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DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
STANFORD, CALIFORNIA 94305

LARRY L. SCHUMAKER

Vanderbilt University

Clearly the fitting of functions of more than two variables is an important problem, and it is nice to see that statisticians are willing to tackle it. Mathematicians have tended to concentrate on the bivariate case (perhaps because even there, much remains to be done). The recently published bibliography [5] provides a fairly up-to-date list of what approximation theorists have been doing. Some of this work does deal with the many variable case. In particular, the papers [1], [2], [3], [4] and [11] deal with adaptive fitting of piecewise polynomials, much in the spirit of the paper under discussion. These papers also deal with the problem of giving error bounds.

Approximation theorists have also recently been interested in the problem of approximating multivariate functions by sums of univariate functions. In this connection I would like to cite [6], [7] (see also the bibliography [5]). Other references can be found in the book [6].

Next, a few comments on the paper under discussion. I am a bit puzzled by the assertion in Section 2.4.2 and later in Section 3.2 that lack of smoothness of the approximating functions limits the accuracy of the approximation. Generally it is true that lack of smoothness of the function to be approximated limits accuracy, while for the approximating functions it is the degree of the polynomials used which is critical. Similarly, I do not understand the discussion of end effects in Section 3.7. The classical natural splines perform badly near the boundaries precisely because they smoothly match linear functions there; i.e., they are constrained at the endpoints in the wrong way. The author uses a basis of piecewise linear functions which are smoothed out to be C^1. If one does not need C^1 functions, it seems it would be better to simply use linear splines to begin with. As far as I know, the approximation properties of the modified basis function are not understood. Do they approximate with the same power as linear splines? Surely they not do as well as quadratic ones. My next remark relates to the basis functions being used. As noted in Section 3.9, the one-sided truncated power basis is well known to be very badly conditioned whereas the classical B-splines are very well-conditioned. Why not use the latter? Updating might even be easier.

The idea of simplifying the model by removing knots (recombining pieces) strikes me as very important. This idea has recently been discovered by approximation theorists in connection with general spline fitting. The papers [8]–[10] are representative.

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DEPARTMENT OF MATHEMATICS
VANDERBILT UNIVERSITY
NASHVILLE, TENNESSEE 37240

CHARLES J. STONE

University of California, Berkeley

This pioneering paper successfully combines creative breakthroughs (especially, *not* removing the parent basis function) with numerous techniques developed over the years by the author and his collaborators and others