coupling results are assembled, in §2 we construct a coupling which implies Theorem One, in §3 we give some applications of Theorem One to cases not covered by theorem Two.

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§1

In this section we establish crude bounds on how boundary behaviour can propagate. We fix a particular particle system η_t with flip functions c satisfying the hypotheses of Theorem One. Let $R \in Z_+$ be such that c_T is identically zero if T has length greater than R and such that $c_T(\nu, \eta)$ only depends on spins of η at sites within R of every site in T. Let N(T) be the set of sites within R of every site of T. We say a process η performs a T jump at time t if $\eta_t(y) = \eta_{t-}(y)$ if and only if y is outside T.

We describe a particle system η' as a system restricted to interval I, with boundary conditions 0 (without loss of generality 0 is in D), if all sites outside I are fixed at value 0 and all sites in I flip according to functions $c_T(.,.)$ for $T \subset I$. Given two particle systems η and η', both of range R, we say they are naturally coupled if whenever $\eta_{t-} = \eta'_{t-}$ on N(T), then η performs a T jump at time t if and only if η' performs a T jump at time t and in this case $\eta_t = \eta'_t$ on T. Such couplings exist (see e.g. Durrett (1988), Liggett (1985)).

Lemma 1.1. *Let η_t and η'_t be naturally coupled where*

i) $\eta_0 = \eta'_0$ *on* $[-n, n]$,

ii) η' *is a particle system restricted to an interval containing* $[-n, n]$ *with zero boundary conditions.*

Then there exists a Poisson Process $V(t)$ of rate $2R$ such that for all times t $\eta_t = \eta'_t$ on $[-n+RV(t), n-RV(t)]$

Proof: We call a site x vulnerable at time t if there exists $s \in [0, t]$ and $|y-x| \leq R$ such that $\eta_s(y)$ is not equal to $\eta'_s(y)$. Let $L(t) = \sup\{x < R+1 : x$ is vulnerable at time $t\}$ and let $R(t) = \inf\{x > -(R+1) : x$ is vulnerable at time $t\}$. The following are obviously true:

i) $\eta_t = \eta'_t$ on $[L(t) - (R-1), R(t) + (R-1)]$,

ii) $L(0) < -n+R$, $R(0) > n-R$.

L(t) can only increase if there is a T jump (for η or η') with T having right endpoint in $[L(t) - (R-1), L(t)]$ flips, at which time L(t) will increase by at most R. By our bounded flip rates assumption, $\sum_T \sum_{\nu \in D^T} c_T(\nu, \eta) < R$, where we sum over T with right endpoint in the interval $[L(t) - (R-1), L(t)]$. Similarly for the way R(t) decreases. The lemma follows.

Corollary 1.1. *Let η_t and η'_t be naturally coupled and*
i) $\eta_0=\eta'_0$ *on* $[-3R^2S, 3R^2S]$,
ii) η' *is restricted to* $[-3R^2S, 3R^2S]$ *with zero boundary conditions.*
For fixed $M\in Z_+$, *the chance that* $\eta_S=\eta'_S$ *on* $[-M, M]$ *tends to one as S tends to infinity.*

We now define (separately for each t) another process $\zeta_s^{K,t}$, $0\leq s\leq t+K$ which will be defined on an essentially finite state space with the property that for fixed M, as t tends to infinity the spins of $\zeta_{t+K}^{K,t}$ on $[-M, M]$ will be close to those of η_{t+K}, provided their initial configurations are close. The process $\zeta^{K,t}$ is defined in three steps.

1. First take a rate one Poisson process. Let its first $[t\text{-}t^{2/3}]$ points be $\tau_1<\tau_2...\tau_{t-t^{2/3}}$ (with slight abuse of notation for the last term). (As will be seen in the technical details we chose the term $t^{2/3}$ simply because 2/3 is in the interval $(1/2, 1)$.)
2. On the interval $[\tau_i, \tau_{i+1})$ (τ_0 is taken to be 0), we let $\zeta_s^{K,t}$ evolve as a particle system restricted to $[-4R^2(t-i), 4R^2(t-i)]$ with zero boundary conditions. We take $\zeta_{\tau_i}^{K,t}$ to be the configuration $\zeta_{\tau_i-}^{K,t}$ restricted to this interval. We take $\zeta_0^{K,t}$ to be previously defined.
3. If $\tau_{t-t^{2/3}}$ is greater than or equal to t+K, then $\zeta^{K,t}$ is fully defined on the interval [0, t+K] and no further definition are required. If not then we let $\zeta_s^{K,t}$ evolve on $[\tau_{t-t^{2/3}}, t+K]$ as a particle system restricted to $[-4R^2(t-[t-t^{2/3}]), 4R^2(t-[t-t^{2/3}])]$ with boundary conditions equal to 0 and such that $\zeta_{\tau_{t-t^{2/3}}}^{K,t}$ is equal to $\zeta_{\tau_{t-t^{2/3}}-}^{K,t}$ on $[-4R^2(t-[t-t^{2/3}]), 4R^2(t-[t-t^{2/3}])]$.

Note by regarding $\zeta^{K,t}$ as being in the state space $D^{[-4R^2(t-i),4R^2(t-i)]}$ on time interval $[\tau_i, \tau_{i+1})$, we can think of $\zeta^{K,t}$ as being a continuous time Markov chain on the finite state space $\bigcup_{i=0}^{[t-t^{2/3}]} D^{[-4R^2(t-i),4R^2(t-i)]}$ which jumps (where this makes sence) from $D^{[-4R^2(t-i),4R^2(t-i)]}$ to $D^{[-4R^2(t-i-1),4R^2(t-i-1)]}$ at constant rate 1.

Lemma 1.2. *For fixed* $M\in Z_+$ *and* $K\in R_+$ *if* $\zeta_0^{K,t}=\eta_0$ *on interval of site* $[-4R^2t, 4R^2t]$ *and the two processes are naturally coupled, then the chance that* $\zeta_{t+k}^{K,t}=\eta_{t+k}$ *on* $[-M, M]$ *goes to one as t tends to infinity.*

Proof. Lemma 1.1 together with the Markov property applied to times τ_i gives the following: for every $\varepsilon>0$, there exists a V such that

$$\inf_{t,i\leq t-t^{2/3}} P[\zeta_{\tau_i}^{K,t}=\eta_{\tau_i} \text{ on } [-4R^2(t-i)+V, 4R^2(t-i)-V]]>1-\varepsilon$$

The Central Limit Theorem implies that

$$\frac{t+K-\tau_{t-t^{2/3}}}{t^{2/3}} \to 1 \text{ in probability}$$

Putting these facts together we see that with probability tending to one as t tends to infinity,

$$t+K-\tau_{t-t^{2/3}} \leq \frac{6}{5}t^{2/3} \text{ and } \eta_{\tau_{t-t^{2/3}}} = \zeta^{K,t}_{\tau_{t-t^{2/3}}} \text{ on } \left[-3R^2\frac{6}{5}t^{2/3},\ 3R^2\frac{6}{5}t^{2/3}\right]$$

The result now follows from Corollary 1.1 and the Markov property.

§2

In this section we prove Theorem One. The essential step is to establish

Proposition 2.1. *For any fixed $M \in Z_+$ and $T \in R_+$ there exist a coupling of $\zeta^{T,t}_s$ and $\zeta^{0,t}_s$ such that as t tends to infinity, the probability that $\zeta^{T,t}_{T+t} = \zeta^{0,t}_t$ tends to one.*

Given this result Theorem One follows quickly and simply, therefore we feel it makes sense to show how it follows from Proposition 2.1 before descending into the technicalities necessary for the proof of Proposition 2.1. In the sequel we will drop the superscript 0 and write ζ^t_s for $\zeta^{0,t}_s$.

Prrof of Theorem One. Let t_n tending to infinity be such that η_{t_n} converges to v in distribution. It is sufficient to show that for each cylinder function f and each $T \in R_+$, $\int f(\omega)v(d\omega) = \int S(T)f(\omega)v(d\omega)$. Here, as before, S(t) is the semigroup corresponding to the interacting particle system. Let f have support in $[-M, M]$.

First by definition of weak convergence

$$\int f(\omega)v(d\omega) = \lim_{n\to\infty} E[f(\eta_{t_n})] \ .$$

Lemma 1.2 implies that

$$\lim_{n\to\infty} E[f(\eta_{t_n})] = \lim_{n\to\infty} E[f(\zeta^{t_n}_{t_n})] \ .$$

Proposition 2.1 yields the equality

$$\lim_{n\to\infty} E[f(\zeta^{t_n}_{t_n})] = \lim_{n\to\infty} E[f(\zeta^{T,t_n}_{t_n})] \ .$$

We apply Lemma 1.2 to obtain

$$\lim_{n\to\infty} E[f(\zeta^{T,t_n}_{t_n+T})].= \lim_{n\to\infty} E[f(\eta_{t_n+T})].= \lim_{n\to\infty} E[S(T)f(\eta_{t_n})] \ .$$

But S(T) f is a continuous function since, as previously noted, η_t is Feller, so this last limit must equal $\int S(T)f(\omega)v(d\omega)$ and we are done

It only remains to prove Proposition 2.1. The ideas for the proof are very

close to those of Mountford (1993). We can and will regard the processes ζ_s^t, $\zeta_s^{T,t}$ $0\leq s\leq t+T$ as finite state Markov chains on the set $\bigcup_{i=0}^{t-t^{2/3}} D^{[-4R^2(t-i),4R^2(t-i)]}$.
We wish to couple the processes so that with probability tending to one as t tends to infinity,

$$\zeta_s^t=\zeta_{s+T}^{T,t} \text{ for all } s\in[\tau_{t-t^{2/3}}, t]$$

The process ζ^t ($\zeta^{T,t}$) can be constructed in two steps as follows
FIRST STEPS: Choose the sequence of states that ζ^t jumps to (i.e. pick a realization of the embedded discrete time Markov chain): $X_0,X_1,...X_{N_1},..X_{N_2}...$ where N_i $=\inf\{n: X_n \in D^{[-4R^2(t-i),4R^2(t-i)]}$. X_0 is simply η_0 restricted to the interval $[-4R^2t, 4R^2t]$.
SECOND STEP: Given this sequence X_i, there is, for each state X_i, a jump rate q (X_i). It is obtained by adding all the flip rates of the sites of configuration X_i plus the unit flip rate at which the process moves its boundary inwards if $X_i\in D^{[-4R^2(t-i),4R^2(t-i)]}$ for $i<t-t^{2/3}$. Choose independently of each other exponential random variables $e_0, e_1, e_2,...$ of means $\frac{1}{q(X_0)}, \frac{1}{q(X_1)},...$
We obatain ζ^t by taking $\zeta_s^t=X_i$ for $s\in[\sum_{j=-1}^{i-1} e_j, \sum_{j=-1}^{i} e_j)$, here e_{-1} is taken to be 0.
The above recipe can also be used to construct the process ζ^t.
We will construct a coupling of two processes ζ and ζ^T by employing the above construction for both processes and choosing the same embedded discrete time Markov chain realization $X_0, X_1, X_2,...$ but different exponential random variables $(e_0,e_1,...)$, $(e_0^T, e_1^T,..)$ to construct ζ^t and $\zeta^{T,t}$. The exponential random variables will be chosen with the aim of ensuring that for all i large enough $\sum_0^i e_j=\sum_{j=0}^i e_j^T-T$. We will shortly describe the coupling explicity, but first we require a simple lemma.

Lemma 2.1. *Given $x>0$, we can couple together two mean v exponential random variables X and Y so that with probability $e^{-x/v}$, $X=Y+x$.*

Proof. Let Z be a random variable, independent of X with distribution equl to a Y. Then by memoryless properties of exponential random variables

$$Y\equiv I_{\{X>x\}}(X-x)+I_{\{X<x\}}Z$$

is an exponential random variable of mean v with the desired property.
We can now describe the coupling of two sequences of independent exponential random variables we employ. We call it the ABL coupling because it is a continuous version of a discrete coupling employed by Andjel, Bramson and

Liggett (1988).

We write the sequences of exponential random variables that we wish to couples as $\{e_i\}$, $\{e_i^T\}$. We will choose the pairs (e_n, e_n^T) so that

$$M_n \equiv \sum_{j=-1}^{n-1} e_j^T - e_j$$

is a martingale. (e_{-1} and e_{-1}^T are taken to be zero.) We define the following stopping times for the martingale M:

$V_0 = 0$,
for i odd, $V_i = \inf\{n > V_{i-1}: M_n \geq T\}$
for i even, $V_i = \inf\{n > V_{i-1}: M_n \leq T\}$,

We choose (e_i, e_i^T) independently until $i = V_i$; then we choose (e_i, e_i^T) so that with probability $e^{(T-M_{v_1})/E(e_1)}$ $M_{V_1+1} = T$. By Lemma 2.1 this is possible. If M_{V_1+1} is equal to T, then thereafter we chose $e_i = e_i^T$, ensuring that thereafter $M_n = \mathrm{T}$. If M_{V_1+1} is not equal to one, we choose (e_i, e_i^T) independently until time V_2; then (again as with Lemma 2.1), we choose $(e_{V_2}, e_{V_2^T})$ so that with probability $e^{(M_{V_2}-T)/E(e_{V_2})}$ $M_{V_2+1} = T$. Again if $M_{V_2+1} = \mathrm{T}$, we choose subsequent e_i, e_i^Ts so that M_n remains equal to T. Otherwise we continue with independent e_i, e_i^T until V_3 and so on. We say the ABL couples successfully by time n if $M_{n+1} = \mathrm{T}$. Of course this implies that M_r is equal to T for all subsequent r.

For this couplig to work to our purpose we must first establish a criterion for the coupling to be successful, and secondly show that this criterion will be relevant to the exponential sequences we use to construct ζ^t and $\zeta^{T,t}$.

Lemma 2.2. *There exists a function $f_c(\ )$ such that $\lim_{x\to\infty} f_c(x) = 1$ and if $e_0, e_1, \ldots e_N$ and $e_0^T, e_1^T, \ldots e_N^T$ are sequences of independent exponential random variables with*

a) $E[e_j] = E[e_i^T] \leq 1$ *for each i and*
b) $E[e_i]/E[e_{i+1}]$ *is less than or equal to $c < \infty$, then the chance that the ABL coupling of the sequences succeeds by time N is at least* $f_c\left(\sum_{i=0}^{N} (E[e_i]^2\right)$.

Proof. Given the memoryless property of exponential random variables, the, is n is not equal to V_i for some i, we can think of M_{n+1} as being obtained from M_n by, with probability 1/2 adding an exponential random variable of mean $\mathrm{E}[e_i]$ and with probability 1/2 subtracting an exponential random variable of mean $\mathrm{E}[e_i]$. Given this characterization and using again the memoryless property of exponential random variables, we see that provided M_{V_n} is not equal to T, then $|M_{V_n} - T|$ is distributed like an exponential random variable of mean E $[e_r]$ on the event $V_n = r$. Therefore, given Lemma 2.1 and property (ii) of the sequence $E[e_i]$, we find that

$$P[M_{V_{n+1}} \neq T | M_{V_n} \neq T,\ V_n < \infty] \leq \int_0^\infty e^{-x} e^{-cx} dx = \frac{1}{1+c}$$

Therefore for any n $\in Z_+$, we have

$$P[M_N \neq T] \leq \left(\frac{1}{1+c}\right)^n + P[V_n > N-1]$$

The desired result clearly follows as $P[V_n > N-1]$ tends to zero as $\sum_{i=0}^{N} (E[e_i])^2$ tends to infinity if all the summed terms are less than one.

In working towards the second objective, we first record some necessary facts about the sequence $q(X_n)$.

Lemma 2.3. *The sequence $q(X_n)$ satisfies*

i) $|q(X_n - q(X_{n+1})| < 8R^2 + 2R + 1$

ii) *For some $K > 0$ not depending on t $q(X_{n+1})/q(X_n) > K$ for all $n < N$.*

Proof. we consider inequality (i). This deals with the difference in jump rate resulting from the jump of ζ^t to X_{n+1} from X_n. There are two kinds of jump to consider. The jump corresponding to a τ_k and the jump corresponding to a flip of ζ^t. In the first case the flip rate at X_{n-1} has no contribution from $8R^2$ sites which contribute to $q(X_n)$. Also the flip rates of at most 2R+1 sites has altered. Since the total flip rate at a single site is at most 1 the result follows.

(ii) follows from (i) as if n<N, then the flip rate, $q(X_n)$ is at least 1.

Lemma 2.4. *Let A_k (k=1, 2,... $t-t^{2/3}$) be the sequence of states the process ζ visits in the random time interval $[\tau_{k-1}, \tau_k)$. (Necessarily) given our coupling, A_k is also equal to the sequence of states visited by ζ^t in the time interval $[\tau^T_{k-1}, \tau^T_k)$.) Let $F_{\tau_{k-1}}$ be the sigma field generated by ζ_t; $t \leq \tau_{k-1}$. Then there exists $h > 0$ (not depending on t) so that for all $k \leq t - t^{2/3}$,*

$$P\left[\sum_{X_1 \in A_k} \frac{1}{q(X_i)}^2 > \frac{h}{t-k} | F_{\tau_{k-1}}\right] > h$$

Proof. Let Y_1 be the configuration taken by $\zeta_{\tau_{k-1}}$.
Let $Y_2, Y_3, \ldots$ be the subsequent sites visitd by ζ. Suppose that Y_1 has jump rate equal to V>30 R^2. It follows from Lemma 2.3 (i) that the jump rates $q(Y_2)$, $q(Y_3)$,...$Y_{V/30R^2}$ are all in the interval [2V/3, 4V/3]. Also if a site Y_i has flip rate G, then the chance that its next jump corresponds to a point τ_k is precisely 1−1/G. Therefore, the chance that $Y_2, Y_3, \ldots Y_{V/30R^2}$ are all in A_k is at least equal to $(1-3/2V)^{V/30R^2}$. If this event of probability bounded away

from 0 occurs then $\sum_{X_i \in Ak} \frac{1}{q(X_i)^2} > \frac{V}{30R^2}\left(\frac{3}{4V}\right)^2 = \frac{3}{160R^2V}$. Now observe that since we assumed that the total flip rate at a given site was at most one, the rate V can be at most $8R^2(t-k)+1$. The result follows.

Corollary 2.4. *Let $B(t)$ denote the sequence of states X_i visited by ζ before $\tau_{t-t^{2/3}}$. As t tends to infinity $\sum_{X_i \in B(t)} \frac{1}{q(X_i)^2}$ tends to infinity in probability*

Proof of Proposition 2.1. Corollay 2.4 establishes that as t tends to infinity $\sum_{X_i \in \in B(t)} \frac{1}{q(X_i)^2}$ tends to infinity in probability. Lemma 2.3 part (ii) shows that we may apply Lemma 2.2 to the sequences e_i, e_i^T. This gives us the Proposition.

§3

In this section we present two applications of Theorem One which are close to the author's interests and which are not covered by Theorem Two.

Corollary 3.1. *Consider an annhilating branching process η_t for which the branching coefficient λ is greater than 1/3 and such that $\eta_0 \neq 0$. Then η_t converges to product measure $\frac{\lambda}{1+\lambda}$ in distribution as t tends to infinity.*

Proof. Neuhauser and Sudbury (1992) showed that the anhilating branching process possessed only two extremal invariant measures: the null measure and product measure $\frac{\lambda}{1+\lambda}$. Furthermore it was shown that if $\eta_t \neq 0$, then the position of the nearest occupied site to the origin at time t was tight as t varied. We establish the Corollary by assuming the converse and obtaining and contradiction. If the Corollary is not true, then we can find t_n increasing to infinity such that η_{t_n} converges in mean to a probability measure v not equal to the product measure. Theorem One tells us that v must be invariant; hence it must be a convex combination of the null measure and the product measure. However since the position of the nearest occupied site of η_t is tight as t varies, v can put no mass on 0. This establishes a contradiction.

Corollary 3.2. *Let η_t be a critical one-dimensional catalytic surface model (possibly with particles moving by exclusion). See Grannen and Swindle* (1990) *or Mountford* (1992) *for details. If $\eta_0 \equiv 0$, then η_t converges in distribution to $\frac{1}{2}\delta_{-1}+\frac{1}{2}\delta_{+1}$.*

Proof. As with Corollary 3.2 we suppose the converse. As before there must exist t_n tending to infinity such that η_{t_n} converges to v in distribution where $v \neq \frac{1}{2}\delta_{-1} + \frac{1}{2}\delta_{+1}$. By Theorem One v must be invariant. Furthermore, given that the flip rates are defined in a translation invariant manner and that $\eta_0 \equiv 0$, must be the case that v is also translation invariant. But Mountford (1992) establishes that the only such measures are of the form $v = \alpha\delta_{-1} + (1 - \alpha)\delta_{+1}$. By the symmetry of 1 and -1, it must be the case that α is equal to 1/2. This contradiction establishes the corollary.

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