Let $F(x_1, x_2)$ be a normal distribution with means m_1, m_2, variances σ_1^2, σ_2^2 and correlation coefficient ρ. The transformation can then be written as

$$F_1(x_1) = \Phi\left(\frac{x_1 - m_1}{\sigma_1}\right),$$

$$F_2(x_2 \mid x_1) = \Phi\left(\frac{x_2 - m_2 + \frac{\rho\sigma_1}{\sigma_2}(x_1 - m_1)}{\sigma_2\sqrt{1 - \rho^2}}\right).$$

REFERENCES

[1] P. Lévy, *Théorie de l'Addition des Variables Aléatoires*, Gauthier-Villars, Paris, 1937, pp. 71–73, 121–123.

[2] Paul B. Simpson, "Note on the estimation of a bivariate distribution function," *Annals of Math. Stat.*, Vol. 22 (1951), pp. 476–478.

ABSTRACTS OF PAPERS

(*Abstracts of papers presented at the Eugene meeting of the Institute, June 19–21, 1952*)

1. **The Auditory Cortex—A Probability Model.** Archie R. Tunturi, University of Oregon Medical School.

The role played by the brain in communication is well known, but in what manner the brain handles information is not understood. Some progress has been made in this direction by studying the anatomy and physiology of the auditory cortex in the anesthetized dog with controlled acoustic signals. Communication may be thought of as making a representation in a space of a representation in another space. In three of the four auditory areas (on one side of the brain), the entire frequency spectrum from 100 to 12800 cps is represented literally spacewise by groups of cells that respond only to a narrow range of frequencies. A special method increases the signal to noise ratio, by augmenting the electrical response of the cells, thereby permitting exact measurements of the characteristic frequency and intensity for each group of cells. This is similar to a narrow band filter, and does not reveal the effect of other frequencies on the information. The information capacity of the system can be inferred if it can be assumed that occurrence of the augmented response for the group of cells follows some probability function. These probabilities for all groups of cells can be assembled into a model representing the behavior of the system as a communication device. If there are 70 groups of cells between 100 and 12800 cps, the probability of any particular combination would be $1/2^{70}$, if the selections were equally probable. The effect of noise on this system will be considered. (Research sponsored in part by the Office of Naval Research.)

2. **Testing Message Diffusion: The Utilization of Mathematical Models.** Stuart C. Dodd, Richard J. Hill, and Susan Huffaker, University of Washington.

In connection with the study of interpersonal verbal communication, the Washington Public Opinion Laboratory designed an experimental procedure which yielded data on the temporal diffusion of thirty-three different messages in a population of 184 individuals. Data (including the recipient of each message, the initiator of communication, and the time of communication) were obtained on 5,522 separate instances of communication. The experimental population was selected so that the assumption could reasonably be made that it was homogeneous relative to the interacting (i.e., each individual had an equal probability of interacting), and uninfluenced by the spatial factor in diffusion. In the attempt to describe the temporal diffusion of the messages, three rational mathematical models were compared with the empirical results. The models utilized were the logistic, the normal ogive, and the binomial distribution, for which the mathematical conditions are well identified. The experiment had been designed to match those social conditions underlying the communication with the mathematical conditions underlying the logistic model (namely, a constant probability of interacting in the population and through time). The higher and lower degrees of descriptive closeness and goodness of fit for the three models, and for ordinal and cardinal time units separately, are reported and discussed.

3. On the Method of Collective Marks. HERMAN RUBIN, Stanford University.

In *Colloques International du Centre National de la Recherche Scientifique, No. 13*, D. van Dantzig considered the application of the method of generating functions to run problems. Let $x_1, \cdots, x_n, \cdots$ be a sequence of nonnegative integer-valued random variables of which only a finite number are nonzero with probability one. Then $E(\Pi_{i=1}^{\infty} t_i^{x_i})$ exists for $|t_i| \leqq 1$. If $0 \leqq t_i \leqq 1$, then we can interpret t_i as the probability that a coin when tossed will come up heads. Let an experiment be performable independently an arbitrary number of times with results a_j with probability p_j, and let us ask questions concerning the numbers of runs of various types obtained. One may compute the generating function of the various quantities involved by performing the experiment n times with probability $\lambda^n(1 - \lambda)$. If y_j is the number of occurrences of a_j and z_{jk} is the number of runs of length k of type j, then $P = E(S^n \Pi u_j^{y_j} \Pi v_{jk}^{z_{jk}}) = 1 - \lambda + \Sigma_j f_j P_j$, where $f_j = \Sigma_k(\lambda p_j S u_j)^k v_{jk}$ and P is the probability that no tail occurs and P_j is the probability that in addition a_j is not the first outcome of the experiment. Then $P_j = P - f_j P_j$ and hence $P = (1 - \lambda)/(1 - \Sigma_j f_j/(1 + f_j))$. Then such questions as to number and type of runs in n experiments can be answered from the conditional generating function. Other types of questions can be answered by letting λ approach 1.

4. On Asymptotic Properties of Estimates. LUCIEN LECAM, University of California, Berkeley.

Let $f(x, \theta)$ be a probability density depending on a parameter θ. Under regularity conditions on $f(x, \theta)$ it is shown that, for a broad class of loss functions and for positive continuous a priori densities: (1) The maximum liklihood (M. L.) estimate is asymptotically equivalent to a Bayes estimate. Moreover all "regular" asymptotic Bayes estimates are equivalent. (2) Given any estimate T_n there exists a randomized estimate T_n' asymptotically equivalent to T_n and depending on the sample point by means of the M. L. estimate only. (3) If an estimate T_n is super-efficient at a point θ_0 there exists a sequence θ_n, $\theta_n \rightarrow \theta_0$ of points at which T_n is worse than the M. L. estimate, except maybe for a finite number of values of n.

5. The Principle of Invariance Applied to a Problem of Classification. ROSEDITH SITGREAVES, Stanford University.

A problem of classification considered by Wald and Anderson is as follows: an individual is to be classified on the basis of p measurements as coming from one of two populations, Π_1 or Π_2. The distribution of the measurements in each population is multivariate normal with the same covariance matrix, but with different expected values. The parameters of the distribution are unknown, but samples of N_1 observations from Π_1 and N_2 observations from Π_2 are available. Both Wald and Anderson proposed classification procedures determined by substituting sample estimates for unknown parameter values in the appropriate Bayes' solution for the case when the parameters are known. Optimum properties for these procedures are difficult to prove because the two alternatives are composite. If the possible procedures are restricted by adopting the principle of invariance, there is obtained a family of classification procedures depending on a single nuisance parameter such that the class of procedures defined for a fixed value of the nuisance parameter is a complete class. In a single case, namely, when $N_1 = N_2$ and the losses due to misclassification are equal, the minimax procedure is independent of the nuisance parameter, and depends upon classification statistics proposed by Wald and Anderson.

6. Optimum Selection Procedures. Edward Paulson, University of Washington.

Let $(Y_1, Y_2, \cdots, Y_k, X)$ be a $(k + 1)$ dimensional random variable, with $Y_1, Y_2, \cdots, Y_k$ denoting the admission scores of an individual on a series of k admission tests and X the performance or achievement score of the same individual. Optimum procedures for selecting or rejecting an individual on the basis of his observed admission scores have been investigated by Birnbaum, Chapman, and Cochran. By considering the X scores as playing the role of a parameter with respect to the conditional distribution of $Y_1, Y_2, \cdots, Y_k$ when X is fixed, two additional procedures are derived, each optimum in a certain well defined sense. The second of these procedures, which is analogous to a Bayes solution and minimizes the average loss in classifying an individual, can easily be extended to the problem of classifying an individual on the basis of his admission scores into one of 3 or more classes, providing that the appropriate loss functions and the conditional probability distribution of X for fixed values of $Y_1, Y_2, \cdots, Y_k$ are known.

7. A Theorem on Convex Cones with Applications to Linear Inequalities. Jerry W. Gaddum, Institute for Numerical Analysis.

I can summarize the results as follows: It is proved that if A is a convex cone in E_n and A^* its polar cone, A and A^* have a non-null vector in common unless A is an E_r. This is applied to obtain the result that $Ax \geqq 0$ has a solution if and only if $AA'y \geqq 0$ has a non-negative solution. Results due to Dines and to Motzkin are then applicable to obtain further existence theorems. The theorem on convex cones also provides a method of solution of systems of inequalities.

8. On the Prediction of Nonstationary Stochastic Processes. R. C. Davis, Bureau of Ordnance.

During a finite time interval T the real valued function $S(t) + N(t)$ is observed, in which $S(t)$ is a signal and $N(t)$ is a linearly superimposed noise disturbance. The problem is to predict the value of a given linear functional of $S(t)$, the predictor formula having certain preassigned "optimum properties" among a certain class of predictors. In the case in which the mean value of $S(t)$ is known, the random components of $S(t)$ and $N(t)$ are strictly stationary, and the time interval T is infinite, a complete solution to this problem has been given by N. Wiener. (In the case of discrete time series, the solution was given by A. Kolmogorov.) This theory has been extended by L. Zadeh and J. Ragazzini to the case in which T is a finite time interval and the mean value of $S(t)$ is unknown but is restricted to be a polynomial in time. The above theory is extended to the case in which the random components of both $S(t)$ and $N(t)$ are nonstationary in time and merely possess finite continuous covariance and cross covariance functions. The analytical methods of probability theory used are those developed by M. Loève and K. Karhunen in their studies of stochastic processes of second order. These techniques are very powerful in the analysis of transient random phenomena in linear systems. Finally an exposition is given of prediction by the method of conditional probabilities, and this method is compared with the least squares method.

9. Power Efficiency Function for Normal Alternatives for Several Nonparametric Tests. W. J. DIXON, University of Oregon.

Power curves are computed for samples of size 3, 4, 5 for the maximum absolute deviation, the rank-sum, and the median tests for normal alternatives. These curves are compared with the power curves for the t-test. (Research sponsored by the Office of Naval Research.)

10. Bounds for Second Moments of the Sample Range. SIGEITI MORIGUTI, University of North Carolina and University of Tokyo.

Let R be the range, that is the difference between the two extreme values, of a sample of size n from a symmetrical population with variance σ^2. This paper deals with the least upper bound for $E(R^2)/\sigma^2$, the greatest lower bound for the coefficient of variation of R, and the greatest lower bound for $V(R)/\sigma^2$. It is shown that these can be obtained as characteristic values of certain integral equations. For n even, these equations can be reduced to linear algebraic equations. Numerical results are given for small values of n.

11. Estimates, Tests, and Tolerance Intervals for Lump Data from a Normal Distribution. HARRY M. HUGHES, University of California, Berkeley.

If N independent samples from a normal distribution with unknown mean μ and unknown variance σ^2 are lumped into m subsamples of known size, and if the sums of the subsample values are observed but not the individual values, then the variance of the maximum likelihood estimate of μ is unchanged but the variance of the maximum likelihood estimate of σ^2 (modified to remove bias) is increased to that of a sample of size m. The effect on confidence intervals and on standard tests such as the t-test and F-test follows immediately.

The effect on tolerance intervals, not heretofore easily available, is presented in the form of a table of factors, depending on m, N and the confidence coefficient γ by which to modify existing tables of the coefficient λ in the expression $\bar{x} \pm \lambda s$. The range covered is $m = 2$ to 100, $N = 3$ to 1000 for $N > m$; confidence coefficients .75, .90 and .99; and all tolerance coefficients (percentage of population to be covered) since table entries are independent of the tolerance coefficient. The resulting increase in the expected length of the tolerance interval is indicated.

12. A Computing Formula for the Power of the Two-Sided t-Test for Even Degrees of Freedom. WESLEY L. NICHOLSON, University of Oregon.

A general formula is exhibited for the power function of the "two-sided" t-test for even degrees of freedom. The form employed is particularly adapted to computation of the power for various alternative means, given fixed arbitrary type I error and even degrees of freedom. The power functions for 2, 4, 6, 8, and 10 degrees of freedom are written out explicitly with an indication of their use for arbitrary type I error.

13. On the Analysis of Data Matched in Pairs. FRANK J. MASSEY, JR., University of Oregon.

We suppose that groups of four observations are drawn under varying environmental conditions for the different groups. In each group two of the individuals are subjected to one treatment and the other two are subjected to a second treatment. This paper discusses various parametric and nonparametric techniques of analyzing the data. Power curves are given for the case where the two populations are normally distributed differing only in mean value.

14. The Power Function of the Haldane-Smith Test. B. M. Bennett, University of Washington.

Haldane and Smith (*Annals of Eugenics*, 1948) have devised a non-parametric test for the existence of a birth order effect in families in which a certain number of sibs are affected by a particular disease. The test consists of summing the ranks of all affected individuals. The distribution has been worked out under the assumption that all ranks are equally likely. It is shown that this test may be regarded more generally as a test for the constancy of the probability of an event from trial to trial when it is known to occur a fixed number of times out of a given number of trials. The power function of the test has been considered with respect to several classes of alternatives concerning the successive probabilities of the event.

15. Spectral Analysis of Stationary Time Series. (Preliminary Report.) Ulf Grenander and Murray Rosenblatt, University of Chicago.

Having observed a sample sequence (finite) of a vector-valued stationary time series, one wishes to make inferential statements about the spectrum of the underlying process on the basis of the sample. Assume that the process is a linear process and that the spectrum satisfies certain regularity conditions. There are three aspects to the work being carried out. 1. Obtaining curve-estimates of the spectral density and spectral distribution function. These estimates have been chosen as different types of quadratic forms in the sample values. 2. Obtaining confidence bands and confidence regions for both the spectral density and spectral distribution function. Certain limit theorems nonparametric with respect to the spectrum have been obtained in connection with this aspect of the problem. 3. These limit theorems have also been used to construct one and two sample tests for time series. We plan to set up a sampling program to test the approach to the limiting distributions. Artificial time series will be generated and submitted to analytic techniques provided by the theory. In this connection, schemes of computation are now being devised. Auxiliary tables are under preparation. (This work is being done under the sponsorship of the Office of Naval Research.)

16. Asymptotically Subminimax and Asymptotically Admissible Statistical Decision Procedures. Jack Laderman, Columbia University.

A study is made of the asymptotic behavior of the minimax decision procedure when the distribution space, Ω, consists of a finite number of absolutely continuous distribution functions and the decision problem is to select the true distribution function when a sample of size n is known. It is seen that "asymptotically subminimax" procedures frequently exist which are clearly more desirable than the minimax procedure when n is sufficiently large. Let $r_i(T_n)$ denote the risk associated with the decision procedure T_n when the i-th distribution is the true distribution. A sequence of decision procedures, $\{T_n\}$, is said to be asymptotically admissible if there does not exist another sequence of decision procedures, $\{T_n'\}$, such that $\overline{\lim}_{n\to\infty} r_i(T_n')/r_i(T_n) \leqq 1$ for all values of i and the strict inequality holds for at least one value of i. A detailed study is made of the class of problems for which Ω consists of k univariate normal distributions all having the same variance but different means. The minimax decision procedure is found to be asymptotically admissible only for very special sets of values for the k means. For other sets of mean values, asymptotically subminimax procedures are obtained which are also asymptotically admissible. For still other sets of mean values, it is shown that no asymptotically subminimax procedure exists which is also asymptotically admissible. (This research was sponsored by the Office of Naval Research.)

17. Note on the Problem of Combining Independent Tests of Significance. ALLAN BIRNBAUM, Columbia University.

The "problem of combining independent tests of significance" is stated and illustrated. A distinction is made between two kinds of problems of combining tests. A definition of admissibility of a method of combining tests is proposed. A simple characterization of a minimal essentially complete class of methods of combining tests for a large class of problems is given (in terms of the proposed definition of admissibility). For a simple common testing problem, the properties of several methods of combining tests which have been recommended in the literature are examined and compared with properties of the optimal test for this problem. It is shown that some of these methods correspond to inadmissible test procedures for the original problem; other methods correspond to admissible procedures, and on the basis of their properties one may base a selection of a method of combination of tests, for use in problems which seem similar to that under consideration.

18. On Judging all Contrasts in the Analysis of Variance. (Preliminary Report.) HENRY SCHEFFÉ, Columbia University.

Under the usual (general linear hypothesis) assumptions, if the conventional F-test of the hypothesis $\mathbf{H}$: $\theta_1 = \theta_2 = \cdots = \theta_k$ at the α level of significance rejects $\mathbf{H}$, what further conclusions are valid about the contrasts among the θ_i (beyond the conclusion that the contrasts are not all zero)? Suppose the F-test has $k - 1$ and ν degrees of freedom. For any $c_1, \cdots, c_k$ with $\Sigma_1^k c_i \theta_i = 0$ write ψ_c for the contrast $\Sigma_1^k c_i \theta_i$, and write $\hat{\psi}_c$ and $\hat{\sigma}^2(\hat{\psi}_c)$ for the usual estimates of ψ_c and the variance of $\hat{\psi}_c$. Then for the totality of contrasts ψ_c the probability is $1 - \alpha$ that they all satisfy (*) $|\psi_c - \hat{\psi}_c| \leqq S\hat{\sigma}(\hat{\psi}_c)$, where S^2 is $(k - 1)$ times the upper α point of the F-distribution with $k - 1$ and ν degrees of freedom. Say that the estimated contrast $\hat{\psi}_c$ is "significantly different from zero" if the interval (*) does not cover the point $\psi_c = 0$. Then the F-test rejects $\mathbf{H}$ if and only if some $\hat{\psi}_c$ are significantly different from zero, and if it does, one can say just which $\hat{\psi}_c$. More generally, the inequalities (*) can be employed for all the contrasts with the obvious frequency interpretation about the proportion of experiments in which all statements are correct. Relations are considered to an earlier method of Tukey using the Studentized range tables ("Allowances for various types of error rates," invited address before IMS meeting at Blacksburg, Va., March 19, 1952; see also *Proc. Fifth Annual Convention A.S.Q.C.*, 1951, pp. 189–197). (Work sponsored by the Office of Naval Research.)

19. Further Moving Average Methods in the Estimation of LD-50. B. M. BENNETT, University of Washington.

Further extensions of the moving average methods for estimating LD-50 proposed by W. R. Thompson and the author (*Jour. Hyg.*, 1952) have been developed in order to include minimum variance and minimum variance unbiassed estimates from any assumed distribution of threshold. Numerical estimates of the efficiency of such procedures are included.

20. Nonparametric Theory: Confidence Regions and Tests for Location and Scale Parameters. (Preliminary Report.) D. A. S. FRASER, University of Toronto.

All unbiased estimates of zero are characterized for the class of probability density functions having the p percentile at the origin. As a consequence, results are obtained for location parameters (percentiles), for scale parameters (interpercentile ranges), and for combination location and scale parameters. All randomized β level confidence regions for a location parameter can be described in a simple functional form. All confidence bounds for a location parameter are almost everywhere equal to the order statistics chosen with fixed probabilities; all distribution-free upper tolerance limits are of the same form, the Robbins condition, $\Pi(f(x_1, \cdots, x_n) - x_i) = 0$ almost everywhere, being necessary but not sufficient (unless f is continuous). Most powerful one-sided and most powerful (shortest) unbiased confidence regions are obtained for the location parameters. Correspondingly, most powerful and most powerful unbiased tests are obtained for the location parameter. For the scale parameter similar or exact confidence regions do not exist. Analogously, similar tests do not exist for the scale parameter. For the scale and location parameter, confidence regions can be obtained; most powerful intervals exist for specific purposes. If the location parameter is known UMV unbiased estimates do not exist for any other parameter.

21. **Random Functions Satisfying Certain Linear Relations. I.** S. G. Ghurye, University of North Carolina.

Let $X(t)$ be a random function of the real parameter t, and let there exist k continuous functions $\alpha_1(h), \cdots, \alpha_k(h)$, of $h > 0$, such that for any t and any $h > 0$, the sequence $\{Y(t, h; n), n = 0, \pm 1, \cdots\}$, defined below, satisfies certain assumptions about independence. Here $Y(t, h; n) = X(t + (n + k)h) + \alpha_1(h)X(t + (n + k - 1)h) + \cdots + \alpha_k(h)X(t + nh)$. In other words, it is postulated that any sequence of observations made at equidistant t-points on the X-process satisfies a hypothesis of the type commonly expressed in terms of linear stochastic difference equations. Certain consequences of this assumption are studied in this paper. For instance, in all the cases considered, it is shown that the possible forms of the $\alpha_j(h)$ are restricted by the property that the roots of the following equation in x can be written in the form $e^{\lambda_i h}$, the λ_i being independent of h: $x^k - \alpha_1(h)x^{k-1} - \cdots - \alpha_k(h) = 0$. It is also shown that, under certain conditions, the Y-sequence is $(k - 1)$-dependent for all h, but can be independent for all h only if the X-process is deterministic.

22. **Computational Experience in Solving Linear Programs.** A. J. Hoffman, National Bureau of Standards.

This is a report of the experience gained in solving linear programs on the SEAC (National Bureau of Standards Eastern Automatic Computer) by the simplex method, relaxation, and fictitious play.

23. **Duality of Multi-valued Means.** Theodore S. Motzkin, University of California, Los Angeles.

In problems of statistical measurement and estimation, especially of angles and similar geometric quantities, and in their application including data preparation for linear programming, the question comes up how to define the mean for periodic variables. For these, as the example of equidistant points on a circumference shows, a unique and rotationally invariant mean cannot be defined. A multivalued and invariant mean is obtained as follows. For a set S on a smaller arc the arithmetic mean is ordinarily found by choosing a zero point on the complementary arc A_1. It is reasonable to invest this mean with a weight proportional to the length of A_1, and the means obtained from zeros on the arcs $A_2, \cdots, A_n$ with corresponding weights. One may try repeating the procedure with respect to the set S' of weighted means in view of possible convergence: however it turns out that $S'' = S$. More generally for every circular distribution S the analogous definition of a mean by a Stieltjes-Lebesque integral leads to a dual or complementary distribution S', and $S'' = S$. Similar facts hold on the torus (two periodic variables). On a sphere, even local associativity cannot be upheld; obtaining the mean of a finite set from the means of its finite subsets by iteration and passing to the limit, or as a minimum for the sum of the squares of spherical distances, are two transcendental (essentially infinitely-valued) definitions.

24. A Note on a Dynamic Leontief Model with Substitution. George B. Dantzig, The Rand Corporation.

In an important class of discrete dynamic Leontief models with substitution, an optimal program for the entire time span can be obtained by determining locally the optimal basis for the first time period alone, followed by determining the optimal basis for the second time period given the first, etc. It follows that the relative weights in the objective function of performing the various activities in any time period *do not affect the choice* of activities in earlier time periods although, of course, they may affect the levels of activities in the early time periods. Thus, optimal bases for a sequence of periods are found in the order: Period 1, Period 2, Period 3, $\cdots$, after which activity levels are found by ordinary back-solution for linear equations in the order Period n, Period $n - 1$, Period $n - 2$, $\cdots$.

25. Confidence Intervals on the Slopes of Regression Lines when Both Variables are Subject to Error. C. O. Junge, Jr., University of Washington.

In the problem of linear regression when both variables are subject to error, Reiersol has shown that if the errors and the "steady" parts are normally distributed then the slope of the desired regression line is not identifiable. If the errors in the two variables are independent, however, a conservative confidence interval on the slope of the regression line can be determined even though the slope is not identifiable. If the "steady" parts are a set of fixed but unknown values, independent estimates of the rank of the steady parts are sufficient to determine a meaningful confidence interval. In case an instrumental variable as defined by Geary is available, a confidence interval can be determined.

26. Statistical Estimation from Time and Population Samples. Julius A. Jahn, Washington State College.

Given a population of individuals, $I:1, \cdots, N_I$, and a variable V_i defined over this population, a random sampling procedure is to be designed to provide unbiased estimates of the mean value of V_i for the population with a minimum variance of the estimated values for certain fixed sample sizes and other cost conditions. When the variable is dependent upon time, the use of estimates from samples enumerated during one period of time to apply to later periods of time can lead to large and increasing bias due to changes with time. The solution proposed in this paper is to sub-divide the total period of time into a series of conveniently short intervals, $T:1, \cdots, N_T$, and to define as the universe to be sampled, the set of points (it) formed by the combinations of the identification numbers for time intervals (t) and those for the individuals in the population (i). Correspondingly, the variable is defined over these points (V_{it}) in such a way that the sum over the time intervals for any one individual is equal to V_i. A sample design incorporating the use of weekly time intervals as "strata" within which were selected a random sample of points in each strata and a constant sampling rate for each strata was applied to derive estimates of the mean value of the total expenditures of students at the University of Washington during the Winter Quarter of 1951.

27. The Analysis of Samples from Mixed Populations. (Preliminary Report.) D. G. Chapman, University of Washington.

Samples from natural populations will frequently include a mixture of several races or age groups. The analysis of such a sample into its component parts is useful for the determination of growth rates, mortalities, or the characteristics of the mixed populations. Several cases arise according to whether or not a simple age identifying factor is available, or whether or not supplementary information is available from other samples. Neyman's extended χ^2 formulae can be applied to these problems to analyze the mixed samples into their component parts, and to estimate characteristics of the mixed population. Auxiliary problems that arise in utilizing the supplementary information are considered.

28. Some Applications of the Log-Normal Distribution to the Study of Survival Times in Chronic Fatal Diseases, Particularly Leukemia. HAROLD TIVEY, University of Oregon Medical School.

Illustrations of some of the types of disease processes in which a log-normal distribution will closely approximate the distribution of survival times will be presented. A summary of survival data on over 2600 cases of leukemia taken from the literature will be briefly presented, with particular emphasis on the statistical problems arising from this investigation. The application of the maximum likelihood method of Boag will be discussed.

29. Field Trial Problems. G. A. BAKER, University of California, Davis.

Theoretical and empirical examinations of the correspondence of certain field trials to conventional mathematical models have been made. The correspondence may be quite good but on the other hand serious defections do exist. It seems apparent that a careful study of field plot design and methods of analysis are necessary in order to obtain more accurate measures of plant performance on definite plots. The classical mathematical models have led to marked advances in the theory and practice of field trials and no attempt is being made here to belittle their importance. The point that is being made is that when finer distinctions between varieties and treatments are attempted even minor deviations from mathematical theory are important. Every effort must be made to improve investigative tools. This can be done in two ways. One is to develop more realistic models. This often fails because of lack of detailed knowledge of uniformity trials, inability to predict the behavior of biological material in complex situations, and because of mathematical difficulties. The other general method of improving field trials is to arrange the plots, select the plants, modify overall conditions, or transform the data so that classical models will clearly indicate differences.

30. Population Forecasts, State of Washington, 1950–1960: Methodological Summary. CALVIN F. SCHMID, VINCENT A. MILLER AND WARREN E. KALBACH, University of Washington.

The cohort-survival method was used to prepare the population forecasts for the State of Washington, April 1, 1955 and April 1, 1960, based on detailed data covering: (1) an age-sex breakdown of the population on which projections are to be based, (2) age-sex-specific mortality trends, (3) age-specific fertility trends for females in child-bearing ages, (4) age-sex-specific migration trends. Estimates are made of the age-sex distribution of the population as of the forecast dates.

31. On the Addition of Chi-Squares. (Preliminary Report.) JACK BORSTING, University of Oregon.

An example is used to compare the power of three tests of a simple hypothesis specifying the cell frequencies in discrete population. The tests are as follows; the chi-square test based on a single sample, a chi-square test based on the addition of five independent chi-squares from separate samples, and a test which rejects the hypothesis if any one of the several chi-squares is significantly large. For the example considered the power of the single sample test was considerably greater than the power of the other two tests. For example, for one alternative the power of the single sample test is 0.41 and the power of the test using the sum of chi-squares is 0.20. Curves are drawn comparing the powers for a set of alternatives.

32. Decisions Based on Incomplete Information. NORMAN DALKEY, The Rand Corporation.

It is shown that the usual requirements of consistency, continuity, and convexity in choices made under incomplete information have the consequence that the decision maker must be guided by a set of weights, analogous to probabilities, for the possible relevant states of nature.

33. Method of Synthesizing an Autonomous Shipbuilding Industry for an Inter-industry Input-Output Model. CAPTAIN J. E. HAMILTON, George Washington University.

The paper presents a data-gathering and reduction program and methods of computation to construct a synthetic shipbuilding industry to provide a vector of inputs for a portion of the national bill of goods. This bill of goods is treated as an autonomous industry for application to a Leontief Input-Output matrix. The purpose of this is to test the Inter-Industry technique as to its suitability for testing the feasibility of military programs. The work consisted of assembling data sufficient to construct a suitable number of "prototype" bills of material for complete ships; of establishing a program of shipbuilding with sufficient breadth to embrace any industrial impact during the four year period under construction; of classifying the units in the program by similarities; of computing multipliers for material and manpower to pantograph the type to one of the prototypes; of developing a formula for computing manpower requirements by time periods; of programming computing machinery to compute material requirements by time periods; of calculating a vector of overhead inputs and a method of computation to include impacts of overhead; and of aggregating material, labor, and overhead into a single time-phased vector of total inputs from each of 192 industries.

34. A Mathematical Model of the Shipbuilding Industry. R. F. WILLIAMS, George Washington University.

This paper is a model of task of data gathering involved in an analysis of the industry-by-industry requirements in a "ship-building program." Such a program, in general, includes a variety of tasks other than actual building of ships, e.g. ship repair, ship conversion, etc. The set of data required for a program, **P**, is a matrix of numbers, $\{X_{ji}\}$, such that each X_{ji} is a *measure* of the material from the jth one of the n industrial segments serving the shipbuilding industry, required by the SBI during a certain (the ith) time segment for the program, **P**. First, as the raw data are necessarily found on a ship-task to ship-task basis, these must be amalgamated to arrive at the matrix $\{X_{ji}\}$. Thus an inquiry was made into the various possible means of amalgamation and a formalization of these techniques is presented. Secondly, the only practical such measure has been found to be the unit known as the "fixed dollar." Therefore, as the *cost* of a ship-task includes cost of material and cost of labor, a special inquiry is made into their respective records of data: bills of goods and wage scales, and a formal analysis of these is presented.

35. Distribution-free Tests of Fit for Continuous Distribution Functions. Z. W. BIRNBAUM, University of Washington.

This expository address deals with distribution-free techniques for deciding whether a sample $x_1, x_2, \cdots, x_n$ of a one-dimensional random variable was obtained from a population with a completely specified continuous cumulative distribution function $H(x)$. A number of such techniques are described and discussed. It is observed that all of them are based on statistics of the form $\phi[H(x_1), \cdots, H(x_n)]$ where $\phi(u_1, \cdots, u_n)$ is a symmetric function. The concepts of a distribution-free statistic and of a strongly distribution-free statistic are introduced, and the theorem is stated that under fairly general assumptions a strongly distribution-free statistic must be of the form $\phi[H(x_1), \cdots, H(x_n)]$. A program is outlined for introducing various metrics in the space of distribution functions and for investigating the power of distribution-free tests with respect to these metrics.

36. Acceptance Inspection by Variables when the Measurements Are Subject to Errors with Known Variance. Edward A. Fay and John E. Walsh, U. S. Naval Ordnance Test Station, China Lake.

The situation considered is that where measurement of the characteristic of interest is not exact but subject to noticeable error. The measurement error is assumed to be independent of the true value of the characteristic measured and to be unbiased with known variance. The variance of the true values of the characteristic measured is assumed to have a known lower limit which is greater than zero. The probability distributions involved are assumed to be normal while the true values and measurement errors each form a random sample. For suitable specified acceptable and unacceptable fractions defective, this paper presents one-sided acceptance inspection criteria which have the property that the producer and consumer risks have specified upper bounds. Two-sided criteria can be obtained by appropriate utilization of the one-sided results.

37. A Two-Sided Criterion for Acceptance Inspection by Variables. Edwin L. Crow, U. S. Naval Ordnance Test Station, China Lake.

It is desired to test whether a submitted lot has an acceptably small proportion defective, where an item is considered defective if a measurable characteristic is either less than a prescribed number or greater than another prescribed number. Two approximate solutions making use of measurements on each item of a sample of fixed size have been given by Wallis and Arnold (*Selected Techniques of Statistical Analysis*, Columbia University Press, 1947, pp. 52–57, 63-64). One of these simply combines two one-sided criteria and yields a triangular acceptance region in the $(\bar{x}, s)$ plane in the usual notation. The other corresponds to an acceptance region within the above triangle. The OC curves (probability of acceptance vs. lot proportion defective) of these criteria for particular values of the parameters are calculated by graphical integration. They depend on the lot mean. By interpolation between bounding ones of these OC curves it is possible to find an acceptance region intermediate to the two mentioned above for which the OC curve varies inappreciably with the lot mean. This two-sided criterion is generalized to the case in which the measurements are subject to error.

38. Acceptance Inspection by Variables in the Presence of Measurement Error. Edward A. Fay, U. S. Naval Ordnance Test Station, China Lake.

The lot acceptance criterion $\bar{x} + ks \leqq U$ for sampling inspection by variables (where U is the upper limit to acceptable quality) is generalized to allow for measurement error. If m measurements are made on each item of a sample of size N, the generalized criterion if of the form $\bar{x} + ks_b \leqq U$, where s_b is the standard deviation of the N item means. Under the assumption that the ratio r of standard deviation of measurement error to standard deviation of product variability is known, a straightforward generalization of the derivation of Wallis (*Selected Techniques of Statistical Analysis*, Columbia University Press, 1947, pp. 59–61) leads to the appropriate value of k for the case that m, N, and one point on the OC curve are preassigned, and to the appropriate values of k and N in the case that m and two points on the OC curve are preassigned. It is shown that measurement error increases the sample size required to achieve the same OC curve in the ratio $1 + 2r^2/[m(k^2 + 2)]$, where k is the value appropriate to the case of no measurement error.

Consequences of departure from the assumption of known r are investigated by comparing OC curves.

39. On Continuous Inspection Plans. (Preliminary Report.) GERALD J. LIEBERMAN, Stanford University.

This paper presents two extensions of the first continuous sampling plan devised by H. F. Dodge (*Annals of Math. Stat.*, Vol. 14 (1943), pp. 264–279). Dodge's scheme provided for 100% inspection until i consecutive non-defective units were observed, at which time partial inspection was instituted, namely, one out of $1/f$ units was inspected. As soon as a defect was observed, 100% inspection was reinstated. It is proven in this paper that whether the process is in a state of statistical control or not, an AOQL is assured. In fact, it is shown that always AOQL $\leqq (1/f - 1)/(i + 1/f)$. The second problem considered was that of obtaining a continuous sampling plan for the individual characteristics constituting an item. This paper presents a continuous inspection scheme similar to Dodge's for r such characteristics, where r is any finite integer. It guarantees an overall AOQL provided the process is in a state of statistical control. The procedure is as follows: units are divided into segments of i items as they are manufactured. The k^{th} characteristic $(k = 1, 2, \cdots, r)$ is inspected 100% until the preceding full segment is found free of defects in characteristic k, at which time partial inspection is instituted, namely, one out of $1/f$ units is inspected for characteristic k. Full inspection is reinstated for any charactistic as soon as a defect in that characteristic is observed.

40. An Acceptance Procedure with Two-Sided Specification Limits. GEORGE J. RESNIKOFF, Stanford University.

A two-sided acceptance test in sampling inspection by variables is devised. The test is based on an estimate of the proportion defective which is optimum in the sense of minimum variance among unbiased estimates. In the one-sided case this estimate leads to the well-known test $\bar{x} + ks \leqq U$, where U is a single upper limit defining quality. Graphically the test is a region in the $(\bar{x}, s)$ plane. The probability of a sample point $(\bar{x}, s)$ falling in the region is a function of the lot mean as well as of the proportion defective, but the resulting band of OC curves is so narrow, for the cases investigated, as to enable it to be considered a single curve insofar as industrial applications are concerned. Numerical investigation indicates that this property is itself virtually independent of the sample size. Since the acceptance region is inherently associated with the one-sided test, $\bar{x} \pm ks$, the OC curve of this one-sided test may be used as an approximation to the OC band of the two-sided test.

NEWS AND NOTICES

Readers are invited to submit to the secretary of the Institute news items of interest

Personal Items

Mr. Richard Berger, formerly with General Aniline & Film Corporation, is a Sales Analyst with General Dyestuff Corporation, New York City and is an Instructor in Statistics at Rutgers University.

Professor D. A. Darling, on leave from the Department of Mathematics, University of Michigan, is Visiting Assistant Professor of Mathematical Statistics at Columbia University for the academic year 1952–53.