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Erratum: Qualitative properties of certain piecewise deterministic Markov processes

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There is a mistake in the proof of Theorem 3.4 which was pointed out to us by Alessandra Faggionato. The definition of the set G_t does not make sense and should be modified as follows.

Let L be a common Lipschitz constant for the fields $\{F^i, i \in E\}$ and $\varepsilon > 0$. For all y let

$$G_t(y) = \left\{v \in \left\{F^i(y), i \in E\right\} : \left\langle v - \dot{\eta}(t), y - \eta(t)\right\rangle < L\left\| y - \eta(t)\right\|^2 + \varepsilon\right\}.$$

Claim 1. *$G_t(y)$ is a nonempty closed set.*

Proof. For all y and t there exists $i \in E$ such that

$$\left\langle F^i\left(\eta(t)\right) - \dot{\eta}(t), y - \eta(t)\right\rangle \le 0$$

because $\dot{\eta}(t) \in \mathrm{co}\{F^i(\eta(t)) : i \in E\}$. Thus

$$\left\langle F^i(y) - \dot{\eta}(t), y - \eta(t)\right\rangle \le \left\langle F^i(y) - F^i\left(\eta(t)\right), y - \eta(t)\right\rangle \le L\left\| y - \eta(t)\right\|^2 < L\left\| y - \eta(t)\right\|^2 + \varepsilon.$$

Being finite, $G_t(y)$ is closed. □

Claim 2. *The set valued map $(t, y) \mapsto G_t(y)$ is* (a) *uniformly bounded*, (b) *lower semicontinuous in y, and* (c) *measurable in t.*

Proof. (a) is obvious since F^i are bounded; (b) follows from the continuity of the F^i (and the definition of l.s.c. for multivalued map). (c) Measurability follows from the fact that for each given y, and each given $v \in \{F^i(y), i \in E\}$ the set $\{t : \langle v - \dot{\eta}(t), y - \eta(t)\rangle < L\|y - \eta(t)\|^2 + \varepsilon\}$ is measurable (and the definition of multivalued measurable maps). □

By Theorem 3.2 in [26], there exists a solution to $\dot{y} \in G_t(y)$ with $y(0) = \eta(0) = x$. Hence,

$$\left\langle \dot{y}(t) - \dot{\eta}(t), y(t) - \eta(t)\right\rangle \le L\left\| y(t) - \eta(t)\right\|^2 + \varepsilon$$

so that, by Gronwall's inequality

$$\left\| y(t) - \eta(t)\right\|^2 \le \left(e^{2Lt} - 1\right)\frac{\varepsilon}{L}.$$

The rest of the proof remains unchanged.