The Annals of Mathematical Statistics
1970, Vol. 41, No. 2, 751–757

ABSTRACTS OF PAPERS

(*Abstracts of papers to be presented at the Central Regional meeting, Dallas, Texas, April* 8–10, 1970. *Additional abstracts will appear in future issues.*)

124-4. Exact partitioning of chi-square using successive likelihood ratios.

WANZER DRANE, RONALD B. HARRIST AND PAUL E. WEST, Southern Methodist University.

Wilks' likelihood ratio chi-square has been reserved primarily for testing a single hypothesis against a single alternative. It is shown here that this need not be the case. In fact, the requirement that when using the likelihood ratio the two parameter spaces be imbedded, one in the other, suggests a succession of parameter spaces, each imbedded in the succeeding one. This imbedding and the method of calculating the chi-square give rise to an exact partitioning of the resulting Total Chi-square. Thus, a Chi-square Table can be constructed very much with the same demeanor as the ANOVA tables are. It is well known that this "Chi-square" is only asymptotic. Its characteristics, however, are dependent primarily on total sample size and the assumed distribution and not, as in the case with contingency tables and the Pearson chi-square, individual cell size.

To illustrate its universality an example is given using non-linear regression with contingency tables. (Received January 5, 1970.)

(*Abstracts of papers to be presented at the Eastern Regional meeting, Chapel Hill, North Carolina, May* 5–7, 1970. *Additional abstracts will appear in future issues.*)

125-4. A central limit theorem with nonparametric applications.

GOTTFRIED E. NOETHER, University of Connecticut.

Many nonparametric test and/or confidence procedures are (or can be) based on statistics of the type $S=\sum_{i\in I}\sum_{j\in J} v_{ij}$ where the v_{ij} are $(0,1)$-variables and two random variables v_{ij} and v_{gh} are known to be independent when no subscript in the (i,j)-pair matches a subscript in the (g, h)-pair. Suppose that the number of elements in I and J depend (linearly) on some index N. We are interested in the limiting distribution of the statistic S as $N\rightarrow\infty$. THEOREM. *A sufficient condition for the asymptotic normality of* S (*under proper normalization*) *is that* $\operatorname{Var} S$ *is of order* N^3. The proof consists in showing that the moments of normalized S converge to the moments of the standard normal distribution. This proof remains valid for arbitrary random variables v_{ij} that are uniformly bounded. (Received November 14, 1969.)

125-5. On bounded length sequential confidence intervals based on one-sample rank-order statistics.

PRANAB KUMAR SEN AND MALAY GHOSH, University of North Carolina.

The problem of finding a bounded length confidence band for the mean of an unknown distribution (having finite second moment) is studied by Anscombe [*Proc. Cambridge Philos. Soc.* **48** (1952) 600–607] and by Chow and Robbins [*Ann. Math. Statist.* **36** (1965) 457–462]. In the present paper, we consider the problem of providing a similar (sequential) confidence interval for the median of a symmetric (but otherwise unknown) distribution based on a general class of one-sample rank order statistics. Of particular interest is the procedure based on the one-sample normal scores statistics. This procedure is shown to be asymptotically (i.e., as the prescribed bound on the width of the confidence interval is made to converge to zero) at least as efficient as the Chow–Robbins procedure for a broad class of parent distributions. In this context, several useful convergence results on the empirical process and a process involving one-sample rank statistics are also established. (Received November 20, 1969.)

125-6. On a method of sum composition of orthogonal latin squares. A. Hedayat and E. Seiden, Cornell University and Michigan State University.

A new method of construction of latin squares and orthogonal latin squares is introduced here. We use a method of sum composition as contrasted with product composition used by other authors. We shall exhibit a method of construction of a latin square of order $n = n_1 + n_2$ given the squares of order n_1 and n_2. Two theorems are proved regarding the construction of a pair of orthogonal squares of order $n = n_1 + n_2$ for $n_1 \geqq 7$ except $n_1 = 13$, where $n_1 = p^\alpha$, p an odd prime or $n_1 = 2^\alpha$, α a positive integer provided that $n_2 = (n_1 - 1)/2$ and $n_2 = n_1/2$ respectively. The construction includes an infinite collection of pairs of orthogonal latin squares of order $4t+2$. For this it is necessary and sufficient that $p \equiv 7 \pmod 8$ and α odd.

Further research is in progress indicating possibilities of obtaining more results in two directions. One consists in changing the relative magnitudes of n_1 and n_2, the other in increasing the number of orthogonal squares whenever the values of n_1 and n_2 allow for it. (Received November 24, 1969.)

125-7. Estimation of expected life in the presence of an outlier observation. S. K. Sinha and B. K. Kale, University of Manitoba.

Consider a situation in which $(X_1, X_2, \cdots X_n)$ are such that $(n-1)$ of them are distributed as $f(x, \sigma) = \sigma^{-1} \exp(-x\sigma^{-1})$ and one has pdf $f(x, \sigma/\alpha)$ and a priori each X_i has probability $1/n$ of being distributed as $f(x, \sigma/\alpha)$ where $0 < \alpha \leqq 1$. In this paper we obtain the pdf of $X_{(i)}$, the ith order statistic and the joint pdf of $X_{(i)}$, $X_{(j)}$. An "optimum" estimator of σ based on linear combinations of order statistics is suggested and its mean squared error is compared against the standard estimators for homogeneous case of $\alpha = 1$. We also consider a Bayes estimator when α is treated as an rv with beta type prior distributions. Similar problems are studied for the case when the pdf are $g(x, \mu) = \exp\{-(x-\mu)\}$ and $g(x, \mu+\delta)$ $\delta \geqq 0$. This paper generalized results contained in a paper by Kale and Sinha [*Technometrics* (in press)]. (Received December 1, 1969.)

125-8. Two applications of a general stochastic approximation theorem. Mark A. Johnson, The Upjohn Company.

An extension of Burkholder's (*Ann. Math. Statist.* **27** (1956) 1044–1059) theorem on a general class of stochastic approximation procedures is given. This extension admits a less restrictive class of norming sequence, allows the observed random variables to be constrained to lie between specified limits, and allows an increasing number of observations to be taken on each step. Two corollaries to the main theorem are given. The first corollary is a straightforward extension of Burkholder's corollary on the asymptotic distribution of the Kiefer–Wolfowitz (*Ann. Math. Statist.* **23** (1952) 462–466) process. The second corollary gives conditions for the asymptotic normality of a modification of the Kiefer–Wolfowitz process which Wilde (*Optimum Seeking Methods*, Prentice Hall (1964)) suspects is less sensitive to the variance of the observations. (Received December 2, 1969.)

125-9. Simultaneous tests for equality of latent roots against certain alternatives II. P. R. Krishnaiah and V. B. Waikar, Aerospace Research Laboratories and Miami University.

Let Λ be a $p \times p$ positive definite symmetric matrix and let $\lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_p > 0$ be the latent roots of Λ. Let the random matrix L be an estimate of Λ and let $l_1 > l_2 > \cdots > l_p > 0$ be its latent roots. In addition, let $H:\lambda_1 = \cdots = \lambda_p$, $A_{ij} = \lambda_i > \lambda_j$ and $A = \bigcup_{i<j} A_{ij}$. In this paper, we propose a procedure for testing H against A. The proposed procedure is to accept H if and only if $f_{ij} \leqq c_{ij\alpha}$ for $i<j$, $i = 1,\cdots,p$ where $f_{ij} = l_i/l_j$ and $c_{ij\alpha}$ are constants such that $P[1 \leqq f_{ij} \leqq c_{ij\alpha};\ i<j,\ i,j = 1,\cdots,p \mid H] = 1-\alpha$. For practical purposes we choose $c_{ij\alpha} = c_\alpha$ for all $i<j$. In this case, the above equation reduces to $P[1 \leqq f_{1p} \leqq c_\alpha \mid H] = 1-\alpha$. In the present paper, we derived the null and non-null distributions of f_{1p} when L is (i) Wishart matrix (ii) $S_1 S_2^{-1}$ (iii) matrix associated with MANOVA and (iv) matrix associated with canonical correlations; here S_1 and S_2 are independently distributed Wishart matrices. The distribution of f_{1p} in various cases was derived starting from the joint distribution of $f_{1p}, f_{2p}, \cdots, f_{p-1\,p}$ which was derived for various cases by the authors in an earlier paper (Simultaneous tests for equality of latent roots against certain alternatives I, *ARL* Tech. Report). For the case of Wishart Matrix, T. Sugiyama (Mimeo. Series No. 131, Dept. of Statistics Purdue University (1967)) derived the null distribution of f_{1p} starting from the null distribution of $f_{21}, f_{31}, \cdots, f_{p1}$. (Received December 9, 1969.)

125-10. The admissibility of a response surface design. LAWRENCE L. KUPPER, University of North Carolina.

The usual concept of admissibility of an experimental design (see Karlin and Studden, *Ann. Math. Statist.* **37** (1966) 808–812) is based solely on variance considerations and no allowance is made for the possible occurrence of bias errors resulting from the use of an incorrect model. This limitation is not entirely desirable in a response surface setting where a graduating function $\hat{y}(\mathbf{x};\mathbf{b})$ such as a polynomial will always fail to represent exactly a response surface $\eta(\mathbf{x};\boldsymbol{\beta})$. In fact, several authors such as Box and Draper (*J. Amer. Statist. Assoc.* **54** (1959) 622–654) have demonstrated in typical situations the overriding importance of bias considerations when selecting a design to minimize expected mean-square error $E(\hat{y}(\mathbf{x};\mathbf{b})-\eta(\mathbf{x};\boldsymbol{\beta}))^2$ averaged over a compact set $\mathscr{X}$. Motivated by their findings, this author first provides a generalization of a result on the minimization of integrated squared bias which appeared in the preceeding paper and then uses it along with some ideas from decision theory (c.f. Ferguson, *Mathematical Statistics: A Decision Theoretic Approach* (1967)) to develop the notions of variance (V-), bias (B-) and mean-square (MS-) admissibility, which have particular appeal in a response surface framework. A theorem is proved characterizing a meaningful situation in which a design can be said to possess one of these types of admissibility. (Received December 10, 1969.)

125-11. Some $\mathscr{G}$-minimax estimators for the normal mean. STEPHEN L. GEORGE, M. D. Anderson Hospital and Tumor Institute.

Some $\mathscr{G}$-minimax estimators (as defined by Blum and Rosenblatt, *Ann. Math. Statist.* **38** (1967) 1671–78) are derived for the mean of a univariate normal distribution for various specifications of the class $\mathscr{G}$. All estimators are shown to be admissible. In one case a $\mathscr{G}$-minimax confidence interval is given that is shorter than the usual confidence interval for all $G \in \mathscr{G}$. Where there also exist prior observations, some asymptotic results are given and it is shown that even a naive use of the past observations can result in a reasonable estimator. (Received January 5, 1970.)

125-12. Optimum property of the partial sequential probability ratio test. CAMPBELL B. READ, Southern Methodist University and Southwestern Medical School of the University of Texas.

In a partial sequential probability ratio test of $H_0:\theta = \theta_0$ vs. $H_1:\theta = \theta_1$ from a sequence of i.i.d.r.v.'s with common pdf $p_\theta(\cdot)$, n observations are first made, and continued beyond stage n one at a time as in a Wald SPRT. The optimum property of the SPRT, obtained by Wald and

Wolfowitz, is shown to apply to PSPRT procedures in the class of tests having at least n observations. In this class the Bayes risk for given fixed losses for wrong decision and fixed cost is minimized by a PSPRT; equivalently, among all tests in this class bounded by the error probabilities of any PSPRT, the PSPRT has smallest ASN when $\theta = \theta_0$ or θ_1. (Received January 9, 1970.)

(*Abstracts of papers to be presented at the Annual meeting, Laramie, Wyoming, August* 25–28, 1970. *Additional abstracts will appear in future issues.*)

126-1. Maximum-likelihood estimates for queues with state-dependent service. TARSAIM L. GOYAL AND CARL M. HARRIS, The George Washington University and Research Analysis Corporation.

Methods are indicated for the maximum-likelihood estimation of the arrival and service parameters of ergodic queuing processes with service times whose distributions are state dependent. Two basic methods of sampling are considered. For the first, only the queue size at each departure point is observed, while for the second, the time between successive departures is noted along with the queue size. Based upon each sampling method, procedures are developed for deriving estimates both when the effect of initial queue size on the likelihood function is ignored and when it cannot be. Three specific models of this semi-Markov process are presented, along with a numerical example illustrating some of the results, based on data generated by simulating a known model. (Received December 10, 1969.)

(*Abstracts of papers contributed by title.*)

70T-12. A test for the comparison of two exponential distributions. S. KUMAR AND H. I. PATEL, University of Wisconsin-Milwaukee.

Let $x_{(1)}^i, x_{(2)}^i, \ldots, x_{(n_i)}^i$ be an ordered sample from the distribution with pdf $f(x; \beta_i, \theta_i) = \theta_i^{-1} \exp\{-\theta_i^{-1}(x-\beta_i)\}$ for $x \geqq \beta_i$ and zero otherwise ($i = 1, 2$). It is assumed that $\theta_1 = \theta_2$ but it is unknown. The problem is to test the hypothesis $\beta_1 = \beta_2$ against the alternative $\beta_1 \neq \beta_2$. A test for this hypothesis based on the first k_i ordered observations from the ith distribution ($2 \leqq k_i \leqq n_i$, $i = 1, 2$) is proposed. The distribution of the test statistic under the null hypothesis is derived. The cut-off points for various values of k_i, $n_i (i = 1, 2)$ are tabulated for $\alpha = .01$ and $.05$. (Received November 21, 1969.)

70T-13. Estimating the parameters of a multivariate exponential using order statistics (preliminary report). RUSSELL MAIK, Horace Mann Educators.

When n machines obeying a multivariate exponential (as defined by Marshall and Olkin in the *J. Amer. Statist. Assoc.* March (1967)) are put in life test at time $T = 0$, we observe not only the first k failure times, but also the component(s) of the machines which fail. A machine is said to fail if *any* one of its components fail or if two or more of its components fail simultaneously. Using the above information, we find the likelihood function of the first k failures. Also we obtain a set of complete sufficient statistics for the parameters of the multivariate exponential and hence the unique minimum variance unbiased estimates of these parameters. Although our method differs considerably from that employed by Arnold (*J. Amer. Statist. Assoc.* September 1968)) our estimates are identical if we observe all n failures. The testing of hypothesis is left for a later communication. (Received December 3, 1969.)

70T-14. An ordering of p-dimension random vectors (preliminary report). RUSSELL MAIK, Horace Mann Educators.

The concept of an order statistics for a univariate distribution is extended to a p-variate distribution in this paper. The random vector X is said to be lth less than or equal to the random vector Y (denoted by $X \leqq_l Y$) if the lth largest coordinate of X is less than or equal to the lth largest coordinate of Y. Using this definition we obtain the joint density of the ith order statistic when the p-dimension vectors are ordered by their lth ($l = 1, \cdots, p$) largest coordinate (denoted by $Xl_{(i,n)}$) and the joint density of the ith and jth order statistics when both vectors are ordered by their lth largest coordinate. The asymptotic distributions of these order statistics are then examined. These results are applied to a multivariate exponential (as defined by Marshall and Olkin in the *J. Amer. Statist. Assoc.* March (1967)) to obtain the marginal densities of the various components of the ith order statistics for the bivariate and trivariate exponentials when the observation vectors are ordered by their smallest coordinate. Then we obtain the expected life of the components of the ith machine that fails when n machines are put in life test simultaneously. (Received December 3, 1969.)

70T-15. Tests of hypothesis concerning means of normal populations (preliminary report). S. R. KULKARNI, Karnatak University.

Suppose we have $2t$ samples—t samples of size n_1 from normal distribution with mean μ_i and variance σ_1^2, $i = 1, 2, \cdots, t$, and t samples of size n_2 from normal distribution with mean μ_i^* and variance σ_2^2, $i = 1, 2, \cdots, t$; the problem is to test the hypothesis $\mu_i = \mu_i^*$, $i = 1, 2, \cdots, t$, when σ_1^2 and σ_2^2 may or may not be equal. Naik (*Ann. Inst. Statist. Math.* **19** (1967) 301–312) has studied the level of the bilateral and unilateral tests for the problem. In this paper the exact power function of the bilateral test is derived when $n_1 = n_2$. It is shown that the level of the bilateral test, which depends only on $R = \sigma_1^2 1(\sigma_1^2 + \sigma_2^2)$, does not exceed the given level of significance α, for all t and R. When $t = 1, 2$, and $0 < \alpha < 1$, the exact level of any right-hand tail test, based on the bilateral statistic and a constant cut off point is shown to be a strictly convex function of R. The bilateral and unilateral tests turn out to be asymptotically equivalent to locally asymptotically most stringent test for the problem. Studying the level as a function of R, interesting conclusions are drawn about the use of these asymptotically equivalent tests. Alternative tests superior to the bilateral test, in some sense, are obtained on the lines of Wald (*Selected Papers in Statistics and Probability* (1955) 669–695) and Welch (*Biometrika* **34** (1947) 28–35). The situation when n_1 may not be equal to n_2 is also studied. (Received December 5, 1969.)

70T-16. Almost mixing. C. FARMER, University of Tulsa.

In the paper we characterize transformations on a probability space which are almost mixing. The notations and methods are those of P. Billingsley, *Ergodic Theory and Information* Wiley, New York (1965). Using as a model the paper of P. Barfai and R. Revesz, On a zero-one law. *Z. Wahrscheinlichkeitstheorie Verw. Gebeite* 7 (1967) 43–44, we obtain examples of transformations which satisfy our mixing for various values of a defining parameter. As in the classical case, the hierarchy of our mixing over our ergodicity is established. Finally, almost mixing is shown to be an invariant under isomorphism. (Received December 9, 1969.)

70T-17. Estimates of logarithms of interactions for saturated s^m designs with an underlying exponential distribution (preliminary report). RUSSELL MAIK, Horace Mann Educators.

The best estimate of the logarithm of the scale parameter θ of an exponential distribution based on the first k order statistics is given by $\ln \{k^{-1} [\sum_{i=1}^k x_i + (n-k)x_k]\}$ where $0 < x_1 < x_2 < \cdots < x_k$.

This result is then used to obtain the best estimates of the logarithms of the general mean, main effects and two-factor interactions in saturated s^m factorial designs under the assumption that each treatment combination corresponds to a scale parameter of an exponential distribution. Our results are then applied to the optimal saturated 2^m factorial designs ($m = 3, 4, 5, 6$) found by J. N. Srivastava and D. Chopra (unpublished) to determine the number of ordered responses we should observe on the various treatment combinations when the total number of responses is held constant. Again, we assume each response obeys an exponential distribution with scale parameter corresponding to a particular treatment combination. (Received December 22, 1969.)

70T-18. Estimation in linear regression with heteroscedastic variances. J. N. K. RAO AND K. SUBRAHMANIAM, University of Manitoba.

The method of Hartley, J. N. K. Rao, and Kiefer [*J. Amer. Statist. Assoc.* **64** (1969) 841–51] is used for estimation of the regression parameters in linear regression with heteroscedastic error variances. C. R. Rao [*J. Amer. Statist. Assoc.* (in press)] has shown that the estimators of error variances obtained by the Hartley *et al.* method possess certain optimality properties. Two special cases are considered: (1) combining k unbiased estimators $u_1, \cdots, u_k$ of a parameter μ with variances $\sigma_1^2, \cdots, \sigma_k^2$ [Cochran and Carroll, *Biometrics* **9** (1953) 447–59]; (2) estimation of parameters α and β in the linear regression model $y_{ij} = \alpha + \beta x_i + e_{ij}$; $j = 1, \cdots, m_i$, $i = 1, \cdots, k$ where e_{ij} are uncorrelated with mean zero and unknown variances σ_i^2 [Jacquez, Mather and Crawford, *Biometrics* **24** (1968) 607–626]. Relative efficiencies of the present estimators compared to those proposed in the literature for cases (1) and (2) are investigated in detail by Monte Carlo and analytical methods. Application of the Hartley *et al.* method to estimation in mixed analysis of variance models is also considered. (Received December 16, 1969.)

70T-19. Nonparametric sequential confidence intervals and tests using iterated logarithms laws (preliminary report). H. S. STEYN, JR. AND J. C. GEERTSEMA, Potchefstroom University.

We derive an iterated logarithm law for U-statistics and in this way obtain two sequences $\{a(n)\}$ and $\{b(n)\}$ of positive integers such that $P(W_{n,a(n)} < \gamma$ for some $n \geqq m) \leqq \alpha$ $P(W_{n,b(n)} > \gamma$ for some $n \geqq m) \leqq \beta$ where $W_{n,1} \leqq W_{n,2} \leqq \cdots \leqq W_{n,\frac{1}{2}n(n+1)}$ are the ordered $\frac{1}{2}(x_i + x_j)$, $1 \leqq i \leqq j \leqq n$ and $x_1, \cdots, x_n$ is a sample from a distribution symmetric about γ. We define a sequential confidence interval procedure for γ as follows: Let N be the first $n \geqq m$ such that $W_{n,a(n)} - W_{n,b(n)} \leqq L$ and choose as confidence interval $I_N = (W_{N,b(N)}, W_{N,a(N)})$. It follows that $P(\gamma \in I_N) \geqq 1 - \alpha - \beta$. We find an asymptotic expression for EN as $L \to 0$ and also find under regularity conditions an asymptotic expression for EN for a confidence interval procedure introduced by Farrell (*Ann. Math. Statist.* **37**, 586–592) and prove that the ratio of these two expressions is the same as the Pitman efficiency between the sign test and the Wilcoxon one-sample test. In the same way as in Darling and Robbins (*Proc. Nat. Acad. Sci. USA* **57**, 1188–1192) we derive a sequential Wilcoxon one-sample test with specified probability of type I error and zero probability of type II error, which may however not terminate under the hypothesis. Properties of the test, such as truncation and expected sample size and of the confidence interval procedures, such as bounds on m are studied. (Received December 17, 1969.)

70T-20. On Riedwyl's goodness of fit statistics. U. R. MAAG AND F. STREIT, University of Montreal and University of Toronto.

Riedwyl (*J. Amer. Statist. Assoc.* **62** (1967) 390–398) introduced modified versions of the classical Kolmogorov-Smirnov and Cramér-von Mises one sample statistics. The distance between the empirical and the theoretical distribution function $F(x)$ is measured at the points $F^{-1}(i/N)$, $i = 1, 2, \cdots, N-1$, where N is the sample size, instead of at the points determined by the observations. Results are derived for the moments and the distributions of these statistics under the null hypothesis for finite sample sizes. The asymptotic distributions are the same as in the corresponding classical cases. The exact power function of the modified tests are calculated for some alternatives. (Received January 5, 1970.)

70T-21. Robust estimation in the asymmetric case (preliminary report). PETER J. BICKEL, University of California at Berkeley.

We show how to construct estimates of location which are $n^{\frac{1}{2}}$ consistent and asymptotically normal whenever the underlying distribution is: (1) Symmetric satisfying weak regularity conditions or (2) A mixture in known proportions of a normal distribution with known variance and any other distribution which places all of its mass outside a (known) interval about the mean of the normal distribution. A similar estimate may be constructed in the multilinear regression problem. Efficiency properties of this estimate are under study. (Received January 5, 1970.)

70T-22. Linear expansions and robust estimation (preliminary report). PETER J. BICKEL, University of California at Berkeley.

In 1964 P. J. Huber proposed a robust estimate of location which can be viewed as the solution of a particular maximum likelihood problem. We show that the problem is sufficiently smooth so that a one step Newton approximation is asymptotically equivalent to that of Huber. This type of estimate is the following. Consider the residuals of the observations from an initial estimate (e.g. $\bar{X}$) which is $n^{\frac{1}{2}}$ consistent, replace each observation whose *residual* is larger than C by C, treat those with residuals less than $-C$ analogously and leave all others unchanged. Divide the sum of the new observations by the number of unchanged observations. The same approach yields estimates which behave like Huber's in multilinear regression and have a similar simple characterization. The trimmed mean has the same optimality properties as Huber's estimate in the location problem. Motivated by a linear expansion argument we show how to construct simple analogues of the trimmed mean using ordered residuals which have similar properties for multilinear regression. (Received January 5, 1970.)

70T-23. An iterated logarithm law for maxima and minima. HERBERT ROBBINS AND DAVID SIEGMUND, Columbia University.

Let $x_1, x_2, \cdots$, be independently and uniformly distributed on $(0, 1)$ and put $y_n = \min(x_1, \cdots, x_n)$ (or $y_n = 1 - \max(x_1, \cdots, x_n)$). Then if c_n is any ultimately increasing sequence of positive constants such that c_n/n is ultimately decreasing, $P(ny_n \geqq c_n \text{ i.o.})$ is 0 or 1 according as the series $\sum_1^\infty (c_n \exp(-c_n)/n)$ converges or diverges; in particular, for $c_n = \log_2 n + 2\log_3 n + \log_4 n + \cdots + (1+\varepsilon)\log_k n$ the preceding probability is 1 for $\varepsilon = 0$ and 0 for $\varepsilon > 0$. Inequalities and asymptotic equalities for boundary crossing probabilities of the form $P(ny_n \geqq f(n/m)$ for some $n \geqq m)$, etc. are derived by methods similar to those in *Proc. Nat. Acad. Sci. USA* **62** (1969), 11–13. For example, for any $\varepsilon > 0$, $P(ny_n \geqq \log(1+\varepsilon n/m)$ for some $n \geqq m) \to \log(1+\varepsilon)/\varepsilon$ as $m \to \infty$. Analogous exact crossing probabilities are derived for a certain continuous time stochastic process which in a sense interpolates the sequence y_n. (Received January 5, 1970.)