REFERENCES

BREIMAN, L. (1989a). Fitting additive models to data. Technical report 210, Dept. Statist., Univ. California, Berkeley.

BREIMAN, L. (1989b). The II-method for estimating multivariate functions from noisy data. *Technometrics*. To appear.

BREIMAN, L. and SPECTOR, P. (1989). Submodel selection and evaluation in regression X-random case. *Internat. Statist. Rev.* To appear.

BUJA, A., HASTIE, T. and TIBSHIRANI, R. (1989). Linear smoothers and additive models (with discussion). *Ann. Statist.* **17** 453–555.

CHONG, G., BATES, D. M., CHEN, Z. and WAHBA, G. (1988). The computation of GCV functions through Householder tridiagonalization with application to the fitting of interaction spline models. Technical report 823, Dept. Statist., Univ. Wisconsin.

FRIEDMAN, J. H. and SILVERMAN, B. W. (1989). Flexible parsimonious smoothing and additive modeling (with discussion). *Technometrics* **31** 3–39.

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J. H. Friedman presents the new recursive method of regression estimation for high dimensional data. This method is very interesting and has very good perspective. The main idea is an adaptive and recursive construction of the system of basis functions. The proposed estimation method has good flexibility and it is convenient for computer realization. We think that this approach is applicable for other nonparametrical estimation problems, for instance in the spectral density estimation for stationary Gaussian data.

The interesting problem connected with the proposed method is the theoretical study of quality of this method for different classes of smooth regression functions. (The reasons for consideration of the classes of smooth functions lie not only in practical importance of such constraints. From our point of view the most important theoretical results are established for these functional classes.) Let us recall some known results in this direction.

1. The best in minimax sense order of the rate of convergence of the L_p, $1 \le p < \infty$, risks to zero for the regression function of the smoothness of β in $\mathbb{R}^k$ is equal to $n^{-\beta/(2\beta+k)}$ [Ibragimov and Hasminskii (1980) and Stone (1982)].
2. Speckman (1985) and Nussbaum (1985) found regression estimators which cannot be improved, not only in the sense of order of the rate of convergence but also in the sense of constant. Impossibility of improvement (in minimax sense) of this constant for special case ellipsoids in the Sobolev spaces and integrated mean-squared error was proved by Nussbaum (1985), who used the results of Pinsker (1980).

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3. Recently, Golubev and Nussbaum (1991) have constructed the estimator, having the best constant and the best order for a priori unknown ellipsoid in the Sobolev space. This estimator is adaptive in this sense and it uses adaptive choice of basis, too. The family of Demmler and Reinsch (1975) bases $\{\varphi_i^\beta\}$ is used for different orders of smoothness β. The estimator has the form

$$\hat{f}_n(x) = \sum_{i=1}^{n} \left[1 - \left(\frac{i}{W}\right)^\beta\right]_+ \langle X_n, \varphi_i^\beta\rangle \varphi_i^\beta(x).$$

Here

$$\langle X_n, \varphi_i^\beta \rangle = \frac{1}{n} \sum_{l=1}^{n} X(t_l^n)\varphi_i^\beta(t_l^n),$$

t_l^n, $l = 1, \ldots, n$, is equidistant observation design. The values of β and bandwidth W are chosen adaptively on base of data.

We think that the interesting thing in the theoretical sense question is: Have Friedman's estimators or some of their modifications analogous asymptotical properties or not?

In conclusion, we would like to repeat that Friedman's estimator is very attractive for applications independent of the answer to this question.

REFERENCES

DEMMLER, A. and REINSCH, C. (1975). Oscillation matrices with spline smoothing. *Numer. Math.* **24** 375–382.

GOLUBEV, G. and NUSSBAUM, M. (1991). Adaptive spline estimators in regression model. *Probab. Appl.* **36**. To appear.

IBRAGIMOV I. and HASMINSKII R. (1980). Asymptotical bounds of quality for nonparametrical regression estimation in L_p. In *Investigation in the Mathematical Statistics, III* **97** 88–101.

NUSSBAUM, M. (1985). Spline smoothing in regression models and asymptotic efficiency in L_2. *Ann. Statist.* **13** 984–997.

PINSKER, M. (1980). Optimal filtering of square integrable signals in white Gaussian noise. (In Russian). *Problems Inform. Transmission* **16** 52–68.

SPECKMAN, P. (1985). Spline smoothing and optimal rates of convergence in nonparametric regression models. *Ann. Statist.* **13** 970–983.

STONE, C. (1982). Optimal global rates of convergence for nonparametric regression. *Ann. Statist.* **10** 1040–1053.

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