ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Central Regional meeting, Lincoln, Nebraska, April 1–3, 1965. Additional abstracts appeared in earlier issues.)

15. On Some c-Sample Nonparametric Tests for Location in the Behrens-Fisher Situation. P. V. RAMACHANDRAMURTY, Case Institute of Technology.

When the variances of the underlying populations differ, the usual rank tests for location in the c-sample problem ($c \geqq 2$) do not have the stated level of significance even asymptotically since the asymptotic variance of the test statistics depends on the unknown variances. An obvious way of overcoming this difficulty is by replacing the asymptotic variance by a consistent nonparametric estimator. The resulting test does not usually have the minimum property of consistency for *all* alternatives because of the indistinguishability of the null hypothesis and the alternative. We show here that for the Kruskal-Wallis test the above method can be applied in such a way that the resulting test is consistent for all alternatives for which the underlying populations are symmetric; in fact, for all generalized Behrens-Fisher alternatives. When there are only two populations, the above method gives a test which is asymptotically equivalent to the studentised Wilcoxon test given by Sen. (*Ann. Inst. Statist. Math. Tokyo* **14**. This test is shown to be better than a conservative test proposed by Pothoff (*Ann. Math. Statist.* **34**).

(Abstracts of papers presented at the Eastern Regional meeting, Tallahassee, Florida, April 29–May 1, 1965. Additional Abstracts appeared in earlier issues.)

12. Adaptive Statistical Procedures in Reliability and Maintenance Problems. JOSEPH L. GASTWIRTH and J. H. VENTER, Johns Hopkins University and Potchefstroom University. (Invited address)

In the present paper the following problem is considered:

A "system" with an exponentially distributed lifetime is to be inspected at times $t_1 < t_2 < \cdots$. If inspection reveals that the system is in-operative, it is repaired, otherwise nothing is done. The problem is to choose the sequence $\{t_i\}$ in an optimal manner. Most of the previous literature is concerned with the choice of the times $\{t_i\}$ when λ (the failure rate) is known. In the present work we assume that λ is unknown and several sequential inspection plans are proposed. These plans use the information as it becomes available from inspection to estimate λ and thereby approach the plan that would be optimum if λ were known. We discuss the problem of finding optimum sequential plans (called adaptive plans) when the objective is either to obtain the maximum limiting average information about the parameter λ or to minimize the average expected loss. In this second case our loss function takes into account the cost of an inspection, the cost of repair or replacement and the cost incurred while the system is inoperative.

13. Principles for Ranking Independent Variables by Order of Importance in the General Linear Hypothesis Model. KLAUS ABT, U. S. Naval Weapons Laboratory.

Three principles for the ranking of individual, or groups of, independent variables ("IV's") by order of importance are stated and justified. These principles are applicable to the general linear hypothesis model, i.e. to ordinary multiple regression problems as well as to non-orthogonal analysis of variance and covariance. The corresponding computational procedures are suitable only for large scale electronic computers. By reference to an NWL-coded IBM STRETCH program, it is illustrated how the combined application of the three principles can be used to screen IV's and, consequently, to establish a "significant model". The First Principle is that of ranking the IV's according to their prediction power for the dependent variable. As a measure of this prediction power, the tail area under the F-distribution curve to the right of the calculated F-value for testing the corresponding nullhypothesis is recommended. The Second Principle is that of executing the ranking process "reversely", i.e. in deleting the IV's from a postulated model by increasing order of prediction power. The concept of a "Compound" is introduced to describe a specified relation between the dependent variable and a set of IV's. The presence of compounds is shown to necessitate this reverse process. The Third Principle is that of "Admissibility" of IV's for the ranking process. For polynomial terms it is recommended to consider a term of a given order for ranking only after the respective higher order terms have been ranked. For general non-orthogonal ANOVA it is shown that main effects and interaction effects can uniquely be ranked only after the respective higher order effects have been ranked.

14. Small Sample Power for the One-Sample Wilcoxon Test for Non-Normal Shift Alternatives. HARVEY J. ARNOLD, Bucknell University.

Klotz [*Ann. Math. Statist.* **34** (1963) 624] computed the power of the one-sample Wilcoxon rank-sum test for the hypothesis that the Median is zero against various shift alternatives for samples drawn from a normal distribution. This paper reports on computations for samples to size 10 from some non-normal distributions. The power for selected Type I errors α are compared with the power of a best signed-rank procedure obtained by sorting the probabilities in decreasing order for all possible sample configurations for fixed size N and adding up the first 100 $\alpha\%$. The non-normal distributions selected for study are the t distributions with degrees of freedom $\frac{1}{2}$, 1, 2 and 4. The one-sample Wilcoxon rank-sum test, although powerful for normal shift alternatives, deteriorates badly in power for the long-tailed distributions studied as does the one sample t test. However the Wilcoxon test remains more powerful than the t test. The sign test is still more powerful than either Wilcoxon or t. No asymptotic results have been obtained.

15. Conditional Distribution of Number of Renewals. D. L. BENTLEY and B. J. PROCHASKA, Pomona College and Clemson University.

The conditional distribution of number of renewals in the interval $(T - a, T - b]$, $0 < b < a$, given a renewal at $T = t$ is derived for a general renewal process. This result is then applied to the case $b = 0^+$ for a type I or non-paralyzable counter with Poisson arrivals and constant dead time.

16. Bayes Sequential Design in the Two Action Case (Preliminary Report). ROBERT BOHRER, Research Triangle Institute and University of North Carolina.

Consideration of Bayes sequential design procedures which depend on observations only through posterior distributions is motivated. For the problem of deciding between two actions, the following design rule χ^* is proposed for use with given stopping and terminal decision rules. If, for a given posterior distribution, sampling is continued, then χ^* uses the experiment which would be preferred if one experiment were to be used for all succeeding trials. Let ρ_n denote the risk of the procedure which follows χ^* through n trials and subsequently performs the, then, best experiment at all remaining trials; let ρ^* be the risk using χ^*. Then $\rho_n \geqq \rho_{n+1}$, and, quite generally, $\rho^* = \lim \rho_n$. For specified stopping and terminal decision rules, χ^* is proved asymptotically (as cost of experimentation approaches zero) optimum.

17. Effects of Sampling Frequency on Estimates of Regression Parameters when Correlation Among Errors is Ignored (Preliminary Report). VICTOR CHEW, RCA Service Company.

In regression analysis with correlated errors, the covariance matrix of the errors must be completely known, except perhaps for a scalar multiplier, if minimum variance unbiased estimates of the regression parameters are desired. Often this information is not available and it is not possible or feasible, as in real time trajectory computation, to estimate the covariance matrix. Standard least squares method, ignoring the correlation, is then used, sometimes at reduced sampling frequency so as to reduce the effects of the correlation. The problem of sampling frequency has been studied by Proschan (*Siam Review* **3** 230–235) for uncorrelated errors. Hoel (*Ann. Math. Statist.* **32** 1042–1047) gives asymptotic results when the errors are generated by a stationary stochastic process. Grenander (*Ann. Math. Statist.* **25** 252–272) has shown that for polynomial regression the standard least squares estimates are asymptotically efficient. The present paper gives small sample results on the effects of sampling frequency on standard least squares estimates when errors follow one of several postulated models.

18. The Variance of the Number of Zeros of a Stationary Normal Process. J. D. CRYER and M. R. LEADBETTER, Research Triangle Institute.

Let $x(t)$ be a separable stationary normal stochastic process with zero mean, covariance function $r(t)$ (with $r(0) = 1$ for convenience), and (integrated) spectrum $F(\lambda)$ having an absolutely continuous component. Let N denote the number of zeros of $x(t)$ for $0 \leqq t < T$, and let $\lambda_2 = \int_0^\infty \lambda^2 \, dF(\lambda)$. It is well known that $\mathcal{E}N = T(\lambda_2)^{\frac{1}{2}}/\pi$, the best conditions being given by Ylvisaker (*Ann. Math. Statist.*, to appear). The second moment has been given by a number of authors but the sufficient conditions given for the validity of the result include the existence of a sixth derivative for $r(t)$. In this paper $\mathcal{E}N^2$ is derived under conditions which are very close to the necessary ones. It is shown that the formula for $\mathcal{E}N^2$ given for example in Rozanov and Volkonskii (*Theor. Probability Appl.*, **6** (1961)) is valid if $\lambda_2 < \infty$ and, for all sufficiently small t, $\lambda_2 + r''(t) \leqq \Psi(t)$ where $\Psi(t)$ decreases as t decreases to zero and is such that $\Psi(t)/t$ is integrable over $[0, T]$.

19. K-Sample Analogues of Rényi's Kolmogorov-Smirnov Type Theorems MIKLÓS CSÖRGÖ, Princeton University.

Let X_{ji}, $i = 1, \cdots, n_j$, $j = 1, \cdots, k$, be k mutually independent samples of mutually independent random variables having a common continuous distribution function $F(x)$. Let $F_{n_j}(x)$, $j = 1, \cdots, k$, be the corresponding empirical distribution functions. We define $N = n_1/(\sum_{j=1}^k n_1/n_j)$ and let $N \to \infty$ mean that $n_j \to \infty$, $j = 1, \cdots, k$, so that $n_1/n_j \to C_j$, $j = 2, \cdots, k$, where C_j's are constant for each j. Under these conditions we derive the limit distribution of the random variables $N^{\frac{1}{2}} \sup_{a \leqq F(x)} (\prod_{j=1}^k F_{n_j}(x) - F^k_{(x)})/F^k_{(x)}$ and

$$N^{\frac{1}{2}} \sup_{a \leqq F(x) \leqq b} \left(\prod_{j=1}^k F_{n_j}(x) - F^k_{(x)}\right)/F^k_{(x)}$$

and give a lower estimate for that of the supremum of the absolute values of these quotients.

20. A Test for Goodness of Fit for a Continuous Distribution. V. P. GODAMBE, Dominion Bureau of Statistics, Ottawa.

Until now no test was put forward to test that the observed sample consists of n independent observations from a *continuous* distribution, with the same statistical logic as that of χ^2-test for multinominal distributions. It is well known that χ^2-test can be obtained by considering the envelop of likelihood-ratio tests for multinominal distributions. By applying likelihood ratio criteria the author [*Amer. Math. Soc.* (1962)] obtained a statistic to test whether two samples have come from the same continuous distribution or not. From this test one sample test is obtained, to test whether the observed sample has come from a given continuous cumulative distribution function F_0, by the technique developed by Moses [*J. Amer. Statist. Assoc.* (1964)] to obtain one sample analogue of two sample tests. The test actually consists of arranging the sample observations according to their magnitudes say $x^{(1)}, \cdots, x^{(n)}$ and computing the statistic $G = \sum_{i=1}^{n+1} \log [F_0(x^{(i)}) - F_0(x^{(i-1)})]$, $x^{(0)}$ denoting $= \infty$ and $x^{(n+1)} + \infty$. Smaller the value of G more doubtful is F_0. The test has an intuitively appealing property that it takes into account the significance of all clustering of the observations.

21. Asymptotic Variances and Covariances of Maximum-Likelihood Estimators, from Censored Samples, of the Parameters of Weibull and Gamma Populations. H. LEON HARTER and ALBERT H. MOORE, Aerospace Research Laboratories and Air Force Institute of Technology.

For ordered samples of size n, with proportions q_1 and q_2 of the sample values censored from below and from above, respectively, from three-parameter Weibull and gamma populations, expressions are given for the elements of the maximum-likelihood information matrices, each element being the negative of one of the second partial derivatives of the likelihood function with respect to the parameters. An important result is that, while one or more of the elements become infinite for values of the shape parameter less than or equal to two when $q_1 = 0$, thus making estimation non-regular, this does not happen for $q_1 > 0$. If one lets $n \rightarrow \infty$ while holding q_1 and q_2 fixed and then inverts the information matrix, the result is the asymptotic variance-covariance matrix whose elements are the asymptotic variances and covariances of the joint maximum-likelihood estimators of the parameters. Tables are given of the coefficients of $(1/n)$ times powers of the scale parameter θ in the asymptotic variances and covariances for the cases $q_1 = 0.000(0.005)0.025$, $q_2 = 0.00(0.25)0.75$ for both Weibull and gamma populations with shape parameters 1, 2, and 3, omitting cases for which $q_1 = 0$ when the shape parameter is 1 or 2.

22. Semifolding s^{n-k} Design. PETER W. M. JOHN, University of California, Davis.

Consider a 2^{n-k} fraction such that the main effect A is not a defining contrast but at least one interaction containing A is a defining contrast. Semifolding on A consists of augmenting the design to a $3(2^{n-k-1})$ fraction by repeating at low A the points previously run at high A. If the original design is a form letter plan semifolding will give estimates of the 2 fi containing A. This procedure will not isolate the 2 fi from a three letter plan.

23. A Chi-Square Decision Procedure Using Prior Information. WILLIAM E. LEVER, Florida State University.

Let U be an observation from the non-central chi square distribution with S degrees of freedom and non-centrality parameter λ. Consider the decision procedure, make decision

d_0 if $\lambda \geqq \Delta$ and make decision d_1 if $\lambda > \Delta$, given the loss function;

$$\begin{aligned} L(\lambda, d_0) &= 0 && \text{if } \lambda \leqq \Delta \\ &= k_0 w_0(\lambda, \Delta) && \text{if } \lambda > \Delta; \\ L(\lambda, d_1) &= k_1 w_1(\lambda, \Delta) && \text{if } \lambda \leqq \Delta \\ &= 0 && \text{if } \lambda > \Delta. \end{aligned}$$

If it is assumed that λ has an *a priori* distribution $f(\lambda)$ such that $w_i(\lambda, \Delta) f(\lambda)$ is bounded (for $(0 \leqq \lambda \leqq \infty)$ and $i = 0, 1$) and that $w_0(\lambda, \Delta) = |w_1(\lambda, \Delta)|$ for $0 \leqq \lambda \leqq \Delta$. Then a decision procedure is developed so that the decision d_0 is taken if $U \leqq C$ and the decision d_1 is taken if $U > C$, where C is an appropriate constant.

24. Functions of Finite Markov Chains. Frederick W. Leysieffer, Florida State University.

Let $X(t)$ be a standard Markov chain having states $0, 1, \cdots, N - 1$, positive initial probabilities and transition probability matrix $P(t) = \exp(\Lambda t)$. Assume Λ has distinct eigenvalues. With these properties, $X(t)$ is termed a basic Markov chain. Let f be a function from the state space of $X(t)$ onto $0, 1, \cdots, M - 1$ where $M \leqq N$. Consider the process $Y(t) = f[X(t)]$. For $n > 0$, let s be a sequence $s_1 < \cdots < s_n$ and $y = \{y_i\}$, $0 \leqq y_i \leqq M - 1$, $1 \leqq i \leqq n$. Then (y, s) is termed a sequence pair for $Y(t)$ and the joint distribution function associated with this sequence pair is given by $p(y, s) = P[Y(s_i) = y_i, 1 \leqq i \leqq n]$. A characterization for $p(y, s)$ is found leading to the definition of processes of exponential type. Such a process is characterized by joint distribution functions which are linear combinations of exponential terms involving distinct quantities ν_k. The order K, of such a process is the minimum number of ν_k appearing in the representation for $p(y, s)$ for all sequence pairs (y, s). It is shown, functions of basic Markov chains are of exponential type. Sufficient conditions for a process to be a function of a basic Markov chain with a state space of K points are given. $Y(t) = f[X(t)]$ is a Markov iff $K = M$.

25. Measures of Dispersion. A. M. Mathai, McGill University.

In this paper an attempt is made to give an axiomatic development of the concept of dispersion in a statistical population. Multivariate analogue and related topics, the concept of distance between two populations and criteria for testing various statistical hypotheses are also examined in the light of the proposed axioms. General principles for estimation and testing hypothesis are also given. Statistical analysis is treated as a study of dispersion in a statistical population.

26. Lattice paths, Kolmogorov-Smirnov Statistics and Compositions. Sri Gopal Mohanty, McMaster University.

In this paper, minimal lattice paths from the origin to a point (m, n) not crossing two fixed boundaries have been enumerated. With the help of this result, the distributions of Kolmogorov-Smirnov statistics $D_{m,n}$ and $D_{m,n}^+$ have been obtained and the number of $(n + 1)$-compositions of m that are dominated [Narayana and Fulton "A note on the compositions of an integer" *Canad. Math. Bull.* **1** (1958)] by a given $(n + 1)$-composition of m and dominate another given $(n + 1)$-composition of m has been determined.

27. A Simple Proof of the Conservative Character of the Wilcoxon Procedures in the Discrete Case. GOTTFRIED E. NOETHER, Boston University.

A discrete sample may be assumed to have arisen by "projection" from a continuous sample. If the method of "projection" is suitably chosen, it follows immediately that a Wilcoxon confidence interval containing the translation parameter of interest for the continuous sample, does so also for the discrete sample. Thus the confidence coefficient associated with the discrete case is at least as large as in the distribution free continuous case.

28. Effect of Non-Normality on Two-Sided Variables Sampling Plans and Two-Sided Tolerance Limits which Control Percentages in Both Tails of the Normal Distribution (Preliminary Report). J. N. K. RAO and D. B. OWEN, Graduate Research Center of the Southwest.

Owen [Control of percentages in both tails of the normal distribution. *Technometrics* **6** (1964) 377–387] has given two-sided variables sampling plans and two-sided tolerance limits which control percentages in both tails of the normal distribution. In this paper a mathematical investigation is made to examine in what way the OC curves of Owen's variables plans and tolerance limits are affected when the characteristic follows a non-normal distribution specified by the third approximation to the law of error (Edgeworth's Form). The approach used here is similar to that of Srivastava [Variables sampling inspection for non-normal Samples. *J. Sci. Engrg. Res.* **5** (1961) 146–152] for one-sided variables sampling plans.

29. On Stochastic Approximation Processes. M. T. WASAN, Queen's University, Ontario.

The mean square error of a stochastic approximation process of type A_3 as defined in (*) has been considered for a finite number of observations by taking a regression function $M(x)$ of the form $\alpha - \beta(x - \theta)^3$. This enables one to make a probability statement about an estimate of θ by Tchebichev's inequality. For computation, the recursion relation is indicated. For given sequence of positive number $\{c_n\}$, the criteria are developed for optimum choice on $\{a_n\}$ a positive number sequence required for definition of A_3. It is also shown that the mean square error tends to zero as the number of observations tends to infinity under suitable conditions. One can easily consider similarly the behaviour of the mean square error for other stochastic approximation processes defined in (*) ["On a class of Stochastic Approximation Processes" (1956) *Ann. Math. Statist.* **27** 1044–1059] by D. L. Burkholder.

30. A Compound Generalized Extreme Value Distribution. SATYA D. DUBEY, Procter and Gamble Co. (By title)

A 3-parameter distribution, defined over the whole real line, has been considered in this paper which includes the extreme value distribution of double exponential type as a special case. This distribution is compounded with a gamma distribution. The resulting compound generalized extreme value distribution is shown to include the logistic distribution of Gupta and Shah [*Ann. Math. Statist.* (1963) 681] as a special case. It is further shown that the compound distribution of this paper yields an intensity function or hazard function which can be recognized as a *generalized logistic curve*. Consequently, this compound distribution may be considered to be a *generalized logistic distribution*. The moment generating function of the generalized logistic distribution has been derived which involves the usual beta function.

31. **The Hyperbolic Secant Distribution.** W. L. HARKNESS and M. L. HARKNESS, Pennsylvania State University. (By title)

The two-parameter family of distributions having characteristic functions $\phi(t) = [\text{sech } \alpha t]^\rho$, $\alpha > 0$, $\rho > 0$, will be called the hyperbolic secant distributions. These distributions have arisen in characterizations of families of distributions in which certain types of statistics have quadratic regression on a second statistic [cf. Laha and Lukacs, Biom. **47** (1960) 335–343 and Bolger and Harkness, *Ann. Math. Statist.* **36** (1965)]. Some of the important properties of these distributions are considered, including moments, cumulants, and estimation of parameters.

32. **Some Necessary Conditions for the Existence of Partially Balanced Arrays.** J. N. SRIVASTAVA, University of Nebraska. (By title)

Partially balanced arrays (PBA) are well known. In Bose and Srivastava [*Sankhyā* **26** (1964)], a detailed theory of connected association schemes and linear association schemes and linear associative algebras is developed. Let T be a PBA with s symbols, m constraints and strength t. For simplicity consider first the case $t = 4$, $s = 2$. Let μ_i be the number of columns in any 4-rowed submatrix of T, such that in each column, the symbol 1 occurs exactly i times, $(i = 0, \cdots, 4)$. A new method for testing the existence of T with given parameters μ_i is as follows. Consider using T for the estimation of all effects involving 2 or fewer numbers of factors in a 2^m factorial and let the normal equations be $M\mathbf{L} = \mathbf{z}$, where $\mathbf{z}$ depends upon the observations and $\mathbf{L}$ is the usual vector of estimates. In this paper, firstly the roots of M are explicitly obtained as functions of the μ_i. Since the existence of T implies that M is at least positive semi-definite each root of M is nonnegative. This gives a set of new and rather stringent conditions to be satisfied by the μ_i, if a PBA with these parameters exists. For, say $t < 4$, submatrices of M give appropriate new results. The same method, with slight restrictions, is useful for any $s > 2$.

33. **A Group-Testing Problem** (Preliminary Report). S. KUMAR, University of Minnesota.

The problem is to classify each of the N given units into one of the three disjoint categories by means of group testing. We shall call the three categories good, mediocre and defective. In group testing, a set of x units is tested simultaneously as a group with one of the three possible outcomes: (i) all the x units are good, (ii) at least one of the x units is mediocre and none is defective, (iii) at least one of the x units is defective. It is assumed that the N units can be represented by independent trinomial random variables with probability q_1, q_2 and $q_3 = 1 - q_1 - q_2$ of being good, mediocre and defective, respectively. The problem is to devise a procedure, for known values of q_1, q_2 and q_3, which minimizes the expected number of tests $E(T)$ required to classify all the N units as good, mediocre or defective. A procedure, which is optimal in a certain class of procedures, is proposed. Furthermore, we prove the following THEOREM: *A necessary and sufficient condition that the optimal group test plan among all procedures is unique and tests one unit at a time is that*

$$q_1 < \tfrac{1}{2}[(5q_2^2 - 6q_2 + 5)^{\frac{1}{2}} - q_2 + 1]. \tag{1}$$

If the inequality is reversed in (1), then $ET < N$ for the optimal group test plan and if $\leqq$ holds then $ET = N$. Some further generalizations are under consideration.

(Abstracts of papers to be presented at the Western Regional meeting, Berkeley, California, July 19–21, 1965. An additional abstract appeared in an earlier issue.)

2. Analysis of the N-Way Classification Design Model: Non-Orthogonal Layout with One or More Concomitant Variables (Preliminary Report). BONITA J. PEURA, U. S. Navy Electronics Laboratory.

This paper is concerned with the analysis of main effects and interactions when concomitant variables are present. The regression coefficients of the concomitant variables are assumed to be different for all treatment combinations, thereby giving a more general form of the analysis of covariance. Statistics are constructed by Hotelling's T^2, which are then used for testing the various hypotheses. The analysis which is derived is applicable to both orthogonal and non-orthogonal layouts.

3. U-Statistics and Combination of Independent Estimators of Regular Functionals. PRANAB KUMAR SEN, University of California, Berkeley.

Suppose we have c independent samples drawn from c different populations with cdf's $F_1, \cdots, F_c$ respectively. We are interested in estimating an estimable functional $\theta = \theta(F)$. Under the homogeneity of the cdf's $F_1, \cdots, F_c$, we may either combine the c different U-statistics (which are the MVU estimators) from the c samples into a single statistic having some optimal properties or we may use the pooled sample U-statistic corresponding to the kernel of $\theta(F)$ (and this will again be the MVU estimate of $\theta(F)$). Here various properties of these two estimators are studied and contrasted in both the situations, where $F_1, \cdots, F_c$ are homogeneous and they are not. Some useful variance inequalities are obtained and their applications considered.

4. On a Test of the Homogeneity of Correlation Coefficients. M. S. SRIVASTAVA, University of Toronto. (By title)

In this paper a test based on the maximum of the squared sample correlation coefficients $r_1^2, \cdots, r_k^2$ is proposed for testing that the correlation coefficients $\rho_1, \cdots, \rho_k$ of k independent bivariate normal populations are all zero. It has been shown that the proposed test has the monotonicity property; a property enjoyed by the likelihood ratio test also. However, it has an added (over the likelihood ratio test) feature of simplicity in calculation; the ease with which the percentage points can be obtained.

Although questionable, but if one restricts to procedures based on $r_m^2 = \max_{1 \leq i \leq k} r_i^2$, then the proposed test is uniformly most powerful among all the tests based on r_m^2.

(Abstracts of papers to be presented at the Annual meeting, Philadelphia, Pennsylvania, September 8–11, 1965. Additional abstracts will appear in future issues.)

1. Some New Families of Partially Balanced Designs of the Latin Square Type. WILLARD H. CLATWORTHY, State University of New York at Buffalo.

Chang and Liu (*Sci. Sinica* **13** (1964), 1493–95) have presented constructions for 21 new partially balanced designs of the Latin square type in the range of ten or fewer replications and block sizes not exceeding ten. These have been generalized, in certain instances, to produce families of designs which have general solutions and which provide identification and construction of larger designs. Letting $i - 2$ be the number of mutually orthogonal Latin squares used to define a Latin square type association scheme of s^2 treatments and using the conventional notation, the four new families of designs are specified as follows:

(A) $v = s^2, b = isC_k^s, n_1 = i(s-1), \lambda_1 = C_{k-2}^{s-2}, r = iC_{k-1}^{s-1}, k = k, n_2 = (s-1)(s-i+1), \lambda_2 = 0$, where $2 \leqq i \leqq s-1$ and $2 \leqq k \leqq s$; (B) $v = s^2, b = iC_2^s, n_1 = i(s-1), \lambda_1 = s+i-2, r = i(s-1), k = 2s, n_2 = (s-1)(s-i+1), \lambda_2 = i$, where $s \neq i$ and $s \neq i-1$; (C) $v = s^2, b = s(s+2c-1), n_1 = 2(s-1), \lambda_1 = c, r = s+2c-1, k = s, n_2 = (s-1)^2, \lambda_2 = 1$, where s is a prime power, $s = p^n > 2$, and c is an integer, $c \geqq 2$; (D) $v = s^2, b = 2s^2, n_1 = 2(s-1), \lambda_1 = 1, r = 2s, k = s, n_2 = (s-1)^2, \lambda_2 = 2$, where s is a prime power, $s = p^n > 2$.

2. **All Admissible Linear Estimates of the Mean Vector** (Preliminary Report). Arthur Cohen, Rutgers-The State University.

Let y be an observation on a $p \times 1$ random vector which is distributed according to the multivariate normal distribution with mean vector θ and covariance matrix I. Consider the problem of estimating θ when the loss function is the sum of the squared errors in estimating the individuals components of θ. Let G be a $p \times p$ matrix. Then it is proved that estimates Gy are admissible if and only if G is symmetric and all its latent roots, g_i say, $i = 1, 2, \cdots, p$ satisfy, $0 \leqq g_i \leqq 1$, with at most two of the roots equal to 1. The proof regarding the latent roots uses essentially the ideas of Karlin (1958) and Stein (1955) while the proof of symmetry utilizes a theorem of Sacks (1963). The result stated answers the question raised by this author, Cohen (1965). Generalizations with regard to the underlying distribution and with regard to restricted classes of estimates are also discussed.

3. **Linear Combinations of Non-Central Chi-Squared Variates.** S. James Press, The RAND Corporation.

Let U, V be positive semi-definite quadratic forms in normal variates. Then U, V are each distributed as a linear combination (with positive coefficients) of non-central chi-squared variates. Let $T = U - V$. This paper is concerned with the determination of the distribution of T for known values of the parameters. A new mixture representation is presented for the cdf of U in terms of Poisson weight coefficients. The cdf of a positive linear combination of non-central F variates is derived and an exact representation of the density function of T is obtained in terms of confluent hypergeometric functions. Finally, it will be shown how to obtain approximate percentage points of T for the tails of the distribution.

(Abstracts of papers not connected with any meeting of the Institute.)

1. **The Queue $GI/M/3$ with Service Rate Depending on the Number of Busy Servers.** U. N. Bhat, University of Western Australia.

Let $t_0, t_1, t_2, \cdots$ be the epochs of arrival in a three server queueing system with a general inter-arrival time distribution and negative exponential service times. Also let the service rate depend on the number of busy servers so that, the service is slowed or accelerated when all the servers are not busy. Let $Q(t)$ be the number of customers in the system at time t and define ${}^oP_{ij}^{(n)}(t) = \Pr\{Q(t) = j, t_n \leqq t < t_{n+1}, Q(t_r) > 0 \ (r = 1, 2, \cdots, n) \mid Q(t_0) = i, t_0 = 0\}$ and $R_i^{(n)}(t) = \Pr\{Q(t_n) = 0, t_n \leqq t, Q(t_r) > 0 \ (r = 1, 2, \cdots, n-1) \mid Q(t_0) = i, t_0 = 0\}$. Transforms of these probabilities are obtained using recurrence relations and they involve the roots of an equation of the type $z^2 = \omega\psi\ (\theta + 3\lambda - 3\lambda z)$ in $|z| < 1$ where $\psi(\theta)$ is the Laplace-Stieltjes transform of the inter-arrival time distribution. The probabilities ${}^oP_{ij}^{(n)}(t)$ and $R_i^{(n)}(t)$ give the busy period and the busy cycle distributions as well as the transient behaviour of the process $Q(t)$. The method can be easily extended to the s server case whose equilibrium behaviour (of the imbedded Markov chain) has been given by D. G. Kendall [*Ann. Math. Statist.* **24** (1953) 338–354].

2. On a Class of Optimal Stopping Problems. Y. S. Chow and H. Robbins, Purdue University and Columbia University.

For fixed $n \geqq 1$ and $\alpha > 0$ let $x_1, \cdots, x_n$ be independently and uniformly distributed on [0, 1]. We observe the x_i sequentially and must stop with some x_i, $1 \leqq i \leqq n$; if we stop with x_j we lose $j^\alpha x_j$. Denote the minimum possible expected loss for all stopping rules by v. It is shown that for $0 \leqq \alpha < 1, v \sim 2(1 - \alpha)n^{\alpha-1}$, and that for $\alpha = 1$, $v \sim 2(\log n)^{-1}$ as $n \to \infty$. For $\alpha > 1$, v tends to a positive limit as $n \to \infty$, which is $\frac{1}{2}$ for α greater than some $1 < \alpha^* < 2$. The existence and value of v for untruncated rules is also considered.

3. Sufficiency for Selection Models. D. A. S. Fraser, University of Toronto.

Minimal sufficiency is examined for selection models: fixed density function truncated to a carrier set $X(\theta)$ that depends on the parameter. The minimal sufficient statistic can be given as $\bigcap X(\theta)$ containing x or as $\bigcap(x) = \{\theta \mid x \varepsilon X(\theta)\}$. Regularity conditions are introduced: the boundary is formed by a finite number of connected surfaces with a normal vector and differentiability with respect to θ. For any x form the vector of derivatives with respect to the coordinates of θ; let $c(x, \theta_0)$ be the related vector of unit length, call it a structural vector. The *rank* of a distribution at θ_0 is the number of different structural vectors if each intersection of differentiable boundary segments is convex and is $+\infty$ otherwise. Theorem: *If a distribution has finite rank s at θ_0 then for a sample of n the local minimal sufficient statistic has dimension at most s, and the dimension s is attained at some point if* $n \geqq s$. This is used to derive some Koopman-Darmois-Pitman theorems and provides the basis for analyzing models having variable carrier and variable relative density.

4. Bayesian Single Sampling Attribute Plans for Discrete Prior Distributions. Anders Hald, University of Copenhagen.

The paper gives a rather complete tabulation and discussion of a system of single sampling attribute plans obtained by minimizing average costs under the assumptions that costs are linear in p, the fraction defective, and that distribution of lot quality is a double binomial distribution. The optimum sampling plan (n, c) depends on 6 parameters $(N, p_r, p_s, p_1, p_2, w_2)$, where N denotes lot size, (p_r, p_s) suitably normalized cost parameters, and (p_1, p_2, w_2) parameters of the prior distribution. A procedure to obtain the exact solution of the problem has been developed in a previous paper and this procedure is used for computing a set of *master tables* in which $p_r = p_s = 0.01$ and 0.10, $w_2 = 0.05$, $(p_1 p_2)$ take on suitably chosen values in relation to the value of p_r, and $1 \leqq N \leqq 200{,}000$. The properties of the optimum plans are studied, and *simple conversion formulas* are derived which makes it possible to find the optimum plan for an arbitrary set of parameters from a plan in the master tables with a "corresponding" set of parameters. The main tool for this investigation is *the asymptotic expressions for the acceptance number and for the sample size*, viz. $c = np_0 + a + o(1)$ and $n = \varphi_0^{-1} (\ln N - \frac{1}{2} \ln \ln N + \ln \lambda + \frac{3}{2} \ln \varphi_0) + o(1)$, where p_0 and φ_0 are functions of (p_1, p_2), only, whereas a and λ depend on the other parameters also. *Efficiency* of various other systems of sampling plans is studied in relation to the present model and some general recommendations are made.

5. Rank Tests for Randomized Blocks when the Alternatives Have *A Priori* Ordering. Myles Hollander, Stanford University.

Let X_{ij} $(i = 1, \cdots, n, j = 1, \cdots, k)$ be independent random variables with X_{ij} having the continuous distribution $F_j(x - b_i)$ where b_i is the nuisance parameter corresponding to "block" i. Also, let $Y_{uv}^{(i)} = |X_{iu} - X_{iv}|$ and $R_{uv}^{(i)}$ = rank of $Y_{uv}^{(i)}$ in the ranking from least to greatest of $[Y_{uv}^{(i)}]_{i=1}^{n}$. Furthermore, let $T_{uv} = \sum_{i=1}^{n} R_{nv}^{(i)} \psi_{uv}^{(i)}$ where $\psi_{uv}^{(i)} = 1$ if $X_{iu} < X_{iv}$, and 0 otherwise. To test $H_0: F_j = F$ (unknown), the statistic $Y = \sum_{u<v}^{k} T_{uv}$ is proposed as one which will be sensitive to the ordered alternatives $H_a: F_1 \geqq F_2 \geqq \cdots \geqq F_k$ where at least one of the inequalities is strict. Under suitable regularity the random variables $T_{uv}(1 \leqq u < v \leqq k)$ have an asymptotic joint normal distribution and hence Y is asymptotically normal. Y is neither distribution-free for finite n, nor asymptotically distribution-free, as the null correlation $\rho_0{}^n(F)$ between T_{uv} and T_{uw}, $u \neq v \neq w$, and $\rho^*(F) = \lim_n \rho_0{}^n(F)$ depend on F (see Abstract 6 these *Annals*). A consistent (and unbiased) estimate of $\rho_0{}^n(F)$ is given and hence $(Y - E_0(Y))/\hat{\sigma}_0(Y)$ is asymptotically $N(0, 1)$ under H_0. General expressions are obtained for the asymptotic relative efficiencies of Y with respect to some of its nonparametric and parametric competitors, viz., (1) Page's L (*J. Amer. Statist. Assoc.* **58** 216–229); (2) Jonckheere's P (*Brit. J. Statist. Psych.* **7** 93–100); (3) the t-test of $\theta = 0$ for the model $X_{ij} = \mathrm{m} + b_i + (j - 1)\theta + U_{ij}$, where the U_{ij} are iid $N(0, \sigma^2)$. In particular, for the shift alternatives $F_j(x) = F(x - b_i - (j - 1)\theta)$ with $\theta \to 0$, when F is normal the ARE of Y with respect to t is .963 for $k = 3$ and $\to .9897$ as $k \to \infty$ These values compare favorably with the corresponding ones of L (.716, .9549) and P (.694 .9549). Necessary and sufficient conditions for the consistency of Y, L and P are established

6. An Asymptotically Distribution-Free Multiple Comparison Procedure—Treatments vs. Control. MYLES HOLLANDER, Stanford University.

Let X_{i0} and X_{ij} $(i = 1, \cdots, n, j = 1, \cdots, k)$ be the independent measurements on the control and jth treatments in the ith block. Nemenyi [Thesis, Princeton University, (1963)] suggests treatment-control comparisons based on the statistic $T = \mathrm{Max}_j\, T_{0j}$ (T_{0j} is defined in Abstract 5, these *Annals*), assuming it to be distribution-free when the treatments and control have a common cdf F. Actually, the null correlation, $\rho_0{}^n(F)$, between T_{0j} and T_{0k}, $j \neq k$, depends on F and hence so does the distribution of T. (This dependence is slight, especially in the upper tail of the distribution.) It is shown that $\rho_0{}^n(F) = [(24\lambda(F) - 6)n^2 + (48\mu(F) - 72\lambda(F) + 7)n + (48\lambda(F) - 48\mu(F) + 1)]/(n + 1)(2n + 1)$ where $\mu(F) = P(X_1 < X_2 ; X_1 < X_5 + X_6 - X_7)$ and $\lambda(F) = P(X_1 < X_2 + X_3 - X_4 ; X_1 < X_5 + X_6 - X_7)$ when $X_1, X_2, \cdots, X_7$ are iid according to F. Bounds on $\rho^*(F) = \lim_n \rho_0{}^n(F)$ are obtained. A consistent (and unbiased) estimate of $\rho_0{}^n(F)$ is used, in conjunction with Gupta's table of the equi-correlated multivariate normal distribution (*Ann. Math. Statist.* **34** 792–828) to provide an asymptotically distribution-free comparison method. Large sample properties of this procedure are investigated by utilizing the asymptotic joint normality of $[T_{0j}]_{j=1}^{k}$ under various treatment-control alternatives.

7. On a Generalization of the Wishart Distribution. D. G. KABE, Northern Michigan University.

The following lemma is proved. Let $Z = (X'Y')'$ be a complex $(p + q) \times N$ matrix whose first p rows X are real and the next q rows Y are complex, D a given $t \times N$ real matrix of rank $t(<N)$, A a real $N \times N$ positive definite symmetrix matrix, $N \geqq p + q + t$. Then $\int_{ZA\bar{Z}=G, D\bar{Z}'=\bar{V}'} f(ZA\bar{Z}', D\bar{Z}')\, dZ = 2^{-(p+q)} \prod_{i=1}^{q} C(2N - 2p - 2q - 2t + 2i) \prod_{i=1}^{p} \cdot C(N - p - t + i)\, |A|^{-(p+2q)/2}\, |DA^{-1}D'|^{-(p+2q)/2} f(G, \bar{V}')\, |G - V(DA^{-1}D')^{-1}\bar{V}'|^{(N-p-q-t)} \cdot |G_{11} - V_1(DA^{-1}D')^{-1}V_1'|^{-(N-p-t+1)/2}$. Here $(G_{11} - V_1(DA^{-1}D')^{-1}V_1')$ denotes the minor of $(G - V(DA^{-1}D')^{-1}\bar{V}')$ formed by the first p rows and columns, and $C(N)$ is the surface area of a unit N dimensional sphere.

8. Asymptotic Efficiency of the Maximum Likelihood Estimator. Sol Kaufman, Cornell University. (Introduced by J. Wolfowitz.)

This paper extends a result $[A]$ by J. Wolfowitz (7th All-Soviet Union Conference on Probability and Mathematical Statistics at Tbilisi, October 7–14, 1963; to appear in *Teoriya Vyeroyatnostey i yeyaw Primeniene* (1965)). Let θ be a k-dimensional parameter and $\mathcal{E}$ the class of all estimators which, when suitably normalized, converge uniformly in distribution. Then, under reasonable regularity conditions, it is shown that the maximum likelihood estimator $\{\hat{\theta}_n\}$ is efficient over $\mathcal{E}$ in the following sense: For any $\{T_n\}$ in $\mathcal{E}$ and any ellipsoid $B \subset R^k$ concentric with the ellipsoid of concentration of the limiting distribution of $n^{\frac{1}{2}}(\hat{\theta}_n - \theta)$, $\lim_{n\to\infty} P_\theta\{n^{\frac{1}{2}}(T_n - \theta) \varepsilon B\} \leqq \lim_{n\to\infty} P_\theta\{n^{\frac{1}{2}}(\hat{\theta}_n - \theta) \varepsilon B\}$. Various continuity properties are proved for the limit distribution, $L(\cdot \mid \theta)$, of $n^{\frac{1}{2}}(T_n - \theta)$ (for any $\{T_n\} \varepsilon \mathcal{E}$). In particular, for each θ, $L(\cdot \mid \theta)$ is absolutely continuous and gives positive probability to any non-degenerate interval of R^k. The latter implies, for the case $k = 1$, that $L(\cdot \mid \theta)$ has a unique median; hence, in the terminology of $[A]$, $l(\theta) \equiv u(\theta)$ and no "turning points" exist.

9. On Convergence of k-Means and Partitions with Minimum Average Variance. J. MacQueen, Western Management Science Institute, University of California, Los Angeles.

For a rv X taking values in E_n with $EX^2 < \infty$, let $V(S) = \sum_{i=1}^k E[(X - M(S_i))^2 \mid X \varepsilon S_i] \cdot P(S_i)$, where $M(S_i) = E(X \mid X \varepsilon S_i)$ and $S = \{S_1, S_2, \cdots, S_k\}$ is a k-partition of $E_n(\bigcup_{i=1}^k S_i = E_n, S_i \cap S_j = \phi$ if $i \neq j)$. Let $V^* = \inf_S V(S)$. A *minimum distance* k-partition relative to a set x of k points in E_n, $\{x_1, x_2, \cdots x_k\}$, is a k-partition $S(x) = \{S_1(x), S_2(x), \cdots, S_k(x)\}$ such that $S_i(x) \subseteq \{y: y \varepsilon E_n, (y - x_i)^2 \leqq (y - x_j)^2$ for $j \neq i\}$. A k-partition is *unbiased* if it is a minimum distance k-partition relative to x and $M(S_i(x)) = x_i$. For a sample sequence $y_1, y_2, \cdots$ representing independent observations on X, define sample k-means $x^n = \{x_1^n, x_2^n, \cdots, x_k^n\}$ with weights $w^n = \{w_1^n, w_2^n, \cdots w_k^n\}$ as follows: $x_i^1 = y_i$, $w_i^1 = 1$, $i = 1, 2, \cdots k$, x^{n+1}, w^{n+1} are formed from x^n, w^n by the rule that if y_{k+n+1} is nearest to x_i^n, then $x_i^{n+1} = (x_i^n w_i^n + y_{k+n+1})/(w_i^n + 1)$, $w_i^{n+1} = w_i^n + 1$, and $x_j^{n+1} = x_j^n$, $w_j^{n+1} = w_j^n$, $j \neq i$, with ties being broken by tossing an appropriate fair coin. In several special cases it is proved that x^n converges a.s. If $x^n \to_{\text{a.s.}} x^*$, then $S(x^*)$ is a.s. unbiased. Sometimes every unbiased S satisfies $V(S) = V^*$; in these cases $x^n \to x^*$ implies $V(S(x^*)) = V^*$ a.s. Examples show there are X such that with positive pr., $x^n \to x^*$ for which $V(S(x^*)) > V^*$. However, it seems likely that $V(S)$ has only countably many distinct values on the space of unbiased S, and that as among the corresponding sets of limits x^*, the set for which $V(S(x^*)) = V^*$ has positive, if not maximal, probability. If this is true, then repeated runs of the k-means would yield a k-partition S with $V(S)$ arbitrarily near V^*. Other applications of k-means occur in pattern recognition, in data reduction of large samples of n-dimensional data, in coding problems, and with certain modifications, in the description of categorizing behavior.

10. On Some Tests of Homogeneity of Variances. M. L. Puri, Courant Institute of Mathematical Sciences.

For testing the equality of c continuous probability distributions on the basis of c independent random samples, various non-parametric tests for dispersions are offered. These tests, include among others, the multi-sample analogues of the two-sample normal-scores test of dispersion and the tests considered by Ansari and Bradley [*Ann. Math. Statist.* (1960)], Klotz [*Ann. Math. Statist.* (1962)], Mood [*Ann. Math. Statist.* (1954)], and Siegel and Tukey [*J. Amer. Stat. Assn.* (1960)]. Under suitable regularity conditions, it is shown that these tests have limiting non-central chi-square distributions with $c - 1$ df. The asymptotic relative efficiencies (in the Pitman sense) of these tests relative to one another and the $\mathfrak{F}$ test (Scheffé, *The Analysis of Variance*, pp. 83–87) are obtained and are shown to be the same as for the two-sample problem [cf. Ansari-Bradley and Klotz cited above].

11. **Orthonormal Bases of Error Spaces and Their Use for Investigating the Normality and the Variances of Residuals.** JOSEPH PUTTER, Volcani Institute of Agricultural Research, Rehovot.

If, under the assumptions of a linear model, all the residual errors are independent normals with the same variance, then the same also holds for the elements of any orthonormal basis of the error space of the model. Therefore, these elements can be used for testing the normality and overall variance homogeneity of the residuals. Special orthonormal bases have additional properties, which make them suitable for investigating problems of partial variance homogeneity. The general considerations involved in this approach are explained and illustrated. The case of the unreplicated two-way layout is investigated in some detail, yielding new estimates of the error variances and an exact test of overall variance homogeneity under the assumption of one-way homogeneity. The results are readily extended to general complete multi-factor layouts. Some orthonormal bases are also constructed for the case of linear regression on one variable.

12. **On a Stochastic Model of an Epidemic.** C. J. RIDLER-ROWE, University College of Swansea.

The model is obtained by specifying the birth and death rates for particular states of the population, and it is the irreducible case of a model considered by M. S. Bartlett (*Hotelling Festschrift*, Stanford Univ. Press (1960) 89–96). From a general theorem (see G. E. H. Reuter, *Proc. Fourth Berkeley Symp. Math. Statist. Prob.* **2** (1961) 421–430) the expectation of the time at which the infectives first die out is the least positive solution of a certain inequality. By finding sufficiently good solutions of this inequality, and by comparing the process with other simpler processes, it is found that the expectation of this extinction time is asymptotically proportional to $\log(m + n)$ as $m + n \to \infty$, where n (>0) and m are the initial numbers of infectives and susceptibles respectively. The stochastic process considered is non-dissipative, i.e. $\sum_j \pi_j = 1$, where π_j is the limit as $t \to \infty$ of the transition probability $p_{ij}(t)$. By considering the equation $\sum_i \pi_i q_{ij} = 0$, where $\{q_{ij}\}$ is the set of birth and death rates of the process, it is found that $\sum_j \pi_j$ converges geometrically on the state space; furthermore, if ν is the birth rate of the susceptibles, then as $\nu \to 0+$, the limits π_j are shown to converge to those for the reducible case where $\nu = 0$.

13. **On a Stochastic Model of the Competition Between Two Species.** C. J. RIDLER-ROWE, University College of Swansea.

This model was suggested by G. E. H. Reuter (*Proc. Fourth Berkeley Symp. Math. Stat. Prob.* **2** (1961) 421–430). Let αm, γmn be the birth and death rates respectively of a species of size m, and let βn, δmn be the birth and death rates respectively of a species of size n. The functions considered are functions of position which are characterized by being the least positive solutions of certain inequalities. By finding sufficiently good solutions of these inequalities, and by comparing the process with other simpler processes, the following results are obtained. If $X(m, n)$ is the probability that the species of size n will survive the other species, then $X(m, n) - \Phi\{(n\gamma - m\delta)/[m\delta(\gamma + \delta)]^{\frac{1}{2}}\} \to 0$ uniformly as $m + n \to \infty$, where $\Phi(t) = (2\pi)^{-\frac{1}{2}}\int_{-\infty}^{t} \exp(-s^2/2)\, ds$. A result of G. E. H. Reuter in his above mentioned paper is extended to show that the expectation of the time at which one of the species first becomes extinct is bounded, independently of the initial state of the process. It is also shown that, if $E(m, n)$ is the expectation of the number of births and deaths that occur before one of the species becomes extinct, where the initial numbers of the species are respectively m and n, then $E(m, n) \sim (\gamma + \delta) \min (m/\gamma, n/\delta)$ uniformly as m and $n \to \infty$.

14. On First Hitting Times of Some Recurrent Two-Dimensional Random Walks. C. J. RIDLER-ROWE, University College of Swansea.

A random walk in discrete time and on a two-dimensional lattice is considered under the assumptions that (i) each step has zero first moment and a finite moment of order greater than two, and (ii) starting from any given point, each point of the lattice may be visited with positive probability. Let the random variable $T(x)$ be the time at which the random walk first visits the origin, starting from the point x. By finding the generating function of the distribution function of $T(x)$, and then applying a Tauberian argument of a type due to J. Karamata, it is found that, for each $\alpha \geqq 2$, $P\{T(x) \leqq |x|^\alpha\} \to 1 - 2\alpha^{-1}$ as $|x| \to \infty$, where $|x|$ denotes the length of the vector x. It is shown that this result may be proved similarly for the random walk in continuous time and on a two-dimensional lattice under certain similar assumptions; and moreover in both cases the result holds if the origin is replaced by any finite set of points. A related property of the asymptotic behaviour of first visits to a fixed line is deduced for a restricted class of random walks in discrete time and on a three-dimensional lattice.

15. Limit Distributions of an Age-Dependent Branching Stochastic Process. ISMAIL N. SHIMI, University of California, Riverside.

An age-dependent branching stochastic process is considered. $X_N(t)$ is the number of particles in the population at time t and N is the initial size of the population. If the parameters of the process change, as $N \to \infty$, in a way similar to the Poisson approximation of a binomial distribution, then if t is fixed (not necessarily large) and N is allowed to tend to infinity a limiting distribution of the stochastic process $X_N(t) - N$ is obtained, and shown to be the distribution of a stochastic process with independent increments. This limiting distribution can be used to find an approximation to the distribution of $X_N(t)$ when t is fixed (not necessarily large) and N is large. This considerably improves a result previously obtained [*Ann. Math. Statist.* **35** 557–565] where age-dependence was not considered.

16. Product Estimators. S. K. SRIVASTAVA, Lucknow University.

The present paper considers the product estimator suggested by Murthy (*Sankhyā* **26** 69–74) for the estimation of the population mean of a finite population of a character y using an auxiliary character x. The product estimator takes the form $\bar{y}\bar{x}/\bar{X}$, where $\bar{y}$ and $\bar{x}$ are simple random sample means of y and x and $\bar{X}$ is the population mean of x. Exact formulae for the bias and mean square error of the estimator have been obtained. The asymptotic distribution of the estimator has been shown to be normal under certain very mild conditions. The results for stratified random sampling and sampling with varying probabilities of selection have also been obtained.

17. On a Property of the Binomial Distribution. K. SUBRAHMANIAM, University of Western Ontario.

We have established a uniqueness property of the binomial distribution in reference to the bivariate discrete distributions. We have studied, in particular, (i) the bivariate Poisson with the probability generating function (pgf) $\phi(z_1, z_2) = \exp[\lambda_1(z_1 - 1) + \lambda_2(z_2 - 1) + \lambda_{12}(z_1z_2 - 1)]$; (ii) the bivariate binomial distribution with the pgf $\phi(z_1, z_2) = [1 + \lambda_1(z_1 - 1) + \lambda_2(z_2 - 1) + \lambda_{12}(z_1z_2 - 1)]^n$; (iii) the bivariate negative binomial with the pgf $\phi(z_1, z_2) = [1 + \lambda_1(z_1 - 1) + \lambda_2(z_2 - 1) + \lambda_{12}(z_1z_2 - 1)]^{-r}$; (iv) the bivariate logarithmic series distribution with the pgf $\phi(z_1, z_2) = \ln[\lambda_0 + \lambda_1(z_1 - 1) + \lambda_2(z_2 - 1) + \lambda_{12}(z_1z_2 - 1)]/\ln[\lambda_0]$. It has been established that the univariate binomial distribution plays a unique role in the conditional distribution of y, given $x = X$, for all the distributions mentioned above. We have also established the following THEOREM: *Let x and y have the bivariate discrete distribution with the* pgf $\phi(z_1, z_2)$. *Also let the conditional distribution of y, given $x = X$, be the convolution of the* rv χ_1 *with the* rv χ_2. *Then* $\phi(z_1, z_2) = \sum_{X=0}^{\infty} P(X)\phi_1(z_2/X)\phi_2(z_2/X)z_1^X$, *where ϕ_1 and ϕ_2 are the* pgf's *of the conditional distributions of χ_1 and χ_2 given x and $P(X)$ is the probability function of the* rv x.

18. Decomposition of a Mixture of Two Poisson Distributions. K. SUBRAHMANIAM and A. K. MD. EHSANES SALEH, University of Western Ontario.

This paper deals with the problem of estimating the parameters (λ_1, λ_2 and p) in a mixture of two Poisson distributions $P(x) = pe^{-\lambda_1}(\lambda_1^x/x!) + (1 - p)e^{-\lambda_2}(\lambda_2^x/x)$ $(0 < \lambda_1 < \lambda_2 < \infty; 0 < p < 1)$ by the method of factorial moments. The variance-covariance expressions of the estimators are obtained. Also the asymptotic relative efficiencies (ARE) of the estimators and the joint asymptotic efficiency (JAE) of the estimation relative to maximum likelihood estimation are presented for several combination values of λ_1, λ_2 and p.

19. Bias of the One-Sample Cramér-von Mises Test. RORY THOMPSON, Massachusetts Institute of Technology.

There are one-sided alternatives to any continuous hypothesized population for which the probability of rejection by a test based on a statistic equivalent to $\sum (F_0(X_k) - E_{kn})^2$ is less than the size for the E_{kn}'s positive.