If possible let $b < v$. Consider the $v \times v$ matrix

$$N = \begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1b} & 0 & \cdots & 0 \\ n_{21} & n_{22} & \cdots & n_{2b} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ n_{v1} & n_{v2} & \cdots & n_{vb} & 0 & \cdots & 0 \end{bmatrix} \tag{2.4}$$

where the last $v - b$ columns of N consist of zeros. It follows from (2.2) and (2.3) that

$$NN' = \begin{bmatrix} r & \lambda & \cdots & \lambda \\ \lambda & r & \cdots & \lambda \\ \cdots & \cdots & \cdots & \cdots \\ \lambda & \lambda & \cdots & r \end{bmatrix} \tag{2.5}$$

where N' denotes the transpose of N.

$$\det (NN') = \{r + \lambda(v - 1)\} (r - \lambda)^{v-1} \tag{2.6}$$

But $\qquad = kr(r - \lambda)^{v-1}$ from (1.1).

$$\det (NN') = \det N \det N' = 0. \tag{2.7}$$

This makes $r = \lambda$, and contradicts (1.2). Hence the assumption $b < v$ is wrong, and we must have

$$b \geq v \tag{2.8}$$

REFERENCES

[1] R. A. Fisher, "An examination of the different possible solutions of a problem in incomplete blocks," *Annals of Eugenics*, London, Vol. 10 (1940), pp. 52–75.

[2] F. Yates, "Incomplete randomised blocks," *Annals of Eugenics*, London, Vol. 7 (1936), pp. 121–140.

ABSTRACTS OF PAPERS

(Presented September 1, 1949 at Boulder at the Twelfth Summer Meeting of the Institute)

1. **Structure of Statistical Elements.** Duane M. Studley, Foundation Research, Colorado Springs, Colorado.

Research in logical semantics and in practical elementation has set forth the proposition that all words and ideas have set form. As a consequence of this universal proposition all notions and conceptions in statistics should be accessible to set-theoretic analysis and interpretation. This paper explains the results of a preliminary analysis performed on statistical notions and conceptions with a view to a proper organization of definitions and conceptions which will, it is hoped, make possible a better and simpler construction of statistics from a system of basic notions.

2. On the Relative Efficiencies of BAN Estimates. LEO KATZ, Michigan State College, East Lansing, Michigan.

J. Neyman, in the Proceedings of the Berkeley Symposium on Mathematical Statistics and Probability, 1949, proved that χ^2 minimum estimates with either of two alternative definitions of χ^2 are efficient, as also are the maximum likelihood estimates. He also raised the question whether some of these estimates were better than others. This paper bears on that question. In making χ^2 minimum estimates, it is often necessary to avoid small frequencies by grouping together at least one tail of the distribution. It is with respect to the parameters of these *modified* distributions that the χ^2 estimates are efficient. Define *relative efficiency* in these circumstances as the ratio of the variance of an efficient estimator in the unmodified case to that of one in the *modified* case. It is shown that, except for a rectangular probability law, the relative efficiency <1 and, further, it decreases as the tail grouping is made wider. Formulae are given for the relative efficiencies of χ^2 minimum estimators for Binomial and Poisson probability laws and some representative values computed to exhibit these effects.

3. Adjustment of an Inverse Matrix Corresponding to Changes in the Elements of a Given Column or a Given Row of the Original Matrix. JACK SHERMAN and WINIFRED J. MORRISON, The Texas Company Research Laboratories, Beacon, New York.

A simple computational procedure is derived for obtaining the elements b'_{ij} of a nth order matrix (B') which is the inverse of (A'), directly from the elements b_{ij} of a matrix (B) which is the inverse of (A), when (A') differs from (A) only in the elements of one column, say the Sth column. The equations which form the basis of the computation are:

$$b'_{Sj} = \frac{b_{Sj}}{\sum_{i=1}^{n} b_{Sr}\, a'_{rS}}, \qquad j = 1, 2, \cdots n.$$

$$b'_{ij} = b_{ij}\, b'_{Sj} \sum_{r=1}^{n} b_{ir}\, a'_{rS}, \qquad \begin{array}{l} i = 1, 2, \cdots S-1, S+1, \cdots n \\ j = 1, 2, \cdots n. \end{array}$$

Analogous equations are derived for the case that A and A' differ in the elements of a given row rather than a column.

4. On the Problem of Optimum Classification. PAUL G. HOEL, University of California at Los Angeles.

Let f_i, $(i = 1, 2, \cdots, k)$, be the probability density function of population i and let p_i be the probability that population i will be sampled. Assume certain differentiability conditions and moment properties. Then, for known parameters, the probability of a correct classification will be maximized by choosing the region M_i, which corresponds to classifying into population i, as that part of variable space where $p_i f_i \geq p_j f_j$, $(j = 1, 2, \cdots, k)$. If the parameters are unknown, an asymptotically optimum set of estimates will be given by the set that minimizes a certain form in the covariances. Among uncorrelated estimates, maximum likelihood estimates are seen to be asymptotically optimum.

If weight functions, W_{ij}, are introduced and the expected value of the loss is minimized, the same methods of proof show that the region M_i becomes that part of variable space where $\sum_{r=1}^{k} p_r f_r (W_{rj} - W_{ri}) \geq 0$, $(j = 1, 2, \cdots, k)$, and that the criterion for an asymptotically optimum set of estimates is of the same form as the preceding criterion.

5. Optimal Linear Prediction of Stochastic Processes whose Covariances are Green's Functions. C. L. Dolph and M. A. Woodbury, University of Michigan, Ann Arbor.

A method of unbiased, minimal variance, linear prediction is developed for problems similar to those of prediction and filtering treated by Wiener. It differs from these in that, the unbiased condition is imposed, only a finite part of the past is employed, and no stationary assumption is used. It is shown that the special stationary case discussed by Cunningham and Hund, "Random Processes in Problems of Air Warfare" (*Supp. Journal Royal Stat. Soc.*, 1946) succeeds because the correlation function, $e^{-\lambda(t-s)}$, well known to that of the process defined by the Langevian equation, is the Green's function of the homogeneous differential equation formed by letting the adjoint differential operator of the Langevian equation operate on the operator of this equation. This relationship is shown to persist for any physically stable linear differential equation driven by "white noise." The well-known equivalence between integral and differential equations is then extended by use of Stieltjes integrals and used to effect the solutions of the integral equations of the first kind which yield the "optimum" linear prediction. The nonstationary example consisting of purely random motion about a mean linear path in the presence of radar type errors is treated in detail.

6. The Integral of the Gaussian Distribution over the Area Bounded by an Ellipse. H. H. Germond, RAND Corporation, Santa Monica, California.

This paper describes the preparation of tables from which to obtain the integral of a bivariate Gaussian distribution over the area of an ellipse. The center of the ellipse need not coincide with the mean of the Gaussian distribution, nor need the axes of the ellipse have any special orientation with respect to the Gaussian distribution.

7. Theorems on Convergency of Compound Distributions with Symmetric Components. (*By title*) Maria Castellani, University of Kansas City.

The purpose of this paper is to present some results obtained when operations of convulution in R_1 are concerned with a specific family of distributions. The compound distribution $K(x) = F(x) * G(x)$ is here obtained combining any d.f. $F(x)$ with a d.f. $G(x)$ under the restriction of symmetry, i.e., $G(x + h) + G(x - h) = 1$ for any $h > 0$.

A generalization of Cantelli's Inequalities will enable us to write a preliminary theorem on the following upper and lower bounds:

$$F(a - h) - 2\int_h^\infty dG(y) < K(a) < F(a + h) + 2\int_h^\infty dG(y),$$

$$K(a - h) - 2\int_h^\infty dG(y) < F(a) < K(a + h) + 2\int_h^\infty dG(y),$$

where a is any point in R_1 and $h > 0$.

The theorem is derived assuming the Stieltjes Integral,

$$K(a) = \int_{-\infty}^{+\infty} F(a - y)\, dG(y),$$

is taken as a sum of three integrals connected with three convenient intervals $(-\infty, -h)$, $(-h, h)$, $(h, +\infty)$. When the symmetric component of the convolution is a member of a family of normal distributions such as $G_\alpha(x) = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{x} e^{-\alpha^2 y^2}\, dy$, where α is an arbitrary parameter, the use of Cantelli's Inequalities give

$$K_\alpha(a - h) - K_\alpha(a) - \frac{2}{\sqrt{\pi}} \int_{dh}^{\infty} e^{-u^2}\, du < F(a) - K_\alpha(a)$$
$$< K_\alpha(a + h) - K_\alpha(a) + \frac{2}{\sqrt{\pi}} \int_{\alpha h}^{\infty} e^{-u^2}\, du,$$

where $K_\alpha(x) = F(x) * G_\alpha(x)$.

The d.f. $K_\alpha(x)$ is a continuous point function in R_1, with a fr. f$\cdot\gamma(x)$ which is everywhere uniformly continuous. For an arbitrarily small $\eta > 0$, a convenient small h and large α may be found which will enable us to prove the following two theorems:

THEOREM 1: Given any d.f. $F(x)$ in R_1, there exists a convenient continuous d.f. $K_\alpha(x)$ which for $\alpha \to \infty$ converges asypmtotically and uniformly almost everywhere to the given d.f. $F(x)$.

THEOREM 2: Given any d.f. $F(x)$ in R_1, there exists in any continuity bordered interval a convenient uniformly convergent series of continuous functions which asymptotically approach the given $F(x)$.

8. Partial Sums of the Negative Binomial in Terms of the Incomplete Beta-Function. (*By title*) JULIUS LIEBLEIN, Statistical Engineering Laboratory, National Bureau of Standards.

In acceptance sampling a certain size sample is taken at random from a lot of items and the lot is accepted if the number of defective items do not exceed a predetermined number characteristic of the sampling plan. The Statistical Engineering Laboratory has been studying the probabilities that a decision to accept or reject can be made before the sample is completely inspected. Such probabilities are found to involve certain sums apparently not previously treated. In this note the author proves a simple identity connecting these sums which greatly facilitates their computation and shows how they may be written in terms of the well-known incomplete beta-function of Karl Pearson, for which extensive tables are available.

9. Large Sample Tests and Confidence Intervals for Mortality Rates. (*By title*) JOHN E. WALSH, RAND Corporation, Santa Monica, California.

In computing mortality rates from insurance data, the unit of measurement used is frequently based on number of policies or amount of insurance rather than on lives. Then the death of one person may result in several units of "death" with respect to the investigation; moreover, the number of units per individual may vary noticeably. Thus the usual large sample methods of obtaining significance tests and confidence intervals for the true value of the mortality rate are not applicable to these situations. If the number of units associated with each person in the ivestigation were known, accurate large sample results could be obtained; however, determination of the number of units associated with each individual would require an extremely large amount of work. This article presents some valid large sample tests and confidence intervals for the mortality rate which do not require much work and are reasonably efficient. The procedure followed consists in first dividing the risks into twenty-six subgroups on the basis of the first letter of the last name of the person insured. Some of the groups are then combined until 10 to 15 subgroups yielding approximately the same number of units are obtained. The fraction consisting of the total number of units paid divided by the total number of units exposed is computed for each subgroup. Asymptotically the resulting observations represent independent observations from continuous symmetrical populations with common median equal to the true value of the rate of mortality. Tests and confidence intervals for the rate of mortality are obtained by applying the results of the paper "Some Significance Tests for the Median which are Valid Under Very General Conditions" (*Annals of Math. Stat.*, Vol. 20 (1949), pp. 64–81) to these observations.

NEWS AND NOTICES

Readers are invited to submit to the Secretary of the Institute news items of interest

Personal Items

Mr. Fred C. Andrews will be a teaching assistant in the Statistical Laboratory, Department of Mathematics, University of California for the academic year 1949–1950.

Dr. Joseph Berkson has been promoted to the rank of Professor in the University of Minnesota Graduate School and Mayo Foundation. He continues as Chief of the Division of Biometry and Medical Statistics of the Mayo Clinic.

Mr. Colin R. Blyth is now a research assistant at the University of California, Statistical Laboratory, Berkeley.

Mr. Clyde A. Bridger is now Director of the Section of Statistics and State Registrar of Vital Statistics for the Division of Health of Missouri.

Mr. Loren V. Burns, formerly with the MFA Milling Company at Springfield, Missouri, has been made Vice-President and Technical Director of the Spear Mills, Inc., Kansas City 6, Missouri.

Professor Douglas Chapman, who obtained his Ph.D. in statistics at the University of California, Berkeley, has accepted an appointment as Assistant Professor at the University of Washington in the Department of Mathematics and the Laboratory of Statistical Research.

Dr. Andrew Laurence Comrey, who received his doctor's degree from the University of Southern California last June, has accepted an assistant professorship in the Department of Psychology at the University of Illinois.

Dr. Donald A. Darling has been appointed to an instructorship in the Department of Mathematics, University of Michigan.

Dr. Paul M. Densen resigned his position as Chief of the Division of Medical Research Statistics of the Department of Medicine and Surgery of the Veterans Association as of July 1, 1949 to join the staff of the Graduate School of Public Health, University of Pittsburgh, as an Associate Professor of Biostatistics.

Mr. Amron H. Katz has been promoted to the position of Chief Physicist of the Photographic Laboratory, Engineering Division, Air Material Command, Wright Patterson Air Force Base, Dayton, Ohio.

Associate Professor Louis Guttmann, who had been on leave for two years from the Department of Sociology of Cornell University conducting a research program in Israel, was invited to remain in Israel for another year to direct the